Integraltafeln,

oder

Sammlung von Integralformeln.

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——— Fortsetzung hiervon, 1ter Theil, oder Sammlung von Aufgaben aus der Theorie der algebraischen Gleichungen, 8. 1 rthlr. 16 gr.

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Weitere Ausführung hiervon, siehe unter: Puissant.

——— der Geometrie. Nebst einer Abhandlung über die in diesen Anfangsgründen zu befolgende Ordnung, über die Art sie abzufassen und über die Methode in den mathematischen Wissenschaften, mit 7 Kupf. gr. 8. 1 rthlr. 16 gr.

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Ide, J. J. A., Théorie der Bewegung der Weltkörper unsers Sonnensystems und ihrer elliptischen Figur. Nach de la Place frei bearbeitet; mit einer Vorrede von Kästner, gr. 8. 2 rthlr.

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Krause, G. F., Handbuch der mathematischen Forstwissenschaften, mit Kupf. und Tabellen, gr. 8 2 rthlr.

Monge's, G., Anfangsgründe der Statik. A. d. Franz. übers. und mit Erläuterungen versehen von E. M. Hahn, mit 5 Kupf. gr. 8. 20 gr.

Puissant, L., Sammlung verschiedener Aufgaben der Geometrie, aufgelöset und bewiesen durch die algebraische Analysis; als weitere Ausführung zu Lacroix Trigonometrie. A. d. Franz. übers. von E. M. Hahn, m. Kupf. gr. 8. 16 gr.

Rhode, J. G., Theorie der Verbreitung des Schalls für Bauküunſtler, m. 1 Kupf. 8. 6 gr.

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Integraltafeln,

oder

Sammlung von Integralformeln.

Von

Meier Hirsch.

Berlin,
bei Duncker und Humblot.
1810.

Vorrede.

Tafeln für häufig wiederkehrende Fälle sind zu allen Zeiten als ein wirksames Mittel angesehen worden, theils die Uebersicht zu erleichtern und dem Gedächtnisse zu Hülfe zu kommen, theils die Mühe des öftern Aufsuchens zu ersparen. Das Bedürfniſs solcher Tafeln wird desto dringender, je öfterer die nämlichen Fälle wiederkehren, und je gröſser die Mühe ist, welche das Aufsuchen verursacht. Am fühlbarsten aber wird ihr Mangel in der Analysis, wo die Menge der Formeln von Tag zu Tag anwächst, und die Uebersicht so sehr erschwert, daſs es selbst dem geübtesten Analysten nicht mehr möglich ist, sie alle immer bey der Hand zu haben. Hat man sich auch durch ein vieljähriges Studium mit den allgemeineren vertraut gemacht, so müssen doch die speciellen, welche man zu irgend einem Zwecke brauchen will, immer erst in einem oder dem andern Werke aufgesucht, oder, wenn sie sich nicht finden sollten, berechnet werden. Das letztere ist aber nicht die Sache eines jeden, und überdies muſs bey dem ausübenden Theile der Mathematiker noch der damit verbundene Zeitaufwand in Anschlag gebracht werden. Wer weiſs, ob nicht vielleicht manche Untersuchung, die für die Wissenschaft selbst, oder für ihre Anwendung hätte Gewinn werden können, bloſs dieserhalb unterblieben ist? — Was schon Leibnitz, der selbst sich vorsetzte analytische Tafeln zu verfertigen, von ihrem Nutzen sagt, ist bekannt genug, und es würde nicht schwer

seyn, noch mehrere Autoritäten anzuführen, wenn der Gegenstand ihrer bedürfte.

Was hier von dem Nutzen analytischer Tafeln im Allgemeinen gesagt worden, gilt von Integraltafeln insbesondere in einem hohen Grade. In unsern Lehrbüchern, und selbst in Eulers und Lacroix's ausführlichen Werken, findet man, und zwar mit Recht, nur diejenigen speciellen Formeln angeführt, welche zur Erläuterung der vorgetragenen Sätze dienen können. Stöſst man daher in der Ausübung, wie es gar nicht selten zu geschehen pflegt, auf ein Integral, welches sich weder in diesen, noch in anderen Werken findet, so ist man genöthigt es selbst zu suchen. Integrale sind aber nicht so leicht gefunden, wie etwa die einzelnen Potenzen eines Binoms aus der Binomialformel; es wird dazu schon weit mehr Gewandtheit und Fertigkeit in der Behandlung analytischer Formeln erfordert, als man bey den meisten voraussetzen darf. Und doch behauptet die Integralrechnung den bedeutendsten Platz in der reinen Analysis, sie dringt so mächtig in das Gebiet der Anwendung ein, daſs selbst der ausübende Mathematiker, der sich etwas über das Gewöhnliche erheben will, ihre Hülfe nicht mehr entbehren kann. Der Verfasser glaubt daher in der Ausarbeitung des vorliegenden Werkes, wo man alle, sowohl allgemeine als specielle Integralformeln, die muthmaſslich bey analytischen Untersuchungen oder in der Ausübung gebraucht werden möchten, nicht bloſs berechnet, sondern auch so geordnet findet, wie es ihm zum Nachschlagen am bequemsten schien, etwas Verdienstliches unternommen zu haben. Er wurde in dieser Meinung durch die Aufmunterung mehrerer sehr schätzbarer Männer bestärkt, und er fürchtet nichts so sehr, als daſs die Ausfüh-

rung ihren Erwartungen nicht entsprechen möchte. Besonders hält er es für seine Pflicht, dem geheimen Oberbaurathe Herrn Eytelwein, für so manche Winke in Hinsicht auf die bessere Einrichtung dieser Tafeln, seinen verbindlichsten Dank zu sagen.

Für Richtigkeit der Rechnung und des Druckes ist die möglichste Sorge getragen worden; sollten sich jedoch hier und da noch Fehler finden, welche dem Verf. entgangen sind, so würde ihm durch die Anzeige derselben eine dankeswerthe Gefälligkeit erwiesen werden.

Am Schlusse dieser Vorrede muſs ich noch bemerken, daſs es ein Irrthum war, wenn ich in meiner Samml. von Aufgaben aus der Theorie der Gleichungen die allgemeine Auflösung derselben nicht nur für möglich hielt, sondern sie sogar gefunden zu haben glaubte. Man wird daher das achte Capitel, wie auch das, was in der Vorrede von diesem Gegenſtande gesagt worden, mit Miſstrauen lesen müssen. Zwar habe ich die Auflösung einer Menge sehr merkwürdiger unzerlegbarer Gleichungen gefunden, aber keineswegs die allgemeine Auflösung derselben, in dem Sinne der Euler, Lagrange und anderer groſsen Analysten, von deren Unmöglichkeit ich gegenwärtig überzeugt bin. Der Fehler entsprang aus Uebereilung, und ist so leicht zu entdecken, daſs jeder, der bis dahin gekommen ist, ihn von selbst finden wird.

Berlin, im May 1810.

Meier Hirsch.

Einleitung.

Die Einrichtung und Anordnung der Tafeln kann man am besten aus einer flüchtigen Durchsicht kennen lernen. Es soll daher hier bloſs das Nöthige über den Gebrauch und die richtige Anwendung der Formeln selbst angezeigt werden.

1) Die Zeichen, welche in diesem Werke gebraucht worden, sind die überall üblichen, das Differentialzeichen ∂ etwa ausgenommen. Der Kürze wegen wurden auch die, nun schon in Deutschland hinlänglich bekannten Hindenburgschen Zeichen, ${}^{m}\mathfrak{A}$, ${}^{m}\mathfrak{B}$, ${}^{m}\mathfrak{C}$, etc., ${}^{\frac{p}{q}}\mathfrak{A}$, ${}^{\frac{p}{q}}\mathfrak{B}$, ${}^{\frac{p}{q}}\mathfrak{C}$, etc., ${}^{-\frac{p}{q}}\mathfrak{A}$, ${}^{-\frac{p}{q}}\mathfrak{B}$, ${}^{-\frac{p}{q}}\mathfrak{C}$, etc., für die Binomialcoefficienten der Potenzen m, $\frac{p}{q}$, $-\frac{p}{q}$ gebraucht. Bey den bestimmten Integralen wurde dem Integralzeichen $\int$ oben ein Strich angehängt, um diese Integrale von den andern zu unterscheiden. Die Logarithmen, welche hier vorkommen, sind durchgängig die natürlichen oder hyperbolischen, und die Kreisbogen sämmtlich auf den Halbmesser $= 1$ bezogen. V. Z. heiſst Verkürzungszeichen

2) Man weiſs, daſs bey jeder Integration zu dem gefundenen Integrale noch eine willkührliche Constante gefügt werden muſs. Diese Constante ist, um den Platz nicht zu verengen, in den Tafeln weggelassen worden, weil sie sonst durchgängig hätte gesetzt werden müssen. Bey denjenigen Formeln, welche das Integralzeichen noch enthalten, ist dies nicht nöthig, jedoch muſs es immer nach der vollständigen Entwickelung geschehen. Wie die Constante bestimmt wird, findet man in allen Lehrbüchern angezeigt.

3) Die Coefficienten in den Integralen würden fast durchgängig unbestimmt gelassen. Wie überall in der Analysis, muſs man auch hier, wenn die Formeln auf einzelne Fälle angewandt werden sollen, für die unbestimmten Gröſsen die bestimmten setzen,

welche ihnen nach dem jedesmaligen Fall zukommen. So findet man S. 68, $\int \frac{\partial x}{x^4 X} = -\frac{1}{3ax^3} + \frac{b}{2a^2x^2}$ — etc. (für $X = a + bx + cx^2$).

Wollte man nun z. B. $\int \frac{\partial x}{x^4(-3+2x-x^2)}$ haben, so darf man nur $a = -3$, $b = 2$, $c = -1$ setzen; hierdurch erhält man

$$\int \frac{\partial x}{x^4 X} = \frac{1}{9x^3} + \frac{1}{9x^2} + \frac{1}{27x} + \frac{2}{81}\log\frac{x^2}{X} - \frac{7}{81}\int\frac{\partial x}{X}$$

worin $X = -3 + 2x - x^2$. Das Integral von $\frac{\partial x}{X}$ findet sich S. 61, und da hier $k = 4ac - b^2 = 8$, also positiv, so hat man

$$\int \frac{\partial x}{X} = \frac{2}{\sqrt{8}}\,\text{Arc Tang}\,\frac{2-2x}{\sqrt{8}} = \frac{1}{\sqrt{2}}\,\text{Arc Tang}\,\frac{1-x}{\sqrt{2}}.$$

Wird dieser Werth substituirt, so erhält man

$$\int \frac{\partial x}{x^4(-3+2x-x^2)} = \frac{1}{9x^3} + \frac{1}{9x^2} + \frac{1}{27x} + \frac{2}{81}\log\frac{x^2}{-3+2x-x^2}$$
$$- \frac{7}{81\sqrt{2}}\,\text{Arc Tang}\,\frac{1-x}{\sqrt{2}} + \text{Const.}$$

Gesetzt man wollte $\int \frac{x^2\partial x}{(2-3x+4x^2)^{\frac{5}{2}}}$ haben, so muſs man die Tafel für $\int \frac{x^m \partial x}{(a+bx+cx^2)^{\frac{5}{2}}}$ aufschlagen (S. 190). Hier findet man, wenn $a = 2$, $b = -3$, $c = 4$ gesetzt wird,

$$\int \frac{x^2\partial x}{X^{\frac{5}{2}}} = \left(-\frac{x}{8} - \frac{1}{64}\right)\frac{1}{X\sqrt{X}} + \frac{41}{128}\int\frac{\partial x}{X^{\frac{5}{2}}};$$

ferner auf derselben Seite, da hier $k = 4ac - b^2 = 23$,

$$\int \frac{\partial x}{X^{\frac{5}{2}}} = \left(\frac{1}{69X} + \frac{32}{1587}\right)\frac{16x-6}{\sqrt{X}}.$$

Wird dieser Werth substituirt, so erhält man,

$$\int \frac{x^2\partial x}{X^{\frac{5}{2}}} = -\frac{8x+1}{64X\sqrt{X}} + \frac{41}{128}\left(\frac{1}{69X} + \frac{32}{1587}\right)\frac{16x-6}{\sqrt{X}} + \text{Const.}$$

4) Bisweilen ereignet es sich, daſs für gewisse Werthe der Coefficienten ein Nenner in der Integralformel $= 0$, mithin die Formel unbrauchbar wird. In einem solchen Falle kann man im-

mer gewifs seyn, dafs das gegebene Differential einer Umformung fähig ist, weshalb es nicht mehr zu derjenigen Gattung von Differentialen gehört, zu welcher man es zählte. So z. B. findet man, wenn $\int \frac{\partial x}{x^3(3-6x+3x^2)^{\frac{3}{2}}}$ S. 189 aufgesucht wird, dafs $\int \frac{\partial x}{x^3 X^{\frac{3}{2}}}$ die Formel $\int \frac{\partial x}{X^{\frac{3}{2}}}$ enthalte, und S. 188, dafs diese letztere Formel die Gröfse $k = 4ac - b^2 = 0$ im Nenner habe. Es läfst sich nämlich dem Integral $\int \frac{\partial x}{x^3(3-6x+3x^2)^{\frac{3}{2}}}$ die Form $\int \frac{\partial x}{3^{\frac{3}{2}} x^3 (1-x)^3}$ $= \frac{1}{3\sqrt{3}} \int \frac{\partial x}{x^3(1-x)^3}$ geben, und in dieser Form gehört es zu der Gattung $\int \frac{\partial x}{x^m(a+bx)^3}$. Man findet S. 42, wenn $a = 1$, $b = -1$ gesetzt wird,

$$\frac{1}{3\sqrt{3}} \int \frac{\partial x}{x^3(1-x)^3} = \frac{1}{3\sqrt{3}} \left(-\frac{1}{2x^2} - \frac{2}{x} + 9 - 6x \right) \frac{1}{(1-x)^2}$$
$$- \frac{2}{\sqrt{3}} \log \frac{1-x}{x} + \text{Const.}$$

5) Bey denjenigen Integralformeln, welche keine andere involviren, und unmittelbar durch Logarithmen und Kreisbogen ausgedruckt werden müssen, (man pflegt sie, obgleich etwas uneigentlich, Elementarintegrale zu nennen) erscheinen hier gröfstentheils in mehreren Formen, unter welchen man nach Belieben wählen kann. Der Verfasser wurde hierzu durch die nicht unwichtigen Gründe bewogen, dafs erstens in gewissen Fällen die eine Form vor der andern einen wirklichen Vorzug hat, und dafs es zweitens die Vergleichung mit andern Werken erleichtert, wo bisweilen ausschliefsend die eine oder die andere Form gebraucht wird. Aufser den angeführten giebt es aber noch unendlich viele andere Formen, wohin auch diejenigen zu rechnen sind, welche aus der Veränderung der Constante entspringen. Die vorzüglichsten darunter können am besten hier ihren Platz finden. Wenn nämlich X irgend eine Function von x bezeichnet, so kann man

anstatt	$\text{Arc Sin}\, X$ + Const.	setzen	$-$	$\text{Arc Cos}\, X$	+ Const.
....	$\text{Arc Cos}\, X$ + Const.	....	$-$	$\text{Arc Sin}\, X$	+ Const.
....	$\text{Arc Tang}\, X$ + Const.	....	$-$	$\text{Arc Cot}\, X$	+ Const.
....	$\text{Arc Cot}\, X$ + Const.	....	$-$	$\text{Arc Tang}\, X$	+ Const.
....	$\text{Arc Sec}\, X$ + Const.	....	$-$	$\text{Arc Cosec}\, X$	+ Const.
....	$\text{Arc Cosec}\, X$ + Const.	....	$-$	$\text{Arc Sec}\, X$	+ Const.
....	$\log -X$ + Const.	....		$\log X$	+ Const.
....	$\log \sqrt{-X}$ + Const.	....		$\log \sqrt{X}$	+ Const.

6) Mit Hülfe dieser Tafeln lassen sich auch unendlich viele Integrale durch bloſses Zusammensetzen finden, wie sich am leichtesten an einem Beispiele zeigen läſst.

Es werde das Integral von

$$\partial Z = \frac{(3x^{12} - 2x^9 - 5x^6 + 2x^4 + 9)\partial x}{x^6(3-2x^2)^{\frac{7}{2}}}$$

gesucht. Setzt man der Kürze wegen $3 - 2x^2 = X$, so ist

$$Z = 3\int\frac{x^6\partial x}{X^{\frac{7}{2}}} - 2\int\frac{x^3\partial x}{X^{\frac{7}{2}}} - 5\int\frac{\partial x}{X^{\frac{7}{2}}} + 2\int\frac{\partial x}{x^2 X^{\frac{7}{2}}} + 9\int\frac{\partial x}{x^6 X^{\frac{7}{2}}}.$$

Werden diese Integrale in den Tafeln für

$$\int\frac{x^m\partial x}{(a+bx^2)^{\frac{7}{2}}},\quad \int\frac{\partial x}{x^m(a+bx^2)^{\frac{7}{2}}},$$

(S. 150. 151) aufgesucht, so findet man für $a = 3$, $b = -2$,

$$3\int\frac{x^6\partial x}{X^{\frac{7}{2}}} = \left(\frac{23x^5}{10} - \frac{21x^3}{4} + \frac{27x}{8}\right)\frac{1}{X^2\sqrt{X}} - \frac{3}{8}\int\frac{\partial x}{\sqrt{X}}$$

$$-2\int\frac{x^3\partial x}{X^{\frac{7}{2}}} = \left(-\frac{x^2}{3} + \frac{1}{5}\right)\frac{1}{X^2\sqrt{X}}$$

$$-5\int\frac{\partial x}{X^{\frac{7}{2}}} = -5\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$+2\int\frac{\partial x}{x^2 X^{\frac{7}{2}}} = -\frac{2}{3x}\cdot\frac{1}{X^2\sqrt{X}} + 8\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$+9\int\frac{\partial x}{x^6 X^{\frac{7}{2}}} = \left(-\frac{3}{5x^5} - \frac{4}{3x^3} - \frac{64}{9x}\right)\frac{1}{X^2\sqrt{X}} + \frac{256}{5}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

mithin

$$Z = \left(\frac{23x^5}{10} - \frac{21x^4}{4} - \frac{x^2}{3} + \frac{27x}{8} + \frac{1}{5} - \frac{70}{9x} - \frac{4}{3x^3} - \frac{3}{5x^5}\right)\frac{1}{X^2 \sqrt{X}}$$
$$- \frac{5}{8}\int \frac{\partial x}{\sqrt{X}} + \frac{265}{9}\int \frac{\partial x}{X^{\frac{7}{2}}}$$

Es ist aber

$$\int \frac{\partial x}{X^{\frac{7}{2}}} = \left(\frac{32x^5}{405} - \frac{8x^3}{27} + \frac{x}{3}\right)\frac{1}{X^2 \sqrt{X}}, \int \frac{\partial x}{\sqrt{X}} = \frac{1}{\sqrt{2}} \text{Arc Sin}\, x\sqrt{\tfrac{2}{3}};$$

werden daher diese Werthe substituirt, so erhält man nach der gehörigen Reduction,

$$Z = \left\{\begin{matrix} \frac{33727x^5}{7290} - \frac{13583x^3}{972} - \frac{x^2}{3} + \frac{2849x}{216} \\ + \frac{1}{5} - \frac{70}{9x} - \frac{4}{3x^3} - \frac{3}{5x^5} \end{matrix}\right\} \frac{1}{X^2 \sqrt{X}}$$
$$- \frac{3}{8\sqrt{2}} \text{Arc Sin}\, x\sqrt{\tfrac{2}{3}} + \text{Const.}$$

7) Sollte man auch ein Integral nicht unmittelbar in diesen Tafeln finden, so wird es sich doch immer, wenn überhaupt die vollständige Integration möglich ist; sehr leicht auf eines oder das andere der vorhandenen reduciren lassen. So z. B. findet sich das Integral $\int \frac{x^{\frac{2n+1}{2}} \partial x}{(a+bx)^{\frac{3}{2}}}$ in dieser Form nicht hier, wohl aber in der Form $\int \frac{x^{n+2} \partial x}{(ax+bx^2)^{\frac{3}{2}}}$, welche aus jener erhalten wird, wenn man den Zähler und Nenner des Bruches mit $x^{\frac{3}{2}}$ multiplicirt.

Kurze

Darstellung der Methoden

zur

Zerlegung der gebrochenen rationalen Functionen in Partialbrüche,

mit den

nöthigen Erläuterungen und Beispielen.

Es sey $\frac{U}{V}$ der zu zerlegende Bruch; U, V, bezeichnen zwey ganze rationale Functionen von nachstehender Form:

$$U = \mathfrak{A}x^m + \mathfrak{B}x^{m-1} + \mathfrak{C}x^{m-2} + \mathfrak{D}x^{m-3} + \text{etc.}$$
$$V = x^\mu + \alpha x^{\mu-1} + \beta x^{\mu-2} + \gamma x^{\mu-3} + \text{etc.}$$

und $m < \mu$; die Coefficienten positiv oder negativ, oder auch zum Theil $= o$. Es wird angenommen, daſs man den Nenner V in lauter reelle Factoren von den folgenden vier Formen zerlegen könne:

$$\text{I. } x + a, \qquad \text{II. } (x + a)^n$$
$$\text{III. } x^2 + ax + b, \quad \text{IV. } (x^2 + ax + b)^n$$

Es wird gefordert, den Bruch $\frac{U}{V}$ in solche Brüche zu zerlegen, deren Nenner diese Formen haben.

Da jeder dieser Factoren eine eigene Behandlungsart erfordert, so entspringen daraus vier verschiedene Fälle, für welche nun die Methoden mit den nöthigen Erläuterungen und Beispielen gegeben werden sollen.

Erster Fall.

Der Nenner V habe den Factor $x + a$, und enthalte denselben nur Einmal. Es sey

$$V = (x + a)\,\varrho$$

so ist ϱ eine bekannte ganze und rationale Function; denn es ist $\varrho = \frac{V}{x + a}$. Man setze

$$\frac{U}{V} = \frac{A}{x + a} + \frac{P}{\varrho}.$$

Das noch unbekannte A bezeichne eine constante Grösse, das ebenfalls unbekannte P eine ganze rationale Function von x. Es wird alsdann A nnd P nach den folgenden Methoden bestimmt.

Erste Methode.

Man setze $x + a = o$, also $x = - a$. Man substituire diesen Werth des x in den beiden bekannten Functionen U, Q, und bezeichne die constanten Grössen, worin sie sich hierdurch verwandeln, mit U', Q'. Es ist alsdann immer

$$A = \frac{U'}{Q'}.$$

Hat man auf diese Weise A bestimmt, so erhält man P aus der Formel

$$P = \frac{U - AQ}{x + a},$$

wenn die angezeigte Division wirklich verrichtet wird; der Zähler $U - AQ$ wird immer durch den Nenner $x + a$ ohne Rest theilbar seyn.

Zweite Methode.

Es sey $\partial V = Z\partial x$, also Z eine bekannte Function von x; ferner U', Z', das, worin sich die Functionen U, Z, verwandeln, wenn man darin $-a$ für x setzt: so ist immer

$$A = \frac{U'}{Z'}.$$

Die Function P wird wie bey der ersten Methode gefunden.

Bemerkungen.

1) Ist x selbst ein Factor des Nenners V, so muſs man, um aus den Functionen U, Q, Z, die constanten Gröſsen U', Q', Z', zu erhalten, o für x ſetzen.

2) Enthält der Nenner V, auſser dem Factor $x + a$, noch andere dergleichen Factoren, $x + a'$, $x + a''$, $x + a'''$, etc., so läſst sich aus jedem derselben ein eigener Partialbruch bilden, und die Zähler dieser Brüche A', A'', A'', etc., lassen sich auf die nämliche Art finden, wie der Zähler A für den Factor $x + a$.

3) Ist der Nenner V aus lauter solchen Factoren $x+a$, $x+a'$, $x+a''$, $x+a'''$, etc., zusammengesetzt, so läſst sich der Bruch $\frac{U}{V}$ in lauter Partialbrüche von der Form $\frac{A}{x+a}$ zerlegen, und die Summe derselben wird alsdann dem Bruche $\frac{U}{V}$ gleich seyn.

4) Diese Methoden sind jedoch nur alsdann anwendbar, wenn die Factoren ſämmtlich von einander verschieden sind; denn im entgegengesetzten Falle wird für denjenigen Factor, welcher mehrere Mal vorkommt, sowohl Q' als $Z'=o$, und daher nach beiden Methoden $A=\frac{U'}{o}=\infty$.

Beispiel.

Es sey $\frac{U}{V}=\frac{2x+3}{x^3+x^2-2x}=\frac{2x+3}{(x-1)(x+2)x}$ der zu zerlegende Bruch, also

$U=2x+3$; $V=x^3+x^2-2x=(x-1)(x+2)x$.

Für den ersten Factor $x-1$ ist nun $Q=(x+2)x$, und $x-1=o$, giebt $x=1$. Man findet also, wenn dieser Werth subſtituirt wird, $U'=5$, $Q'=3$, und daher nach der ersten Methode $A=\frac{U'}{Q'}=\frac{5}{3}$. Auch ist $\partial V=(3x^2+2x-2)\partial x$, und daher $Z=3x^2+2x-2$; mithin $Z'=3$, und daher nach der zweiten Methode $A=\frac{U'}{Z'}=\frac{5}{3}$, wie vorher. Der Partialbruch für den Factor $x-1$ ist demnach $\frac{\frac{5}{3}}{x-1}=\frac{5}{3(x-1)}$.

Für den Factor $x+2$ ist $Q=(x-1)x$, und $x+2=o$ giebt $x=-2$. Demnach ist, wenn -2 für x substituirt wird, $U'=-1$, $Q'=6$, daher nach der ersten Methode $A=\frac{U'}{Q'}=-\frac{1}{6}$. Da ferner $Z=3x^2+2x-2$, so ist, wenn -2 für x gesetzt wird, $Z'=6$, und folglich $A=\frac{U'}{Z'}=-\frac{1}{6}$, wie vorher. Der Partialbruch ist demnach $-\frac{1}{6(x+2)}$.

Für den Faktor x ist $Q=(x-1)(x+2)$, und also, wenn

$x = o$ gesetzt wird, $U' = 3$, $Q' = -2$; daher nach der ersten Methode, $A = \frac{U'}{Q'} = -\frac{3}{2}$. Ferner ist $Z = 3x^2 + 2x - 2$, und für $x = o$, $Z' = -2$; daher nach der zweiten Methode $A = \frac{U'}{Z'} = -\frac{3}{2}$, wie vorher. Der Partialbruch ist also $-\frac{3}{2x}$.

Aus allem diesen ergiebt sich, daſs

$$\frac{U}{Z} = \frac{5}{3(x-1)} - \frac{1}{6(x+2)} - \frac{3}{2x}.$$

Zweiter Fall.

Der Nenner V des Bruches $\frac{U}{V}$ enthalte den Factor $x + a$ mehrere Mal, und es sey $V = (x+a)^n Q$; mithin Q eine bekannte ganze rationale Function von x. Man setze

$$\frac{U}{V} = \frac{A}{(x+a)^n} + \frac{A'}{(x+a)^{n-1}} + \frac{A''}{(x+a)^{n-2}} + \ldots\ldots + \frac{A^{(n-2)'}}{(x+a)^2} + \frac{A^{(n-1)'}}{x+a} + \frac{P}{Q}.$$

Es wird gefordert, die constanten Zähler A, A', A'', etc., zu bestimmen.

Erste Methode.

Wenn Q', U', U'_1, U'_2, U'_3, etc., das bezeichnen, worin sich die mit Q, U, U_1, U_2, U_3, etc., bezeichneten Functionen verwandeln, wenn man $x + a = o$, also $x = -a$ setzt: so hat man zur Bestimmung von A, A', A'', A''', etc., nachstehende Formeln:

(1) $A = \frac{U'}{Q'}$	(2) $\frac{U - AQ}{x+a} = U_1$
(3) $A' = \frac{U'_1}{Q'}$	(4) $\frac{U_1 - A'Q}{x+a} = U_2$
(5) $A'' = \frac{U'_2}{Q'}$	(6) $\frac{U_2 - A''Q}{x+a} = U_3$
(7) $A''' = \frac{U'_3}{Q'}$	(8) $\frac{U_3 - A'''Q}{x+a} = U_4$
etc.	etc.

Es wird nämlich zuerst, sowohl in U als in ϱ, für x sein Werth $-a$ gesetzt, und dadurch der Werth der constanten Grössen U', ϱ', gefunden. Die Formel (1) giebt hierauf den Werth von A; und wird dieser Werth in der Formel (2) substituirt, und die angezeigte Division durch $x+a$ wirklich verrichtet, so giebt dieses für U_1 eine ganze Function. Wird in derselben $-a$ für x gesetzt, so erhält man U'_1, und hieraus, vermittelst der Formel (3), den Werth von A'. Wird dieser Werth in der Formel (4) substituirt, und durch $x+a$ wirklich dividirt, so erhält man für U_2 eine ganze Function. Aus derselben erhält man ferner durch die Substitution von $-a$ für x den Werth der constanten Gröſse U'_2, und die Formel (5) giebt alsdann den Werth von A''. Durch die Substitution dieses Werthes in der Formel (6) erhält man U_3, und somit auch den Werth der constanten Gröſse U'_3, und die Formel (7) giebt den Werth von A'''. Mit dieser Operation wird so lange fortgefahren, bis die sämmtlichen Zähler A, A', A'', A''' .... $A^{(n-1)'}$ bestimmt sind.

Ist $A^{(n-1)'}$ gefunden, so läſst sich auch der Zähler P des ergänzenden Bruches $\frac{P}{\varrho}$ finden. Denn aus $A^{(n-1)'}$ erhält man auf dem angegebenen Wege U_n; und es ist alsdann immer $P = U_n$.

Zweite Methode.

Es ist immer

$$A = \frac{U}{\varrho}$$

$$A' = \frac{1}{1 \cdot \partial x} \partial \cdot \frac{U}{\varrho}$$

$$A'' = \frac{1}{1 \cdot 2 \cdot \partial x^2} \partial^2 \cdot \frac{U}{\varrho}$$

$$A''' = \frac{1}{1 \cdot 2 \cdot 3 \cdot \partial x^3} \partial^3 \cdot \frac{U}{\varrho}$$

$$A^{IV} = \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot \partial x^4} \partial^4 \cdot \frac{U}{\varrho}$$

und im Allgemeinen

$$A^{m'} = \frac{1}{1 \cdot 2 \cdot 3 \cdot \ldots\ldots m \cdot \partial x^m} \partial^m \cdot \frac{U}{\varrho}$$

wenn in den erhaltenen Resultaten $-a$ für x gesetzt wird.

Um daher die Gröſsen, A', A'', ... $A^{(n-1)'}$ zu bestimmen, muſs man die Function $\frac{U}{Q}$ $n-1$ Mal nach einander differentiiren, die gefundenen Differentiale, in der Ordnung, wie sie einander folgen, durch $1 . \partial x$, $1 . 2 . \partial x^2$, $1 . 2 . 3 . \partial x^3$, ... $1 . 2 . 3 \ldots . n-1 . \partial x^{n-1}$ dividiren, und in den $n-1$ erhaltenen Resultaten $-a$ für x setzen.

Beispiel.

Es sey der zu zerlegende Bruch

$$\frac{U}{V} = \frac{3x^2 + x - 2}{(x-1)^3 (x^2+1)}$$

und es werde gesetzt:

$$\frac{U}{V} = \frac{A}{(x-1)^3} + \frac{A'}{(x-1)^2} + \frac{A''}{x-1} + \frac{P}{Q};$$

man soll die constanten Gröſsen A, A', A'', wie auch die Function P finden.

Rechnung nach der ersten Methode.

Es ist hier $U = 3x^2 + x - 2$, $Q = x^2 + 1$; ferner giebt $x - 1 = 0$, $x = 1$. Man hat daher $U' = 2$, $Q' = 2$; folglich, nach Formel (1), $A = \frac{U'}{Q'} = 1$.

Nach der Formel (2) ist daher

$$U_1 = \frac{U - 1 . Q}{x-1} = 2x + 3;$$

mithin $U'_1 = 5$, und folglich nach der Formel (3), $A' = \frac{U'_1}{Q'} = \frac{5}{2}$.

Hieraus erhält man nach der Formel (4)

$$U_2 = \frac{U_1 - \frac{5}{2}Q}{x-1} = -\tfrac{5}{2}x - \tfrac{1}{2}$$

mithin $U'_2 = -3$, und daher nach der Formel (5), $A'' = \frac{U'_2}{Q'} = -\frac{3}{2}$.

Soll nun noch P bestimmt werden, so hat man nach der Formel (6)

$$U_3 = \frac{U_2 + \frac{3}{2}Q}{x-1} = \frac{3}{2}x - 1;$$

und daher $P = U_3 = \frac{3}{2}x - 1$.

Es ist demnach

$$\frac{U}{V} = \frac{1}{(x-1)^3} + \frac{5}{2(x-1)^2} - \frac{3}{2(x-1)} + \frac{3x-2}{2(x^2+1)}.$$

Rechnung nach der zweiten Methode.

Hier ist

$$\frac{U}{Q} = \frac{3x^2 + x - 2}{x^2 + 1}$$

$$\frac{1}{1 \cdot \partial x} \partial \cdot \frac{U}{Q} = \frac{-x^2 + 10x + 1}{(x^2+1)^2}$$

$$\frac{1}{1 \cdot 2 \cdot \partial x^2} \partial^2 \cdot \frac{U}{Q} = \frac{(x^2+1)^2(10-2x) + (x^2 - 10x - 1)(x^2+1)4x}{2(x^2+1)^4}.$$

Setzt man in den zweiten Theilen dieser Gleichungen $x = 1$, so erhält man für A, A', A'', die nämlichen Werthe wie vorher.

Dritter Fall.

Es wird angenommen, daſs der Nenner V des Bruches $\frac{U}{V}$ den trinomischen Factor $x^2 + ax + b$ habe, so daſs $V = (x^2 + ax + b) Q$ und Q eine ganze Function sey. Es wird ferner angenommen, daſs das Trinom $x^2 + ax + b$ sich nicht in zwey reelle Factoren von der Form $x + a$ zerfällen lasse, welches Statt hat, wenn die Wurzeln der Gleichung $x^2 + ax + b = 0$ imaginär sind, also, wenn $a^2 - 4b$ negativ wird. Man verlangt den Bruch $\frac{U}{V}$ in zwey andere so zu zerlegen, daſs

$$\frac{U}{V} = \frac{A + Bx}{x^2 + ax + b} + \frac{P}{Q};$$

A, B, sollen constante Gröſsen, und P eine ganze Function von x seyn.

Erste Methode.

Die Gleichung $x^2 + ax + b = o$, wird, der Voraussetzung gemäſs, zwey imaginäre Wurzeln haben; es mögen $h \pm k\sqrt{-1}$ diese Wurzeln seyn. Man bilde die Function

$$U-(A+Bx)\rho = Y;$$

setze hierauf in der Function Y durchgängig $h+k\sqrt{-1}$ für x, so wird dieselbe einen Werth von der Form $M+N\sqrt{-1}$ erhalten, und M, N, werden zwey Constanten seyn, welche die gegenwärtig noch unbekannten Constanten A, B, enthalten. Man mache hierauf die beiden Gleichungen

$$M=0,\quad N=0;$$

sie werden beide vom ersten Grade in Hinsicht auf A und B seyn. Man löse sie auf, so erhält man A und B.

Die Function P erhält man alsdann unmittelbar aus der Formel

$$P=\frac{U-(A+Bx)\rho}{x^2+ax+b}$$

wenn für A, B, ihre gefundenen Werthe gesetzt, und die angezeigte Division wirklich ausgeführt wird.

Zweite Methode.

Man setze in der Function Y, $\frac{1}{2k\sqrt{-1}}\cdot\frac{dV}{dx}$ anstatt ρ, und verfahre hierauf wie bey der ersten Methode.

Anmerk. Die zweite Methode ist nur in gewissen Fällen, welche an ihrem Orte angezeigt werden sollen, der ersten vorzuziehen.

Beispiel.

Es sey der zu zerlegende Bruch

$$\frac{U}{V}=\frac{2x+1}{(x^2+2x+5)(x^2+x+1)(x^2+1)}.$$

Für den Factor x^2+2x+5 ist $\rho=(x^2+x+1)(x^2+1)$. Da nun $U=2x+1$, so ist

$$Y=2x+1-(A+Bx)(x^2+x+1)(x^2+1).$$

Die Gleichung $x^2+2x+5=0$ giebt $x=-1+2\sqrt{-1}$. Wird dieser Werth in Y substituirt, so erhält man

$$Y=-1+4\sqrt{-1}-(A-B+2B\sqrt{-1})(-2+16\sqrt{-1})$$
$$=2A+30B-1+(20B-16A+4)\sqrt{-1};$$

mithin ist $M=2A+30B-1$, $N=20B-16A+4$. Man hat demnach die beiden Gleichungen:

$$2A+30B-1=0,\quad 20B-16A+4=0,$$

und diese geben $A=\frac{7}{26}$, $B=\frac{1}{65}$.

Für den Factor x^2+x+1 ist $\varrho=(x^2+2x+5)(x^2+1)$, ıd daher

$$Y=2x+1-(A+Bx)(x^2+2x+5)(x^2+1).$$

ie Gleichung $x^2+x+1=o$ giebt $x=-\frac{1}{2}+\frac{\sqrt{3}}{2}\sqrt{-1}$. urch die Substitution dieses Werthes erhält man

$$Y=\sqrt{3}.\sqrt{-1}-(A-\tfrac{1}{2}B+\tfrac{B\sqrt{3}}{2}\sqrt{-1})(\tfrac{5}{2}-\tfrac{3}{2}\sqrt{3}.\sqrt{-1})$$
$$=-\tfrac{5}{2}A-B+(\tfrac{3}{2}A-2B+1)\sqrt{3}.\sqrt{-1}$$

ithin ist $M=-\frac{5}{2}A-B$, $N=(\frac{3}{2}A-2B+1)\sqrt{3}$. Man t also die beiden Gleichungen:

$$\tfrac{5}{2}A+B=o,\quad \tfrac{3}{2}A-2B+1=o$$

ıd diese geben $A=-\frac{2}{13}$, $B=\frac{5}{13}$.

Für den dritten Factor x^2+1 ist $\varrho=(x^2+2x+5)$ $^2+x+1)$; also

$$Y=2x+1-(A+Bx)(x^2+2x+5)(x^2+x+1).$$

e Gleichung $x^2+1=o$ giebt $x=\sqrt{-1}$. Wird dieser Werth bstituirt, so erhält man

$$Y=1+2\sqrt{-1}-(A+B\sqrt{-1})(-2+4\sqrt{-1})$$
$$=2A+4B+1+(2B-4A+2)\sqrt{-1}$$

ithin ist $M=2A+4B+1$, $N=2B-4A+2$. Man hat dar die beiden Gleichungen:

$$2A+4B+1=o,\ 2B-4A+2=o,$$

ıd diese geben $A=\frac{3}{10}$, $B=-\frac{2}{5}$.

Es ist demnach

$$\frac{U}{V}=\frac{\frac{7}{26}+\frac{1}{65}x}{x^2+2x+5}+\frac{-\frac{2}{13}+\frac{5}{13}x}{x^2+x+1}+\frac{\frac{3}{10}-\frac{2}{5}x}{x^2+1}.$$

Vierter Fall.

Es enthalte der Nenner V des Bruches $\frac{U}{V}$ den Factor x^2+ax+b ehrere Mal, so dafs $V=(x^2+ax+b)^n\varrho$ und ϱ eine ganze ınction von x sey. Es läſst sich alsdann dieser Bruch, auf eine ınliche Art wie bey dem zweiten Falle, immer so zerlegen, ıſs

$$\frac{U}{V} = \frac{A + Bx}{(x^2 + ax + b)^n} + \frac{A' + B'x}{(x^2 + ax + b)^{n-1}} + \ldots\ldots$$
$$\ldots\ldots + \frac{A^{(n-1)'} + B^{(n-1)'}x}{x^2 + ax + b} + \frac{P}{Q};$$

und es kommt nunmehr nur darauf an, die constanten Größsen A, B, A', B', etc. zu bestimmen.

Methode.

Man bilde successive die Functionen U_1, U_2, U_3, etc. nach dem folgenden Schema:

$$(1)\quad U_1 = \frac{U - (A + Bx)Q}{x^2 + ax + b}$$
$$(2)\quad U_2 = \frac{U_1 - (A' + B'x)Q}{x^2 + ax + b}$$
$$(3)\quad U_3 = \frac{U_2 - (A'' + B''x)Q}{x^2 + ax + b}$$
$$(4)\quad U_4 = \frac{U_3 - (A''' + B'''x)Q}{x^2 + ax + b}$$

etc.

und bestimme aus U, U_1, U_2, U_3, U_4, etc., die Constanten A, B; A', B'; A'', B''; A''', B'''; A^{IV}, B^{IV}, etc., ganz nach der ersten Methode des dritten Falles.

Es werden nämlich zuerst die Constanten A, B, ganz auf die nämliche Art wie bey dem dritten Falle gefunden. Man substituire nun ihre Werthe in der Formel (1), und verrichte die angezeigte Division durch $x^2 + ax + b$ wirklich, so wird man für U_1 eine ganze Function finden. Man verfahre nun mit U_1 gerade so wie vorher mit U, und bestimme dadurch die Constanten A', B'. Die Werthe derselben substituire man in der Formel (2), so wird man für U_2, wenn die angezeigte Division wirklich ausgeführt wird, eine ganze Function finden, aus welcher nun wieder durch dasselbe Verfahren die Constanten A'', B'', bestimmt werden. Diese Operation setze man so lange fort, bis die sämmtlichen Constanten A, B; A', B'; A'', B''; A''', B''', ... $A^{(n-1)'}$, $B^{(n-1)'}$ gefunden sind.

Verlangt man noch überdies P zu bestimmen, so suche man
den nun bekannten Größen $A^{(n-1)'}$, $B^{(n-1)'}$, die Function
; es ist alsdann $P = U_n$.

Beispiel.

Es sey der zu zerlegende Bruch

$$\frac{U}{V} = \frac{2x^5+7x^2-4x}{(x^2+1)^3\,(2x^4-5)}$$

r ist $U = 2x^5+7x^2-4x$, $Q = 2x^4-5$; also

$$Y = 2x^5+7x^2-4x-(A+Bx)(2x^4-5).$$

zt man $x^2+1=o$, so erhält man $x=\sqrt{-1}$, und die Sub-
ution dieses Werthes verwandelt die Function Y in

$$-7-2\sqrt{-1}+3(A+B\sqrt{-1})$$

n hat daher die beiden Gleichungen $3A-7=o$, $3B-2=o$,
l diese geben $A=\frac{7}{3}$, $B=\frac{2}{3}$.

Substituirt man diese Werthe in der Formel (1), so erhält
n

$$U_1 = \frac{2x^5+7x^2-4x-(\frac{7}{3}+\frac{2}{3}x)(2x^4-5)}{x^2+1}$$

$$= \tfrac{1}{3}(2x^3-14x^2-2x+35).$$

n behandle nun U_1, wie vorhin U; so hat man

$$Y = \tfrac{1}{3}(2x^3-14x^2-2x+35)-(A'+B'x)(2x^4-5)$$

diese Function verwandelt sich, wenn $x=\sqrt{-1}$ gesetzt
d, in

$$\tfrac{49}{3}-\tfrac{4}{3}\sqrt{-1}+3(A'+B'\sqrt{-1})$$

oder $3A'+\frac{49}{3}+(3B'-\frac{4}{3})\sqrt{-1}$.

n hat daher die beiden Gleichungen: $3A'+\frac{49}{3}=o$, $3B'-\frac{4}{3}=o$,
diese geben $A'=-\frac{49}{9}$, $B'=\frac{4}{9}$.

Hieraus findet man nun wieder, vermittelst der Formel (2)

$$U_2 = \frac{\frac{1}{3}(2x^3-14x^2-2x+35)-(-\frac{49}{9}+\frac{4}{9}x)(2x^4-5)}{x^2+1}$$

$$= \tfrac{1}{9}(-8x^3+98x^2+14x-140)$$

hier

$$Y = \tfrac{1}{9}(-8x^3+98x^2+14x-140)-(A''+B''x)(2x^4-5)$$

wenn $\sqrt{-1}$ für x gesetzt wird,

$$Y = -\tfrac{238}{9}+\tfrac{22}{9}\sqrt{-1}+3(A''+B''\sqrt{-1});$$

also $3A'' - \frac{238}{9} = o$, $3B'' + \frac{22}{9} = o$; mithin $A'' = \frac{238}{27}$, $B'' = -\frac{22}{27}$

Aus A'' und B'' erhält man endlich:

$$U_3 = \frac{\frac{1}{9}(-8x^3 + 98x^2 + 14x - 140) - (\frac{238}{27} - \frac{22}{27}x)(2x^4 - 5)}{x^2 + 1}$$

$$= \tfrac{1}{27}(44x^3 - 476x^2 - 68x + 770)$$

und dieses ist die Function P.

Es ist demnach

$$\frac{U}{V} = \frac{7 + 2x}{3(x^2+1)^3} + \frac{-49 + 4x}{9(x^2+1)^2} + \frac{238 - 22x}{27(x^2+1)}$$
$$+ \frac{44x^3 - 476x^2 - 68x + 770}{27(2x^4 - 5)}$$

Tafeln

der

Reductionsformeln für das Integral

$$\int x^{m-1} \partial x \, (a + bx^n + cx^{2n} + dx^{3n} + \text{etc.})^p.$$

Wenn man alle die verschiedenen Methoden, deren man sich in der Integralrechnung bedient, um das Integral einer vorgelegten Differentialfunction zu finden, einer näheren Prüfung unterwirft, so wird man einsehen, dafs es hierbey einzig und allein auf die folgenden zwey Erfordernisse ankommt:

1) Auf die Kenntnifs der Elementarintegrale, d. h. solcher Integrale, welche entweder wirklich in der einfachsten Gestalt erscheinen, deren sie fähig sind, oder wenigstens so angesehen werden, wie etwa $\int x^m \partial x$, $\int \frac{\partial x}{x}$, $\int \frac{\partial x}{1+x^2}$, $\int \frac{\partial x}{V(1+x^4)}$, etc.

2) Auf die Methode ein vorgelegtes Integral auf eines oder das andere dieser Elementarintegrale zu reduciren.

Die Reduction eines vorgelegten Integrals auf andere kann auf mehrere Arten geschehen; nämlich:

a) Durch die Zerlegung des gegebenen Differentials in andere; mithin durch partielle Integration.

b) Durch die Einführung einer neuen veränderlichen Gröfse; also durch Substitution.

c) Durch die Anwendung gewisser Formeln involutorischer Art, mit deren Hülfe ein vorgelegtes Integral, ohne eine Substitution, oder irgend ein anderes Mittel, auf ein einfacheres, dieses wieder auf ein einfacheres, u. s. w. zurückgeführt wird. Diese Formeln sollen ausschliefsend Reductionsformeln genannt werden.

d) Durch die Anwendung einiger oder aller dieser Methoden zugleich.

Die allgemeinste Reductionsformel ist folgende:

$$(\odot) \quad \int XY\partial x = X\int Y\partial x - \int \partial X \int Y\partial x$$

wo X, Y, zwey willkührliche algebraische oder transcendente Functionen von x bezeichnen. Aus derselben lassen sich mit Hülfe gewisser Kunstgriffe alle die Reductionsformeln ableiten, welche hier für algebraische Functionen gegeben werden.

Der Gebrauch dieser Formeln erstreckt sich auf das vielumfassende Integral $\int x^{m-1}\partial x X^p$ ($X = a + bx^n + cx^{2n} + dx^{3n} +$ etc.), wo für m, n, p, alle mögliche Zahlen, positive oder negative, ganze oder gebrochene, angenommen werden können. Taf. I. giebt sie für das Binom $a + bx^n$; Taf. II. für das Trinom $a + bx^n + cx^{2n}$; Taf. III. für das Quadrinom $a + bx^n + cx^{2n} + dx^{3n}$; Taf. IV. für das Polynom. Mit Hülfe derselben ist man im Stande die Exponenten m und p nach Belieben zu erhöhen oder zu erniedrigen, bis man zu solchen Integralen kommt, welche sich durch schickliche Substitutionen auf Elementarintegrale zurückführen lassen, die alsdann weiter entweder vollständig integrirt, oder durch Reihen ausgedruckt werden müssen.

Die gedachten Formeln sind übrigens nur so lange anwendbar, als die Nenner der Bruchcoefficienten, welche darin vorkommen, nicht verschwinden; wie dies z. B. bey den Formeln I. und V. Taf. I. geschiehet, wenn $m = o$, oder bey den Formeln III. und IV. Taf. I., wenn $m + np = o$ wird.

Tafel I.

Reductionsformeln für das Integral

$$\int x^{m-1} \partial x\, (a + bx^n)^p$$

VZ. $a + bx^n = X$

I.

$$\int x^{m-1} \partial x X^p = \frac{x^m X^p}{m} - \frac{pnb}{m} \int x^{m+n-1} \partial x X^{p-1}$$

II.

$$\int x^{m-1} \partial x X^p = \frac{x^{m-n} X^{p+1}}{(p+1)nb} - \frac{m-n}{(p+1)nb} \int x^{m-n-1} \partial x X^{p+1}$$

III.

$$\int x^{m-1} \partial x X^p = \frac{x^{m-n} X^{p+1}}{(m+np)b} - \frac{(m-n)a}{(m+np)b} \int x^{m-n-1} \partial x X^p$$

IV.

$$\int x^{m-1} \partial x X^p = \frac{x^m X^p}{m+np} + \frac{pna}{m+np} \int x^{m-1} \partial x X^{p-1}$$

V.

$$\int x^{m-1} \partial x X^p = \frac{x^m X^{p+1}}{ma} - \frac{(m+n+np)b}{ma} \int x^{m+n-1} \partial x X^p$$

VI.

$$\int x^{m-1} \partial x X^p = -\frac{x^m X^{p+1}}{(p+1)na} + \frac{m+n+np}{(p+1)na} \int x^{m-1} \partial x X^{p+1}$$

Tafel I.

Reductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^n)^p$$

VZ. $a+bx^n = X$

VII.

$$\int x^{m-1}\partial x X^p =$$

$$\left\{\begin{matrix} Ax^{m-n} - Bx^{m-2n} + Cx^{m-3n} - Dx^{m-4n} + \text{etc.} \\ \pm Kx^{m-(i-1)n} \mp Lx^{m-in} \end{matrix}\right\} X^{p+1}$$

$$\pm L(m-in)a\int x^{m-in-1}\partial x X^p$$

$$A = \frac{1}{(m+np)b},\quad B = \frac{(m-n)a}{(m-n+np)b}A,\quad C = \frac{(m-2n)a}{(m-2n+np)b}$$

$$D = \frac{(m-3n)a}{(m-3n+np)b}C,\quad E = \frac{(m-4n)a}{(m-4n+np)b}D,\ \text{etc.}$$

$$L = \frac{[m-(i-1)n]a}{[m-(i-1)n+np]b}K$$

VIII.

$$\int x^{m-1}\partial x X^p =$$

$$\left\{\begin{matrix} AX^p + BX^{p-1} + CX^{p-2} + DX^{p-3} + EX^{p-4} + \text{etc.} \\ + KX^{p-i+2} + LX^{p-i+1} \end{matrix}\right\} x^m$$

$$+L(p-i+1)na\int x^{m-1}dx X^{p-i}$$

$$A = \frac{1}{m+np},\quad B = \frac{pna}{m-n+np}A,\quad C = \frac{(p-1)na}{m-2n+np}B$$

$$D = \frac{(p-2)na}{m-3n+np}C,\quad E = \frac{(p-3)na}{m-4n+np}D,\ \text{etc.}$$

$$L = \frac{(p-i+2)na}{m-(i-1)n+np}K.$$

Tafel I.

Reductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^n)^p$$

VZ. $a+bx^n = X$

IX.

$$\int x^{m-1}\partial x X^p =$$

$$\left\{\begin{matrix} Ax^m - Bx^{m+n} + Cx^{m+2n} - Dx^{m+3n} + Ex^{m+4n} - \text{etc.} \\ \pm Kx^{m+(i-2)n} \mp Lx^{m+(i-1)n} \end{matrix}\right\} X^{p+1}$$

$$\pm L(m+in+np)b\int x^{m+in-1}\partial x X^p$$

$$A = \frac{1}{ma},\quad B = \frac{(m+n+np)b}{(m+n)a}A,\quad C = \frac{(m+2n+np)b}{(m+2n)a}B,$$

$$D = \frac{(m+3n+np)b}{(m+3n)a}C,\quad E = \frac{(m+4n+np)b}{(m+4n)a}D,\ \text{etc.}$$

$$L = \frac{[m+(i-1)n+np]b}{[m+(i-1)n]a}K.$$

X.

$$\int x^{m-1}dx X^p =$$

$$-\left\{\begin{matrix} AX^{p+1} + BX^{p+2} + CX^{p+3} + DX^{p+4} + EX^{p+5} + \text{etc.} \\ + KX^{p+i-1} + LX^{p+i} \end{matrix}\right\} x^m$$

$$+ L(m+in+np)\int x^{m-1}\partial x X^{p+i}$$

$$A = \frac{1}{(p+1)na},\quad B = \frac{m+n+np}{(p+2)na}A,\quad C = \frac{m+2n+np}{(p+3)na}B,$$

$$D = \frac{m+3n+np}{(p+4)na}C,\quad E = \frac{m+4n+np}{(p+5)na}D,\ \text{etc.}$$

$$L = \frac{m+(i-1)n+np}{(p+i)na}K.$$

Tafel II.

Reductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^n+cx^{2n})^p$$

VZ. $a+bx^n+cx^{2n}=X$.

I.

$$\int x^{m-1}\partial x X^p =$$
$$\frac{x^m X^p}{m}-\frac{pnb}{m}\int x^{m+n-1}\partial x X^{p-1}$$
$$-\frac{2pnc}{m}\int x^{m+2n-1}\partial x X^{p-1}$$

II.

$$\int x^{m-1}\partial x X^p =$$
$$\frac{x^{m-2n}X^{p+1}}{(m+2pn)c}-\frac{(m-2n)a}{(m+2pn)c}\int x^{m-2n-1}\partial x X^p$$
$$-\frac{(m-n+pn)b}{(m+2pn)c}\int x^{m-n-1}\partial x X^p$$

III.

$$\int x^{m-1}\partial x X^p =$$
$$\frac{x^m X^p}{m+2pn}+\frac{2pna}{m+2pn}\int x^{m-1}\partial x X^{p-1}$$
$$+\frac{pnb}{m+2pn}\int x^{m+n-1}dx X^{p-1}$$

Tafel II.

Reductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^n+cx^{2n})^p$$

VZ. $a+bx^n+cx^{2n}=X$

IV.

$$\int x^{m-1}\partial x X^p =$$
$$\frac{x^m X^{p+1}}{ma} - \frac{(m+n+pn)b}{ma}\int x^{m+n-1}\partial x X^p$$
$$-\frac{(m+2n+2pn)c}{ma}\int x^{m+2n-1}\partial x X^p$$

V.

$$\int x^{m-1}\partial x X^p =$$
$$\frac{Ax^m+Bx^{m+n}}{K}X^{p+1}+\frac{1}{K}\int(Cx^{m-1}+Dx^{m+n-1})\partial x X^{p+1}$$

$A = 2ac - b^2$
$B = - bc$
$C = n(p+1)(b^2-4ac)-m(2ac-b^2)$
$D = (2pn+3n+m)bc$
$K = (p+1)(b^2-4ac)na$

Tafel III.

Reductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^n+cx^{2n}+dx^{3n})^p$$

VZ. $a+bx^n+cx^{2n}+dx^{3n}=X$

I.

$$\int x^{m-1}\partial x X^p =$$
$$\frac{x^m X^p}{m}-\frac{pnb}{m}\int x^{m+n-1}\partial x X^{p-1}$$
$$-\frac{2pnc}{m}\int x^{m+2n-1}\partial x X^{p-1}-\frac{3pnd}{m}\int x^{m+3n-1}\partial x X^{p-1}$$

II.

$$\int x^{m-1}\partial x X^p =$$
$$\frac{x^{m-3n}X^{p+1}}{(m+3pn)d}-\frac{(m-3n)a}{(m+3pn)d}\int x^{m-3n-1}\partial x X^p$$
$$-\frac{(m-2n+pn)b}{(m+3pn)d}\int x^{m-2n-1}\partial x X^p-\frac{(m-n+2pn)c}{(m+3pn)d}\int x^{m-n-1}\partial x X^p$$

III.

$$\int x^{m-1}\partial x X^p =$$
$$\frac{x^m X^p}{m+3pn}+\frac{3pna}{m+3pn}\int x^{m-1}\partial x X^{p-1}$$
$$+\frac{2pnb}{m+3pn}\int x^{m+n-1}\partial x X^{p-1}+\frac{pnc}{m+3pn}\int x^{m+2n-1}\partial x X^{p-1}$$

Tafel III.

eductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^n+cx^{2n}+dx^{3n})^p$$

VZ. $a+bx^n+cx^{2n}+dx^{3n}=X$

IV.

$$\int x^{m-1}\partial x X^p =$$

$$\frac{x^m X^{p+1}}{ma}-\frac{(m+n+pn)b}{ma}\int x^{m+n-1}\partial x X^p$$

$$\frac{+2n+2pn)c}{ma}\int x^{m+2n-1}\partial x X^p-\frac{(m+3n+3pn)d}{ma}\int x^{m+3n-1}\partial x X^p$$

V.

$$\int x^{m-1}\partial x X^p =$$

$$(Ax^m+Bx^{m+n}+Cx^{m+2n})X^{p+1}+$$

$$\int(Dx^{m-1}+Ex^{m+n-1}+Fx^{m+2n-1})X^{p+1}\partial x$$

oefficienten A, B, C, sind durch die drey Gleichungen

$$bdA-3adB+acC=\frac{-bd}{(p+1)na}$$

$$(bc-3ad)A-2acB+2abC=\frac{ad-bc}{(p+1)na}$$

$$(b^2-2ac)A-abB+3a^2C=\frac{ac-b^2}{(p+1)na}$$

en, und aus diesen Coefficienten erhält man

$$D=\frac{1}{a}-mA$$

$$E=-\frac{(p+1)nb}{a}A-(m+n)B-\frac{b}{a^2}$$

$$F=-(m+5n+3pn)C.$$

Tafel IV.

Reductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^n+cx^{2n}+\ldots\ldots+tx^{kn})^p$$

Verkürzungszeichen

$$a+bx^n+cx^{2n}+dx^{3n}+\text{etc.}+sx^{(k-1)n}+tx^{kn}=X$$

$\int x^{m+n-1}\partial x X^{p-1} = S'$	$\int x^{m-n-1}\partial x X^{p} = S'_1$
$\int x^{m+2n-1}\partial x X^{p-1} = S''$	$\int x^{m-2n-1}\partial x X^{p} = S''_1$
$\int x^{m+3n-1}\partial x X^{p-1} = S'''$	$\int x^{m-3n-1}\partial x X^{p} = S'''_1$
$\int x^{m+4n-1}\partial x X^{p-1} = S''''$	$\int x^{m-4n-1}\partial x X^{p} = S''''_1$
$\ldots\ldots\ldots\ldots$	$\ldots\ldots\ldots\ldots$
$\int x^{m+kn-1}\partial x X^{p-1} = S^{k\prime}$	$\int x^{m-kn-1}\partial x X^{p} = S^{k\prime}_1$

$\int x^{m-1}\partial x X^{p-1} = S_2$	$\int x^{m+n-1}\partial x X^{p} = S'_3$
$\int x^{m+n-1}\partial x X^{p-1} = S'_2$	$\int x^{m+2n-1}\partial x X^{p} = S''_3$
$\int x^{m+2n-1}\partial x X^{p-1} = S''_2$	$\int x^{m+3n-1}\partial x X^{p} = S'''_3$
$\int x^{m+3n-1}\partial x X^{p-1} = S'''_2$	$\int x^{m+4n-1}\partial x X^{p} = S''''_3$
$\ldots\ldots\ldots\ldots$	$\ldots\ldots\ldots\ldots$
$\int x^{m+kn-1}\partial x X^{p-1} = S^{k\prime}_2$	$\int x^{m+kn-1}\partial x X^{p} = S^{k\prime}_3$

T a f e l IV.

eductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^n+ex^{2n}+\ldots\ldots+tx^{kn})^p$$

I.

$$m\int x^{m-1}\partial x X^p =$$

$$x^m X^p - pnbS' - 2pncS'' - 3pndS''' - 4pneS''''$$
$$- 5pnfS''''' - \text{etc.} - kpntS^{k'}.$$

II.

$$(m+kpn)t\int x^{m-1}\partial x X^p =$$

$$^{-kn}X^{p+1} - (m-kn)aS_1^{k'} - [m-(k-1)n+pn]bS_1^{(k-1)'}$$
$$-(k-2)n+2pn]cS_1^{k-2)'} - [m-(k-3)n+3pn]dS_1^{(k-3)'}$$
$$- [m-2n+(k-2)pu]sS_1'' - [m-n+(k-1)pn]tS_1'.$$

III.

$$(m+kpn)\int x^{m-1}\partial x X^p =$$

$$x^m X^p + kpnaS_2 + (k-1)pnbS_2' + (k-2)pncS_2''$$
$$+ (k-3)pndS_2''' + \text{etc.} + pnsS_2^{(k-1)'}.$$

Tafel IV.

Reductionsformeln für das Integral

$$\int x^{m-1}\partial x(a+bx^{n}+cx^{2n}+\ldots\ldots+tx^{kn})^{p}$$

IV.

$$ma\int x^{m-1}\partial x X^{p} =$$

$$x^{m}X^{p+1}-(m+n+pn)bS'_{3}-(m+2n+2pn)cS''_{3}$$
$$-(m+3n+3pn)dS'''_{3}-(m+4n+4pn)eS''''_{3}$$
$$-\text{etc.}-(m+kn+kpn)tS^{k'}_{3}.$$

V.

$$\int x^{m-1}\partial x X^{p} =$$

$$(Ax^{m}+Bx^{m+n}+Cx^{m+2n}+\text{etc.}+Tx^{m+(k-1)n})X^{p+1}+$$
$$\int(A'x^{m-1}+B'x^{m+n-1}+C'x^{m+2n-1}+\text{etc.}+T'x^{m+(k-1)k-1})X^{p+1}\partial x$$

Aus dieser Gleichung können mit Hülfe des Differentials die Coefficienten A, B, C, ... T; A', B', C', ... T', bestimmt werden. Die allgemeinen Werthe derselben lassen sich zwar angeben, aber nicht wohl in der Kürze darstellen.

Integraltafeln

für

ationale Differentiale.

Die Integrale der rationalen Differentialfunctionen können, in Hinsicht auf die Operationen, durch welche sie gefunden werden, füglich in die nachstehenden drey Classen geordnet werden:

1) Integrale von $X\partial x$, wenn die Function X entweder an sich schon eine begränzte Reihe von der Form $a + bx^i + cx^k$ dx^l + etc. ist, oder durch die Entwickelung der darin vorhandenen binomischen und polynomischen Potenzen und ihrer Producte unmittelbar darauf gebracht werden kann.

2) Integrale von $X\partial x$, wenn X eine solche Function von x ist, bey welcher dieses nicht Statt hat.

3) Integrale von $X\partial x$, wenn dieses Differential aus zwey Theilen $Y\partial x$, $Z\partial x$, zusammen gesetzt ist, von welchen der eine zur ersten, der andere zur zweiten Classe gehört.

Die dritte Classe hat nichts Eigenes, und kann daher ganz übergangen werden; denn es ist immer $\int X\partial x = \int Y\partial x + \int Z\partial x$.

Erste Classe der Integrale.

1) Wenn k die willkührliche und nach Willkühr zu bildende Constante bezeichnet, so ist, m mag positiv oder negativ seyn,

$$\int ax^m \partial x = \frac{ax^{m+1}}{m+1} + \mathrm{k}.$$

Ausgenommen hiervon ist der Fall, wo $m = -1$; denn alsdann ist

$$\int ax^{-1}\partial x = \int \frac{a\partial x}{x} = a \log x + \mathrm{k} = a \log x + a \log \mathrm{k}$$

$$= a \log \mathrm{k}x = \log \mathrm{k}^a x^a = \log \mathrm{k}x^a.$$

2) Hieraus ergiebt sich

$$\int(a+bx+cx^2+dx^3+\text{etc.})\partial x = ax+\tfrac{1}{2}bx^2+\tfrac{1}{3}cx^3+\tfrac{1}{4}dx^4+\text{etc.}$$

$$\int\left(\frac{a}{x}+\frac{b}{x^2}+\frac{c}{x^3}+\frac{d}{x^4}+\text{etc.}\right)\partial x = a\log x-\frac{b}{x}-\frac{c}{2x^2}-\frac{d}{3x^3}-\text{etc.}$$

$$\int(ax^i+bx^k+cx^l+\text{etc.})\,\partial x = \frac{ax^{i+1}}{i+1}+\frac{bx^{k+1}}{k+1}+\frac{cx^{l+1}}{l+1}+\text{etc.}$$

$$\int\left(\frac{a}{x^i}+\frac{b}{x^k}+\frac{c}{x^l}+\text{etc.}\right)\partial x = -\frac{a}{(i-1)x^{i-1}}-\frac{b}{(k-1)x^{k-1}}-\frac{c}{(l-1)x^{l-1}}-\text{etc.}$$

3) Hieher gehören auch alle Integrale von der Form $\int X^m\partial x$, $\int X^m Y^n\partial x$, $\int X^m Y^n Z^p\partial x$, etc., $\int\frac{X^m\partial x}{x^h}$, $\int\frac{X^m Y^n\partial x}{x^h}$, etc., wenn X, Y, Z, Functionen von der Form $a+bx^i+cx^k+dx^l+\text{etc.}$, und m, n, p, etc., ganze positive Zahlen sind; denn jede solche Potenz wie X^m, Y^n, Z^n, etc., läſst sich mit Hülfe des binomischen oder polynomischen Satzes in eine begränzte Reihe verwandeln, welche aus lauter Gliedern von der Form ax^m bestehet, und das Product mehrerer derselben wird daher auch aus solchen Gliedern zusammengesetzt seyn. So z. B. ist

$$\int(a+bx)^2\partial x = a^2x+abx^2+\tfrac{1}{3}b^2x^3$$

$$\int(ax+bx^2)^3\partial x = \tfrac{1}{4}a^3x^4+\tfrac{3}{5}a^2bx^5+\tfrac{1}{2}ab^2x^6+\tfrac{1}{7}b^3x^7$$

$$\int\left(ax^2+\frac{b}{x^3}\right)^2\partial x = \tfrac{1}{5}a^2x^5+2ab\log x-\frac{b^2}{5x^5}$$

$$\int\left(a+bx+\frac{c}{x}\right)^2\partial x = a^2x+abx^2+2ac\log x+\tfrac{1}{3}b^2x^3+2bcx-\frac{c^2}{x}$$

$$\int(a^2+x^2)(a+x)^2x^3\partial x = \tfrac{1}{4}a^4x^4+\tfrac{2}{5}a^3x^5+\tfrac{1}{3}a^2x^6+\tfrac{2}{7}ax^7+\tfrac{1}{8}x^8$$

$$\int\frac{(a-x)^2(a+x)\partial x}{x^h} = -\frac{a^3}{(h-1)x^{h-1}}+\frac{a}{(h-3)x^{h-3}}+\frac{a^2}{(h-2)x^{h-2}}-\frac{1}{(h-4)x^{h-4}}.$$

Da die Integrale, welche zu dieser Classe gehören, sehr leicht zu finden sind, so bedarf es dazu keiner Hülfstafeln.

Zweite Classe der Integrale.

Zu dieser Classe gehören diejenigen Integrale, welche unter der allgemeinen Form

$$\int \frac{Ax^m + Bx^{m-1} + Cx^{m-2} + Dx^{m-3} + \text{etc.}}{ax^k + bx^{k-1} + cx^{k-2} + dx^{k-3} + \text{etc.}} \partial x$$

begriffen sind; oder wenn Zähler und Nenner durch a dividirt wird, unter folgender:

$$\frac{1}{a}\int \frac{\mathfrak{A}x^m + \mathfrak{B}x^{m-1} + \mathfrak{C}x^{m-2} + \mathfrak{D}x^{m-3} + \text{etc.}}{x^k + \alpha x^{k-1} + \beta x^{k-2} + \gamma x^{k-3} + \text{etc.}} \partial x.$$

Man kann der Sache unbeschadet annehmen, dafs $m < k$ sey; denn wenn $m > k$ oder $m = k$, so wird man durch die Division den Bruchcoefficienten in zwey Theile, nämlich in eine ganze Function Y und in eine Bruchfunction $\frac{U}{V}$ auflösen können, so dafs der höchste Exponent des x im Zähler U kleiner ist, als im Nenner V, und es braucht daher, da $\int Y\partial x$ sich finden läfst, nur $\int \frac{U}{V}\partial x$ gefunden zu werden. Wie dieses mit Hülfe der im Vorhergehenden gegebenen Methoden durch die Zerlegung des Bruches $\frac{U}{V}$ in Partialbrüche geschiehet, wird in allen Lehrbüchern gezeigt. Da aber die hierzu erforderlichen Rechnungen für einzelne Integrale sehr beschwerlich sind, so hat es der Verf. für nützlich gehalten, die hier folgenden Tafeln zu verfertigen, wo man die Formeln für alle die Integrale, welche muthmafslich in der Ausübung vorkommen können, sogleich vollständig berechnet, und in der einfachsten Gestalt, deren sie fähig sind, vorfindet. Die allgemeineren Formeln, nebst den bey der successiven Berechnung gebrauchten Reductionsformeln, sind am Ende angehängt, weil sich in den Tafeln selbst kein schicklicher Platz dazu fand.

Taf. I.

$$\int \frac{x^m \partial x}{a + bx}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X} = \frac{1}{b} \log X = \log X^{\frac{1}{b}} \text{ *)}$$

$$\int \frac{x \partial x}{X} = \frac{x}{b} - \frac{a}{b^2} \log X$$

$$\int \frac{x^2 \partial x}{X} = \frac{x^2}{2b} - \frac{ax}{b^2} + \frac{a^2}{b^3} \log X$$

$$\int \frac{x^3 \partial x}{X} = \frac{x^3}{3b} - \frac{ax^2}{2b^2} + \frac{a^2 x}{b^3} - \frac{a^3}{b^4} \log X$$

$$\int \frac{x^4 \partial x}{X} = \frac{x^4}{4b} - \frac{ax^3}{3b^2} + \frac{a^2 x^2}{2b^3} - \frac{a^3 x}{b^4} + \frac{a^4}{b^5} \log X$$

$$\int \frac{x^5 \partial x}{X} = \frac{x^5}{5b} - \frac{ax^4}{4b^2} + \frac{a^2 x^3}{3b^3} - \frac{a^3 x^2}{2b^4} + \frac{a^4 x}{b^5} - \frac{a^5}{b^6} \log X$$

$$\int \frac{x^6 \partial x}{X} = \frac{x^6}{6b} - \frac{ax^5}{5b^2} + \frac{a^2 x^4}{4b^3} - \frac{a^3 x^3}{3b^4} + \frac{a^4 x^2}{2b^5} - \frac{a^5 x}{b^6} + \frac{a^6}{b^7} \log$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^7}{7b} - \frac{ax^6}{6b^2} + \frac{a^2 x^5}{5b^3} - \frac{a^3 x^4}{4b^4} + \frac{a^4 x^3}{3b^5} - \frac{a^5 x^2}{2b^6} + \frac{a}{b}$$

$$- \frac{a^7}{b^8} \log X$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^8}{8b} - \frac{ax^7}{7b^2} + \frac{a^2 x^6}{6b^3} - \frac{a^3 x^5}{5b^4} + \frac{a^4 x^4}{4b^5} - \frac{a^5 x^3}{3b^6} + \frac{a^6}{2b}$$

$$- \frac{a^7 x}{b^8} + \frac{a^8}{b^9} \log X$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^9}{9b} - \frac{ax^8}{8b^2} + \frac{a^2 x^7}{7b^3} - \frac{a^3 x^6}{6b^4} + \frac{a^4 x^5}{5b^5} - \frac{a^5 x^4}{4b^6} + \frac{a^6}{3b}$$

$$- \frac{a^7 x^2}{2b^8} + \frac{a^8 x}{b^9} - \frac{a^9}{b^{10}} \log X$$

$$\text{*)} \int \frac{\partial x}{X} = \frac{1}{b} \log X + \mathrm{k} = \frac{1}{b} \log X + \frac{1}{b} \log \mathrm{k} = \frac{1}{b} \log \mathrm{k}X$$

$$= \log \mathrm{k}^{\frac{1}{b}} X^{\frac{1}{b}} = \log \mathrm{k}X^{\frac{1}{b}}$$

$$\int \frac{x^m \partial x}{(a+bx)^2}$$ *Taf. II.*

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^2} = -\frac{1}{bX}$$

$$\int \frac{x\partial x}{X^2} = \frac{a}{b^2 X} + \frac{1}{b^2} \log X$$

$$\int \frac{x^2\partial x}{X^2} = \left(\frac{x^2}{b} - \frac{2a^2}{b^3}\right)\frac{1}{X} - \frac{2a}{b^3} \log X$$

$$\int \frac{x^3\partial x}{X^2} = \left(\frac{x^3}{2b} - \frac{3ax^2}{2b^2} + \frac{3a^3}{b^4}\right)\frac{1}{X} + \frac{3a^2}{b^4} \log X$$

$$\int \frac{x^4\partial x}{X^2} = \left(\frac{x^4}{3b} - \frac{2ax^3}{3b^2} + \frac{2a^2x^2}{b^3} - \frac{4a^4}{b^5}\right)\frac{1}{X} - \frac{4a^3}{b^5} \log X$$

$$\int \frac{x^5\partial x}{X^2} = \left(\frac{x^5}{4b} - \frac{5ax^4}{12b^2} + \frac{5a^2x^3}{6b^3} - \frac{5a^3x^2}{2b^4} + \frac{5a^5}{b^6}\right)\frac{1}{X} + \frac{5a^4}{b^6} \log X$$

$$\int \frac{x^6\partial x}{X^2} = \left(\frac{x^6}{5b} - \frac{3ax^5}{10b^2} + \frac{a^2x^4}{2b^3} - \frac{a^3x^3}{b^4} + \frac{3a^4x^2}{b^5} - \frac{6a^6}{b^7}\right)\frac{1}{X} - \frac{6a^5}{b^7} \log X$$

$$\int \frac{x^7\partial x}{X^2} = \left(\frac{x^7}{6b} - \frac{7ax^6}{30b^2} + \frac{7a^2x^5}{20b^3} - \frac{7a^3x^4}{12b^4} + \frac{7a^4x^3}{6b^5} - \frac{7a^5x^2}{2b^6} + \frac{7a^7}{b^8}\right)\frac{1}{X} + \frac{7a^6}{b^8} \log X$$

$$\int \frac{x^8\partial x}{X^2} = \left(\frac{x^8}{7b} - \frac{4ax^7}{21b^2} + \frac{4a^2x^6}{15b^3} - \frac{2a^3x^5}{5b^4} + \frac{2a^4x^4}{3b^5} - \frac{4a^5x^3}{3b^6} + \frac{4a^6x^2}{b^7} - \frac{8a^8}{b^9}\right)\frac{1}{X} - \frac{8a^7}{b^9} \log X$$

$$\int \frac{x^9\partial x}{X^2} = \left(\frac{x^9}{8b} - \frac{9ax^8}{56b^2} + \frac{3a^2x^7}{14b^3} - \frac{3a^3x^6}{10b^4} + \frac{9a^4x^5}{20b^5} - \frac{3a^5x^4}{4b^6} + \frac{3a^6x^3}{2b^7} - \frac{9a^7x^2}{2b^8} + \frac{9a^9}{b^{10}}\right)\frac{1}{X} + \frac{9a^8}{b^{10}} \log X$$

Taf. III.

$$\int \frac{x^m \partial x}{(a+bx)^3}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^3} = -\frac{1}{2bX^2}$$

$$\int \frac{x\partial x}{X^3} = -\left(\frac{x}{b} + \frac{a}{2b^2}\right)\frac{1}{X^2}$$

$$\int \frac{x^2\partial x}{X^3} = \left(\frac{2ax}{b^2} + \frac{3a^2}{2b^3}\right)\frac{1}{X^2} + \frac{1}{b^3}\log X$$

$$\int \frac{x^3\partial x}{X^3} = \left(\frac{x^3}{b} - \frac{6a^2x}{b^3} - \frac{9a^3}{2b^4}\right)\frac{1}{X^2} - \frac{3a}{b^4}\log X$$

$$\int \frac{x^4\partial x}{X^3} = \left(\frac{x^4}{2b} - \frac{2ax^3}{b^2} + \frac{12a^3x}{b^4} + \frac{9a^4}{b^5}\right)\frac{1}{X^2} + \frac{6a^2}{b^5}\log X$$

$$\int \frac{x^5\partial x}{X^3} = \left(\frac{x^5}{3b} - \frac{5ax^4}{6b^2} + \frac{10a^2x^3}{3b^3} - \frac{20a^4x}{b^5} - \frac{15a^5}{b^6}\right)\frac{1}{X^2} - \frac{10a^3}{b^6}\log X$$

$$\int \frac{x^6\partial x}{X^3} = \left(\frac{x^6}{4b} - \frac{ax^5}{2b^2} + \frac{5a^2x^4}{4b^3} - \frac{5a^3x^3}{b^4} + \frac{30a^5x}{b^6} + \frac{45a^6}{2b^7}\right)\frac{1}{X^2} + \frac{15a^4}{b^7}\log X$$

$$\int \frac{x^7\partial x}{X^3} = \left(\frac{x^7}{5b} - \frac{7ax^6}{20b^2} + \frac{7a^2x^5}{10b^3} - \frac{7a^3x^4}{4b^4} + \frac{7a^4x^3}{b^5} - \frac{42a^6x}{b^7} - \frac{63a^7}{2b^8}\right)\frac{1}{X^2} - \frac{21a^5}{b^8}\log X$$

$$\int \frac{x^8\partial x}{X^3} = \left(\frac{x^8}{6b} - \frac{4ax^7}{15b^2} + \frac{7a^2x^6}{15b^3} - \frac{14a^3x^5}{15b^4} + \frac{7a^4x^4}{3b^5} - \frac{28a^5x^3}{3b^6} + \frac{56a^7x}{b^8} + \frac{42a^8}{b^9}\right)\frac{1}{X^2} + \frac{28a^6}{b^9}\log X$$

$$\int \frac{x^9\partial x}{X^3} = \left(\frac{x^9}{7b} - \frac{3ax^8}{14b^2} + \frac{12a^2x^7}{35} - \frac{3a^3x^6}{5b^4} + \frac{6a^4x^5}{5b^5} - \frac{3a^5x^3}{b^6} + \frac{12a^6x^3}{b^7} - \frac{72a^8x}{b^9} - \frac{54a^9}{b^{10}}\right)\frac{1}{X^2} - \frac{36a^7}{b^{10}}\log X$$

$$\int \frac{x^m \partial x}{(a+bx)^4}$$ Taf. IV.

VZ. $a + bx = X$

$$\frac{\partial x}{X^4} = -\frac{1}{3bX^3}$$

$$\frac{x\partial x}{X^4} = -\left(\frac{x}{2b}+\frac{a}{6b^2}\right)\frac{1}{X^3}$$

$$\frac{x^2\partial x}{X^4} = -\left(\frac{x^2}{b}+\frac{ax}{b^2}+\frac{a^2}{3b^3}\right)\frac{1}{X^3}$$

$$\frac{x^3\partial x}{X^4} = \left(\frac{3ax^2}{b^2}+\frac{9a^2x}{2b^3}+\frac{11a^3}{6b^4}\right)\frac{1}{X^3}+\frac{1}{b^4}\log X$$

$$\frac{x^4\partial x}{X^4} = \left(\frac{x^4}{b}-\frac{12a^2x^2}{b^3}-\frac{18a^3x}{b^4}-\frac{22a^4}{3b^5}\right)\frac{1}{X^3}-\frac{4a}{b^5}\log X$$

$$\frac{x^5\partial x}{X^4} = \left(\frac{x^5}{2b}-\frac{5ax^4}{2b^2}+\frac{30a^3x^2}{b^4}+\frac{45a^4x}{b^5}+\frac{55a^5}{3b^6}\right)\frac{1}{X^3} + \frac{10a^2}{b^6}\log X$$

$$\frac{x^6\partial x}{X^4} = \left(\frac{x^6}{3b}-\frac{ax^5}{b^2}+\frac{5a^2x^4}{b^3}-\frac{60a^4x^2}{b^5}-\frac{90a^5x}{b^6}-\frac{110a^6}{3b^7}\right)\frac{1}{X^3} - \frac{20a^3}{b^7}\log X$$

$$\frac{x^7\partial x}{X^4} = \left(\frac{x^7}{4b}-\frac{7ax^6}{12b^2}+\frac{7a^2x^5}{4b^3}-\frac{35a^3x^4}{4b^4}+\frac{105a^5x^2}{b^6}+\frac{315a^6x}{2b^7} + \frac{385a^7}{6b^8}\right)\frac{1}{X^3}+\frac{35a^4}{b^8}\log X$$

$$\frac{x^8\partial x}{X^4} = \left(\frac{x^8}{5b}-\frac{2ax^7}{5b^2}+\frac{14a^2x^6}{15b^3}-\frac{14a^3x^5}{5b^4}+\frac{14a^4x^4}{b^5}-\frac{168a^6x^2}{b^7} - \frac{252a^7x}{b^8}-\frac{308a^8}{3b^9}\right)\frac{1}{X^3}-\frac{56a^5}{b^9}\log X$$

Taf. V.

$$\int \frac{x^m \partial x}{(a+bx)^5}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^5} = -\frac{1}{4bX^4}$$

$$\int \frac{x\partial x}{X^5} = -\left(\frac{x}{3b} + \frac{a}{12b^2}\right)\frac{1}{X^4}$$

$$\int \frac{x^2\partial x}{X^5} = -\left(\frac{x^2}{2b} + \frac{ax}{3b^2} + \frac{a^2}{12b^3}\right)\frac{1}{X^4}$$

$$\int \frac{x^3\partial x}{X^5} = -\left(\frac{x^3}{b} + \frac{3ax^2}{2b^2} + \frac{a^2x}{b^3} + \frac{a^3}{4b^4}\right)\frac{1}{X^4}$$

$$\int \frac{x^4\partial x}{X^5} = \left(\frac{4ax^3}{b^2} + \frac{9a^2x^2}{b^3} + \frac{22a^3x}{3b^4} + \frac{25a^4}{12b^5}\right)\frac{1}{X^4} + \frac{1}{b^5}\log X$$

$$\int \frac{x^5\partial x}{X^5} = \left(\frac{x^5}{b} - \frac{20a^2x^3}{b^3} - \frac{45a^3x^2}{b^4} - \frac{110a^4x}{3b^5} - \frac{125a^5}{12b^6}\right)\frac{1}{X^4} - \frac{5a}{b^6}\log X$$

$$\int \frac{x^6\partial x}{X^5} = \left(\frac{x^6}{2b} - \frac{3ax^5}{b^2} + \frac{60a^3x^3}{b^4} + \frac{135a^4x^2}{b^5} + \frac{110a^5x}{b^6} + \frac{125a^6}{4b^7}\right)\frac{1}{X^4} + \frac{15a^2}{b^7}\log X$$

$$\int \frac{x^7\partial x}{X^5} = \left(\frac{x^7}{3b} - \frac{7ax^6}{6b^2} + \frac{7a^2x^5}{b^3} - \frac{140a^4x^3}{b^5} - \frac{315a^5x^2}{b^6} - \frac{770a^6x}{3b^7} - \frac{875a^7}{12b^8}\right)\frac{1}{X^4} - \frac{35a^3}{b^8}\log X$$

$$\int \frac{x^8\partial x}{X^5} = \left(\frac{x^8}{4b} - \frac{2ax^7}{3b^2} + \frac{7a^2x^6}{3b^3} - \frac{14a^3x^5}{b^4} + \frac{280a^5x^3}{b^6} + \frac{630a^6x^2}{b^7} + \frac{1540a^7x}{3b^8} + \frac{875a^8}{6b^9}\right)\frac{1}{X^4} + \frac{70a^4}{b^9}\log X$$

$$\int \frac{x^m \partial x}{(a+bx)^6}$$ Taf. VI.

VZ. $a + bx = X$

$$\frac{\partial x}{X^6} = -\frac{1}{5bX^5}$$

$$\frac{\partial x}{X^6} = -\left(\frac{x}{4b} + \frac{a}{20b^2}\right)\frac{1}{X^5}$$

$$\frac{{}^{2}\partial x}{X^6} = -\left(\frac{x^2}{3b} + \frac{ax}{6b^2} + \frac{a^2}{30b^3}\right)\frac{1}{X^5}$$

$$\frac{{}^{3}\partial x}{X^6} = -\left(\frac{x^3}{2b} + \frac{ax^2}{2b^2} + \frac{a^2x}{4b^3} + \frac{a^3}{20b^4}\right)\frac{1}{X^5}$$

$$\frac{{}^{4}\partial x}{X^6} = -\left(\frac{x^4}{b} + \frac{2ax^3}{b^2} + \frac{2a^2x^2}{b^3} + \frac{a^3x}{b^4} + \frac{a^4}{5b^5}\right)\frac{1}{X^5}$$

$$\frac{{}^{5}\partial x}{X^6} = \left(\frac{5ax^4}{b^2} + \frac{15a^2x^3}{b^3} + \frac{55a^3x^2}{3b^4} + \frac{125a^4x}{12b^5} + \frac{137a^5}{60b^6}\right)\frac{1}{X^5} + \frac{1}{b^6}\log X$$

$$\frac{{}^{5}\partial x}{X^6} = \left(\frac{x^6}{b} - \frac{30a^2x^4}{b^3} - \frac{90a^3x^3}{b^4} - \frac{110a^4x^2}{b^5} - \frac{125a^5x}{2b^6} - \frac{137a^6}{10b^7}\right)\frac{1}{X^5} - \frac{6a}{b^7}\log X$$

$$\frac{{}^{7}\partial x}{X^6} = \left(\frac{x^7}{2b} - \frac{7ax^6}{2b^2} + \frac{105a^3x^4}{b^4} + \frac{315a^4x^3}{b^5} + \frac{385a^5x^2}{b^6} + \frac{875a^6x}{4b^7} + \frac{959a^7}{20b^8}\right)\frac{1}{X^5} + \frac{21a^2}{b^8}\log X$$

$$\frac{{}^{8}\partial x}{X^6} = \left(\frac{x^8}{3b} - \frac{4ax^7}{3b^2} + \frac{28a^2x^6}{3b^3} - \frac{280a^4x^4}{b^5} - \frac{840a^5x^3}{b^6} - \frac{3080a^6x^2}{3b^7} - \frac{1750a^7x}{3b^8} - \frac{1918a^8}{15b^9}\right)\frac{1}{X^5} - \frac{56a^3}{b^9}\log X$$

Taf. VII. $\int \frac{\partial x}{x^m(a+bx)}$

VZ. $a + bx = X$

$$\int \frac{\partial x}{xX} = \frac{1}{a}\log\frac{x}{X} = -\frac{1}{a}\log\frac{X}{x}\ ^{*)}$$

$$\int \frac{\partial x}{x^2X} = -\frac{1}{ax} + \frac{b}{a^2}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^3X} = -\frac{1}{2ax^2} + \frac{b}{a^2x} - \frac{b^2}{a^3}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{2a^2x^2} - \frac{b^2}{a^3x} + \frac{b^3}{a^4}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{3a^2x^3} - \frac{b^2}{2a^3x^2} + \frac{b^3}{a^4x} - \frac{b^4}{a^5}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{4a^2x^4} - \frac{b^2}{3a^3x^3} + \frac{b^3}{2a^4x^2} - \frac{b^4}{a^5x} + \frac{b^5}{a^6}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{5a^2x^5} - \frac{b^2}{4a^3x^4} + \frac{b^3}{3a^4x^3} - \frac{b^4}{2a^5x^2} + \frac{b^5}{a^6x} - \frac{b^6}{a^7}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^8X} = -\frac{1}{7ax^7} + \frac{b}{6a^2x^6} - \frac{b^2}{5a^3x^5} + \frac{b^3}{4a^4x^4} - \frac{b^4}{3a^5x^3} + \frac{b^5}{2a^6x^2} - \frac{b^6}{a^7x} + \frac{b^7}{a^8}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^9X} = -\frac{1}{8ax^8} + \frac{b}{7a^2x^7} - \frac{b^2}{6a^3x^6} + \frac{b^3}{5a^4x^5} - \frac{b^4}{4a^5x^4} + \frac{b^5}{3a^6x^3} - \frac{b^6}{2a^7x^2} + \frac{b^7}{a^8x} - \frac{b^8}{a^9}\log\frac{X}{x}$$

*) $\log\frac{x}{X} + \mathrm{k} = \log\frac{\mathrm{k}x}{X} = -\log\frac{X}{\mathrm{k}x} = -\log\frac{\mathrm{k}X}{x}$

Taf. VIII.

$$\int \frac{\partial x}{x^m (a+bx)^2}$$

VZ. $a + bx = X$

$$\frac{}{{}^2} = \frac{1}{aX} - \frac{1}{a^2} \log \frac{X}{x}$$

$$\frac{x}{X^2} = \left(-\frac{1}{ax} - \frac{2b}{a^2}\right)\frac{1}{X} + \frac{2b}{a^3} \log \frac{X}{x}$$

$$\frac{x}{X^2} = \left(-\frac{1}{2ax^2} + \frac{3b}{2a^2x} + \frac{3b^2}{a^3}\right)\frac{1}{X} - \frac{3b^2}{a^4} \log \frac{X}{x}$$

$$\frac{x}{X^2} = \left(-\frac{1}{3ax^3} + \frac{2b}{3a^2x^2} - \frac{2b^2}{a^3x} - \frac{4b^3}{a^4}\right)\frac{1}{X} + \frac{4b^3}{a^5} \log \frac{X}{x}$$

$$\frac{x}{X^2} = \left(-\frac{1}{4ax^4} + \frac{5b}{12a^2x^3} - \frac{5b^2}{6a^3x^2} + \frac{5b^3}{2a^4x} + \frac{5b^4}{a^5}\right)\frac{1}{X} - \frac{5b^4}{a^6} \log \frac{X}{x}$$

$$\frac{x}{X^2} = \left(-\frac{1}{5ax^5} + \frac{3b}{10a^2x^4} - \frac{b^2}{2a^3x^3} + \frac{b^3}{a^4x^2} - \frac{3b^4}{a^5x} - \frac{6b^5}{a^6}\right)\frac{1}{X} + \frac{6b^5}{a^7} \log \frac{X}{x}$$

$$\frac{x}{X^2} = \left(-\frac{1}{6ax^6} + \frac{7b}{30a^2x^5} - \frac{7b^2}{20a^3x^4} + \frac{7b^3}{12a^4x^3} - \frac{7b^4}{6a^5x^2} + \frac{7b^5}{2a^6x} + \frac{7b^6}{a^7}\right)\frac{1}{X} - \frac{7b^6}{a^8} \log \frac{X}{x}$$

$$\frac{x}{X^2} = \left(-\frac{1}{7ax^7} + \frac{4b}{21a^2x^6} - \frac{4b^2}{15a^3x^5} + \frac{2b^3}{5a^4x^4} - \frac{2b^4}{3a^5x^3} + \frac{4b^5}{3a^6x^2} - \frac{4b^6}{a^7x} - \frac{8b^7}{a^8}\right)\frac{1}{X} + \frac{8b^7}{a^9} \log \frac{X}{x}$$

$$\frac{x}{X^2} = \left(-\frac{1}{8ax^8} + \frac{9b}{56a^2x^7} - \frac{3b^2}{14a^3x^6} + \frac{3b^3}{10a^4x^5} - \frac{9b^4}{20a^5x^4} + \frac{3b^5}{4a^6x^3} - \frac{3b^6}{2a^7x^2} + \frac{9b^7}{2a^8x} + \frac{9b^8}{a^9}\right)\frac{1}{X} - \frac{9b^8}{a^{10}} \log \frac{X}{x}$$

Taf. IX.

$$\int \frac{\partial x}{x^m (a+bx)^3}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{x X^3} = \left(\frac{3}{2a} + \frac{bx}{a^2}\right)\frac{1}{X^2} - \frac{1}{a^3} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^2 X^3} = \left(-\frac{1}{ax} - \frac{9b}{2a^2} - \frac{3b^2 x}{a^3}\right)\frac{1}{X^2} + \frac{3b}{a^4} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^3 X^3} = \left(-\frac{1}{2ax^2} + \frac{2b}{a^2 x} + \frac{9b^2}{a^3} + \frac{6b^3 x}{a^4}\right)\frac{1}{X^2} - \frac{6b^2}{a^5} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^4 X^3} = \left(-\frac{1}{3ax^3} + \frac{5b}{6a^2 x^2} - \frac{10b^2}{3a^3 x} - \frac{15b^3}{a^4} - \frac{10b^4 x}{a^5}\right)\frac{1}{X^2}$$
$$+ \frac{10b^3}{a^6} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^5 X^3} = \left(-\frac{1}{4ax^4} + \frac{b}{2a^2 x^3} - \frac{5b^2}{4a^3 x^2} + \frac{5b^3}{a^4 x} + \frac{45b^4}{2a^5} + \frac{15b^5 x}{a^6}\right)\frac{1}{X^2}$$
$$- \frac{15b^4}{a^7} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^6 X^3} = \left(-\frac{1}{5ax^5} + \frac{7b}{20a^2 x^4} - \frac{7b^2}{10a^3 x^3} + \frac{7b^3}{4a^4 x^2} - \frac{7b^4}{a^5 x} - \frac{63b^5}{2a^6}\right.$$
$$\left. - \frac{21b^6 x}{a^7}\right)\frac{1}{X^2} + \frac{21b^5}{a^8} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^7 X^3} = \left(-\frac{1}{6ax^6} + \frac{4b}{15a^2 x^5} - \frac{7b^2}{15a^3 x^4} + \frac{14b^3}{15a^4 x^3} - \frac{7b^4}{3a^5 x^2}\right.$$
$$\left. + \frac{28b^5}{3a^6 x} + \frac{42b^6}{a^7} + \frac{28b^7 x}{a^8}\right)\frac{1}{X^2} - \frac{28b^6}{a^9} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^8 X^3} = \left(-\frac{1}{7ax^7} + \frac{5b}{14a^2 x^6} - \frac{12b^2}{55a^3 x^5} + \frac{5b^3}{5a^4 x^4} - \frac{6b^4}{5a^5 x^3}\right.$$
$$\left. + \frac{3b^5}{a^6 x^2} - \frac{12b^6}{a^7 x} - \frac{54b^7}{a^8} - \frac{56b^8 x}{a^9}\right)\frac{1}{X^2} + \frac{56b^7}{a^{10}} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^m (a+bx)^4}$$ *Taf. X.*

VZ. $a + bx = X$

$$\int \frac{\partial x}{x X^4} = \left(\frac{11}{6a} + \frac{5bx}{2a^2} + \frac{b^2x^2}{a^3}\right)\frac{1}{X^3} - \frac{1}{a^4} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^2 X^4} = \left(-\frac{1}{ax} - \frac{22b}{3a^2} - \frac{10b^2x}{a^3} - \frac{4b^3x^2}{a^4}\right)\frac{1}{X^3} + \frac{4b}{a^5} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^3 X^4} = \left(-\frac{1}{2ax^2} + \frac{5b}{2a^2x} + \frac{55b^2}{3a^3} + \frac{25b^3x}{a^4} + \frac{10b^4x^2}{a^5}\right)\frac{1}{X^3} - \frac{10b^2}{a^6} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^4 X^4} = \left(-\frac{1}{3ax^3} + \frac{b}{a^2x^2} - \frac{5b^2}{a^3x} - \frac{110b^3}{3a^4} - \frac{50b^4x}{a^5} - \frac{20b^5x^2}{a^6}\right)\frac{1}{X^3} + \frac{20b^3}{a^7} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^5 X^4} = \left(-\frac{1}{4ax^4} + \frac{7b}{12a^2x^3} - \frac{7b^2}{4a^3x^2} + \frac{35b^3}{4a^4x} + \frac{385b^4}{6a^5} + \frac{175b^5x}{2a^6} + \frac{35b^6x^2}{a^7}\right)\frac{1}{X^3} - \frac{35b^4}{a^8} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^6 X^4} = \left(-\frac{1}{5ax^5} + \frac{2b}{5a^2x^4} - \frac{14b^2}{15a^3x^3} + \frac{14b^3}{5a^4x^2} - \frac{14b^4}{a^5x} - \frac{308b^5}{3a^6} - \frac{140b^6x}{a^7} - \frac{56b^7x^2}{a^8}\right)\frac{1}{X^3} + \frac{56b^5}{a^9} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^7 X^4} = \left(-\frac{1}{6ax^6} + \frac{3b}{10a^2x^5} - \frac{3b^2}{5a^3x^4} + \frac{7b^3}{5a^4x^3} - \frac{21b^4}{5a^5x^2} + \frac{21b^5}{a^6x} + \frac{154b^6}{a^7} + \frac{210b^7x}{a^8} + \frac{84b^8x^2}{a^9}\right)\frac{1}{X^3} - \frac{84b^6}{a^{10}} \log \frac{X}{x}$$

Taf. XI.

$$\int \frac{\partial x}{x^m (a+bx)^5}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{x X^5} = \left(\frac{25}{12a} + \frac{13bx}{3a^2} + \frac{7b^2x^2}{2a^3} + \frac{b^3x^5}{a^4}\right)\frac{1}{X^4} - \frac{1}{a^5} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^2 X^5} = \left(-\frac{1}{ax} - \frac{125b}{12a^2} - \frac{65b^2x}{3a^3} - \frac{35b^3x^2}{2a^4} - \frac{5b^4x^3}{a^5}\right)\frac{1}{X^4} + \frac{5b}{a^6} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^3 X^5} = \left(-\frac{1}{2ax^2} + \frac{3b}{a^2x} + \frac{125b^2}{4a^3} + \frac{65b^3x}{a^4} + \frac{105b^4x^2}{2a^5} + \frac{15b^5x^3}{a^6}\right)\frac{1}{X^4} + \frac{15b^2}{a^7} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^4 X^5} = \left(-\frac{1}{3ax^3} + \frac{7b}{6a^2x^2} - \frac{7b^2}{a^3x} - \frac{875b^3}{12a^4} - \frac{455b^4x}{3a^5} - \frac{245b^5x^2}{2a^6} - \frac{35b^6x^3}{a^7}\right)\frac{1}{X^4} + \frac{35b^3}{a^8} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^5 X^5} = \left(-\frac{1}{4ax^4} + \frac{2b}{3a^2x^3} - \frac{7b^2}{3a^3x^2} + \frac{14b^3}{a^4x} + \frac{875b^4}{6a^5} + \frac{910b^5x}{3a^6} + \frac{245b^6x^2}{a^7} + \frac{70b^7x^3}{a^8}\right)\frac{1}{X^4} - \frac{70b^4}{a^9} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^6 X^5} = \left(-\frac{1}{5ax^5} + \frac{9b}{20a^2x^4} - \frac{6b^2}{5a^3x^3} + \frac{21b^3}{5a^4x^2} - \frac{126b^4}{5a^5x} - \frac{525b^5}{2a^6} - \frac{546b^6x}{a^7} - \frac{441b^7x^2}{a^8} - \frac{126b^8x^3}{a^9}\right)\frac{1}{X^4} + \frac{126b^5}{a^{10}} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^7 X^5} = \left(-\frac{1}{6ax^6} + \frac{b}{3a^2x^5} - \frac{3b^2}{4a^3x^4} + \frac{2b^3}{a^4x^3} - \frac{7b^4}{a^5x^2} + \frac{42b^5}{a^6x} + \frac{875b^6}{2a^7} + \frac{910b^7x}{a^8} + \frac{735b^8x^2}{a^9} + \frac{210b^9x^3}{a^{10}}\right)\frac{1}{X^4} - \frac{210b^6}{a^{11}} \log \frac{X}{x}$$

$$\int \frac{\partial x}{x^m (a+bx)^6} \qquad \textit{Taf. XII.}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{xX^6} = \left(\frac{137}{60a} + \frac{77bx}{12a^2} + \frac{47b^2x^2}{6a^3} + \frac{9b^3x^3}{2a^4} + \frac{b^4x^4}{a^5}\right)\frac{1}{X^5} - \frac{1}{a^6}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^2X^6} = \left(-\frac{1}{ax} - \frac{137b}{10a^2} - \frac{77b^2x}{2a^3} - \frac{47b^3x^2}{a^4} - \frac{27b^4x^3}{a^5} - \frac{6b^5x^4}{a^6}\right)\frac{1}{X^5} + \frac{6b}{a^7}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^3X^6} = \left(-\frac{1}{2ax^2} + \frac{7b}{2a^2x} + \frac{959b^2}{20a^3} + \frac{539b^3x}{4a^4} + \frac{329b^4x^2}{2a^5} + \frac{189b^5x^3}{a^6} + \frac{21b^6x^4}{a^7}\right)\frac{1}{X^5} - \frac{21b^2}{a^8}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^4X^6} = \left(-\frac{1}{3ax^3} + \frac{4b}{3a^2x^2} - \frac{28b^2}{3a^3x} - \frac{1918b^3}{15a^4} - \frac{1078b^4x}{3a^5} - \frac{1316b^5x^2}{3a^6} - \frac{504b^6x^3}{a^7} - \frac{56b^7x^4}{a^8}\right)\frac{1}{X^5} + \frac{56b^3}{a^9}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^5X^6} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^3} - \frac{3b^2}{a^3x^2} + \frac{21b^3}{a^4x} + \frac{2877b^4}{10a^5} + \frac{1617b^5x}{2a^6} + \frac{987b^6x^2}{a^7} + \frac{1134b^7x^3}{a^8} + \frac{126b^8x^4}{a^9}\right)\frac{1}{X^5} - \frac{126b^4}{a^{10}}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^6X^6} = \left(-\frac{1}{5ax^5} + \frac{b}{2a^2x^4} - \frac{3b^2}{2a^3x^3} + \frac{6b^3}{a^4x^2} - \frac{42b^4}{a^5x} - \frac{2877b^5}{5a^6} - \frac{1617b^6x}{a^7} - \frac{1974b^7x^2}{a^8} - \frac{2268b^8x^3}{a^9} - \frac{252b^9x^4}{a^{10}}\right)\frac{1}{X^5} + \frac{252b^5}{a^{11}}\log\frac{X}{x}$$

Taf. XIII.

$$\int \frac{\partial x}{(a+bx^2)^n}$$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X} = \int \frac{\partial x}{X}$$ [Man s. die folgende Seite.]

$$\int \frac{\partial x}{X^2} = \frac{x}{2aX} + \frac{1}{2a}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^3} = \left(\frac{1}{4aX^2} + \frac{3}{8a^2X}\right)x + \frac{3}{8a^2}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^4} = \left(\frac{1}{6aX^3} + \frac{5}{24a^2X^2} + \frac{5}{16a^3X}\right)x + \frac{5}{16a^3}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^5} = \left(\frac{1}{8aX^4} + \frac{7}{48a^2X^3} + \frac{35}{192a^3X^2} + \frac{35}{128a^4X}\right)x + \frac{35}{128a^4}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^6} = \left(\frac{1}{10aX^5} + \frac{9}{80a^2X^4} + \frac{21}{160a^3X^3} + \frac{21}{128a^4X^2} + \frac{63}{256a^5X}\right)x$$
$$+ \frac{63}{256a^5}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^7} = \left(\frac{1}{12aX^6} + \frac{11}{120a^2X^5} + \frac{33}{320a^3X^4} + \frac{77}{640a^4X^3} + \frac{77}{512a^5X^2}\right.$$
$$\left. + \frac{231}{1024a^6X}\right)x + \frac{231}{1024a^6}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^8} = \left(\frac{1}{14aX^7} + \frac{13}{168a^2X^6} + \frac{143}{1680a^3X^5} + \frac{429}{4480a^4X^4} + \frac{143}{1280a^5X^3}\right.$$
$$\left. + \frac{143}{1024a^6X^2} + \frac{429}{2048a^7X}\right)x + \frac{429}{2048a^7}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^9} = \left(\frac{1}{16aX^8} + \frac{15}{224a^2X^7} + \frac{65}{896a^3X^6} + \frac{143}{1792a^4X^5} + \frac{1287}{14336a^5X^4}\right.$$
$$\left. + \frac{429}{4096a^6X^3} + \frac{2145}{16384a^7X^2} + \frac{6435}{32768a^8X}\right)x + \frac{6435}{32768a^8}\int \frac{\partial x}{X}$$

Anmerkung zur vorhergehenden Tafel.

Es ist im Allgemeinen, a und b mögen positiv oder negativ seyn,

$$\int \frac{\partial x}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{Arc Tang}\, x\sqrt{\frac{b}{a}} = \frac{1}{2\sqrt{-ab}} \log \frac{\sqrt{a}+x\sqrt{-b}}{\sqrt{a}-x\sqrt{-b}},$$

nd von diesen beiden Ausdrücken wird jedesmal der gebraucht, welcher ı reeller Form erscheint. Hieraus erhält man

$$\int \frac{\partial x}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{Arc Tang}\, x\sqrt{\frac{b}{a}} = \frac{1}{\sqrt{ab}} \text{Arc Sin}\sqrt{\frac{bx^2}{a+bx^2}}$$

$$= \frac{1}{2\sqrt{ab}} \text{Arc Sin} \frac{2x\sqrt{ab}}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{Arc Cos}\sqrt{\frac{a}{a+bx^2}}$$

$$= \frac{1}{2\sqrt{ab}} \text{Arc Cos} \frac{a-bx^2}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{Arc Cot} \frac{\sqrt{a}}{x\sqrt{b}}$$

$$= \frac{1}{\sqrt{ab}} \text{Arc Sec}\sqrt{\frac{a+bx^2}{a}} = \frac{1}{2\sqrt{ab}} \text{Arc Sec} \frac{a+bx^2}{a-bx^2}$$

$$= \frac{1}{\sqrt{ab}} \text{Arc Cosec}\sqrt{\frac{a+bx^2}{bx^2}} = \frac{1}{2\sqrt{ab}} \text{Arc Cosec} \frac{a+bx^2}{2x\sqrt{ab}}$$

$$= \frac{1}{2\sqrt{ab}} \text{Arc Sin v.} \frac{2bx^2}{a+bx^2}$$

$$\int \frac{\partial x}{a-bx^2} = \frac{1}{2\sqrt{ab}} \log \frac{\sqrt{a}+x\sqrt{b}}{\sqrt{a}-x\sqrt{b}} = \frac{1}{\sqrt{ab}} \log \frac{\sqrt{a}+x\sqrt{b}}{\sqrt{(a-bx^2)}}$$

$$= -\frac{1}{2\sqrt{ab}} \log \frac{\sqrt{a}-x\sqrt{b}}{\sqrt{a}+x\sqrt{b}} = -\frac{1}{\sqrt{ab}} \log \frac{\sqrt{a}-x\sqrt{b}}{\sqrt{(a-bx^2)}}$$

$$\int \frac{\partial x}{-a+bx^2} = -\int \frac{\partial x}{a-bx^2}, \quad \int \frac{\partial x}{-a-bx^2} = -\int \frac{\partial x}{a+bx^2}.$$

ısbesondere ist

$$\int \frac{\partial x}{1+x^2} = \text{Arc Tang}\, x = \text{Arc Sin} \frac{x}{\sqrt{(1+x^2)}} = \tfrac{1}{2} \text{Arc Sin} \frac{2x}{1+x^2}$$

$$= \text{Arc Cos} \frac{1}{\sqrt{(1+x^2)}} = \tfrac{1}{2} \text{Arc Cos} \frac{1-x^2}{1+x^2} = \text{Arc Cot} \frac{1}{x}$$

$$= \text{Arc Sec}\sqrt{(1+x^2)} = \tfrac{1}{2} \text{Arc Sec} \frac{1+x^2}{1-x^2} = \text{Arc Cosec} \frac{\sqrt{(1+x^2)}}{x}$$

$$= \tfrac{1}{2} \text{Arc Cosec} \frac{1+x^2}{2x} = \tfrac{1}{2} \text{Arc Sin v.} \frac{2x^2}{1+x^2}$$

$$\int \frac{\partial x}{1-x^2} = \tfrac{1}{2} \log \frac{1+x}{1-x} = -\tfrac{1}{2} \log \frac{1-x}{1+x}.$$

In diesen sämmtlichen Formeln verschwindet das Integral, wenn $x = 0$. ıll es für $x = h$ verschwinden, so ist

$$\int \frac{\partial x}{a+bx^2} = \frac{1}{\sqrt{ab}} \text{ArcTang} \frac{(x-h)\sqrt{ab}}{a+bhx} = \frac{1}{\sqrt{ab}} \text{Arc Cos} \frac{a+bhx}{\sqrt{(a+bh^2)(a+bx^2)}}, \text{ etc.}$$

Taf. XIV. $\int \frac{x^m \partial x}{a + bx^2}$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X} = \int \frac{\partial x}{X} \text{ (S. 47.)}$$

$$\int \frac{x\partial x}{X} = \frac{1}{2b} \log X$$

$$\int \frac{x^2 \partial x}{X} = \frac{x}{b} - \frac{a}{b} \int \frac{\partial x}{X}$$

$$\int \frac{x^3 \partial x}{X} = \frac{x^2}{2b} - \frac{a}{b} \int \frac{x\partial x}{X}$$

$$\int \frac{x^4 \partial x}{X} = \frac{x^3}{3b} - \frac{ax}{b^2} + \frac{a^2}{b^2} \int \frac{\partial x}{X}$$

$$\int \frac{x^5 \partial x}{X} = \frac{x^4}{4b} - \frac{ax^2}{2b^2} + \frac{a^2}{b^2} \int \frac{x\partial x}{X}$$

$$\int \frac{x^6 \partial x}{X} = \frac{x^5}{5b} - \frac{ax^3}{3b^2} + \frac{a^2 x}{b^3} - \frac{a^3}{b^3} \int \frac{\partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^6}{6b} - \frac{ax^4}{4b^2} + \frac{a^2 x^2}{2b^3} - \frac{a^3}{b^3} \int \frac{x\partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^7}{7b} - \frac{ax^5}{5b^2} + \frac{a^2 x^3}{3b^3} - \frac{a^3 x}{b^4} + \frac{a^4}{b^4} \int \frac{\partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^8}{8b} - \frac{ax^6}{6b^2} + \frac{a^2 x^4}{4b^3} - \frac{a^3 x^2}{2b^4} + \frac{a^4}{b^4} \int \frac{x\partial x}{X}$$

$$\int \frac{x^{10} \partial x}{X} = \frac{x^9}{9b} - \frac{ax^7}{7b^2} + \frac{a^2 x^5}{5b^3} - \frac{a^3 x^3}{3b^4} + \frac{a^4 x}{b^5} - \frac{a^5}{b^5} \int \frac{\partial x}{X}$$

$$\int \frac{x^{11} \partial x}{X} = \frac{x^{10}}{10b} - \frac{ax^8}{8b^2} + \frac{a^2 x^6}{6b^3} - \frac{a^3 x^4}{4b^4} + \frac{a^4 x^2}{2b^5} - \frac{a^5}{b^5} \int \frac{x\partial x}{X}$$

$$\int \frac{x^{12} \partial x}{X} = \frac{x^{11}}{11b} - \frac{ax^9}{9b^2} + \frac{a^2 x^7}{7b^3} - \frac{a^3 x^5}{5b^4} + \frac{a^4 x^3}{3b^5} - \frac{a^5 x}{b^6} + \frac{a^6}{b^6} \int \frac{\partial x}{X}$$

$$\int\frac{x^m\partial x}{(a+bx^2)^2}$$

Taf. XV.

VZ. $a + bx^2 = X$

$$\frac{\partial x}{X^2} = \frac{x}{2aX} + \frac{1}{2a}\int\frac{\partial x}{X}$$

$$\frac{x\partial x}{X^2} = -\frac{1}{2bX}$$

$$\frac{x^2\partial x}{X^2} = -\frac{x}{2bX} + \frac{1}{2b}\int\frac{\partial x}{X}$$

$$\frac{x^3\partial x}{X^2} = \frac{a}{2b^2X} + \frac{1}{2b^2}\log X$$

$$\frac{x^4\partial x}{X^2} = \left(\frac{x^3}{b} + \frac{3ax}{2b^2}\right)\frac{1}{X} - \frac{3a}{2b^2}\int\frac{\partial x}{X}$$

$$\frac{x^5\partial x}{X^2} = \left(\frac{x^4}{2b} - \frac{a^2}{b^3}\right)\frac{1}{X} - \frac{a}{b^3}\log X$$

$$\frac{x^6\partial x}{X^2} = \left(\frac{x^5}{3b} - \frac{5ax^3}{3b^2} - \frac{5a^2x}{2b^3}\right)\frac{1}{X} + \frac{5a^2}{2b^3}\int\frac{\partial x}{X}$$

$$\frac{x^7\partial x}{X^2} = \left(\frac{x^6}{4b} - \frac{3ax^4}{4b^2} + \frac{3a^3}{2b^4}\right)\frac{1}{X} + \frac{3a^2}{2b^4}\log X$$

$$\frac{x^8\partial x}{X^2} = \left(\frac{x^7}{5b} - \frac{7ax^5}{15b^2} + \frac{7a^2x^3}{3b^3} + \frac{7a^3x}{2b^4}\right)\frac{1}{X} - \frac{7a^3}{2b^4}\int\frac{\partial x}{X}$$

$$\frac{x^9\partial x}{X^2} = \left(\frac{x^8}{6b} - \frac{ax^6}{3b^2} + \frac{a^2x^4}{b^3} - \frac{2a^4}{b^5}\right)\frac{1}{X} - \frac{2a^3}{b^5}\log X$$

$$\frac{x^{10}\partial x}{X^2} = \left(\frac{x^9}{7b} - \frac{9ax^7}{35b^2} + \frac{3a^2x^5}{5b^3} - \frac{3a^3x^3}{b^4} - \frac{9a^4x}{2b^5}\right)\frac{1}{X} + \frac{9a^4}{2b^5}\int\frac{\partial x}{X}$$

$$\frac{x^{11}\partial x}{X^2} = \left(\frac{x^{10}}{8b} - \frac{5ax^8}{24b^2} + \frac{5a^2x^6}{12b^3} - \frac{5a^3x^4}{4b^4} + \frac{5a^5}{2b^6}\right)\frac{1}{X} + \frac{5a^4}{2b^6}\log X$$

Taf. XVI. $$\int \frac{x^m \partial x}{(a + bx^2)^3}$$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X^3} = \left(\frac{3bx^3}{8a^2} + \frac{5x}{8a}\right)\frac{1}{X^2} + \frac{3}{8a^2}\int\frac{\partial x}{X}$$

$$\int \frac{x\partial x}{X^3} = -\frac{1}{4bX^2}$$

$$\int \frac{x^2\partial x}{X^3} = \left(\frac{x^3}{8a} - \frac{x}{8b}\right)\frac{1}{X^2} + \frac{1}{8ab}\int\frac{\partial x}{X}$$

$$\int \frac{x^3\partial x}{X^3} = \left(-\frac{x^2}{2b} - \frac{a}{4b^2}\right)\frac{1}{X^2}$$

$$\int \frac{x^4\partial x}{X^3} = \left(-\frac{5x^3}{8b} - \frac{3ax}{8b^2}\right)\frac{1}{X^2} + \frac{3}{8b^2}\int\frac{\partial x}{X}$$

$$\int \frac{x^5\partial x}{X^3} = \left(\frac{ax^2}{b^2} + \frac{3a^2}{4b^3}\right)\frac{1}{X^2} + \frac{1}{2b^3}\log X$$

$$\int \frac{x^6\partial x}{X^3} = \left(\frac{x^5}{b} + \frac{25ax^3}{8b^2} + \frac{15a^2x}{8b^3}\right)\frac{1}{X^2} - \frac{15a}{8b^3}\int\frac{\partial x}{X}$$

$$\int \frac{x^7\partial x}{X^3} = \left(\frac{x^6}{2b} - \frac{3a^2x^2}{b^3} - \frac{9a^3}{4b^4}\right)\frac{1}{X^2} - \frac{3a}{2b^4}\log X$$

$$\int \frac{x^8\partial x}{X^3} = \left(\frac{x^7}{3b} - \frac{7ax^5}{3b^2} - \frac{175a^2x^3}{24b^3} - \frac{35a^3x}{8b^4}\right)\frac{1}{X^2} + \frac{35a^2}{8b^4}\int\frac{\partial x}{X}$$

$$\int \frac{x^9\partial x}{X^3} = \left(\frac{x^8}{4b} - \frac{ax^6}{b^2} + \frac{6a^3x^2}{b^4} + \frac{9a^4}{2b^5}\right)\frac{1}{X^2} + \frac{3a^2}{b^5}\log X$$

$$\int \frac{x^{10}\partial x}{X^3} = \left(\frac{x^9}{5b} - \frac{3ax^7}{5b^2} + \frac{21a^2x^5}{5b^3} + \frac{105a^3x^3}{8b^4} + \frac{63a^4x}{8b^5}\right)\frac{1}{X^2} - \frac{63a^3}{8b^5}\int\frac{\partial x}{X}$$

$$\int \frac{x^m \partial x}{(a+bx^2)^4}$$ **Taf. XVII.**

VZ. $a + bx^2 = X$

$$\frac{\partial x}{X^4} = \left(\frac{5b^2x^5}{16a^3} + \frac{5bx^3}{6a^2} + \frac{11x}{16a}\right)\frac{1}{X^3} + \frac{5}{16a^3}\int\frac{\partial x}{X}$$

$$\frac{{}^{c}\partial x}{X^4} = -\frac{1}{6bX^3}$$

$$\frac{{}^{2}\partial x}{X^4} = \left(\frac{bx^5}{16a^2} + \frac{x^3}{6a} - \frac{x}{16b}\right)\frac{1}{X^3} + \frac{1}{16a^2b}\int\frac{\partial x}{X}$$

$$\frac{{}^{3}\partial x}{X^4} = \left(-\frac{x^2}{4b} - \frac{a}{12b^2}\right)\frac{1}{X^3}$$

$$\frac{{}^{4}\partial x}{X^4} = \left(\frac{x^5}{16a} - \frac{x^3}{6b} - \frac{ax}{16b^2}\right)\frac{1}{X^3} + \frac{1}{16ab^2}\int\frac{\partial x}{X}$$

$$\frac{{}^{5}\partial x}{X^4} = \left(-\frac{x^4}{2b} - \frac{ax^2}{2b^2} - \frac{a^2}{6b^3}\right)\frac{1}{X^3}$$

$$\frac{{}^{6}\partial x}{X^4} = \left(-\frac{11x^5}{16b} - \frac{5ax^3}{6b^2} - \frac{5a^2x}{16b^3}\right)\frac{1}{X^3} + \frac{5}{16b^3}\int\frac{\partial x}{X}$$

$$\frac{{}^{7}\partial x}{X^4} = \left(\frac{3ax^4}{2b^2} + \frac{9a^2x^2}{4b^3} + \frac{11a^3}{12b^4}\right)\frac{1}{X^3} + \frac{1}{2b^4}\log X$$

$$\frac{{}^{8}\partial x}{X^4} = \left(\frac{x^7}{b} + \frac{77ax^5}{16b^2} + \frac{35a^2x^3}{6b^3} + \frac{35a^3x}{16b^4}\right)\frac{1}{X^3} - \frac{35a}{16b^4}\int\frac{\partial x}{X}$$

$$\frac{{}^{9}\partial x}{X^4} = \left(\frac{x^8}{2b} - \frac{6a^2x^4}{b^3} - \frac{9a^3x^2}{b^4} - \frac{11a^4}{3b^5}\right)\frac{1}{X^3} - \frac{2a}{b^5}\log X$$

$$\frac{{}^{10}\partial x}{X^4} = \left(\frac{x^9}{3b} - \frac{3ax^7}{b^2} - \frac{231a^2x^5}{16b^3} - \frac{35a^3x^3}{2b^4} - \frac{105a^4x}{16b^5}\right)\frac{1}{X^3}$$
$$+ \frac{105a^2}{16b^5}\int\frac{\partial x}{X}$$

$$\frac{{}^{11}\partial x}{X^4} = \left(\frac{x^{10}}{4b} - \frac{5ax^8}{4b^2} + \frac{15a^3x^4}{b^4} + \frac{45a^4x^2}{2b^5} + \frac{55a^5}{6b^6}\right)\frac{1}{X^3}$$
$$+ \frac{5a^2}{b^6}\log X$$

Taf. XVIII. $\int\frac{x^m\partial x}{(a+bx^2)^5}$

VZ. $a+bx^2=X$

$$\int\frac{\partial x}{X^5}=\left(\frac{35b^3x^7}{128a^4}+\frac{385b^2x^5}{384a^3}+\frac{511bx^3}{384a^2}+\frac{93x}{128a}\right)\frac{1}{X^4}+\frac{35}{128a^4}\int\frac{\partial x}{X}$$

$$\int\frac{x\partial x}{X^5}=-\frac{1}{8bX^4}$$

$$\int\frac{x^2\partial x}{X^5}=\left(\frac{5b^2x^7}{128a^3}+\frac{55bx^5}{384a^2}+\frac{73x^3}{384a}-\frac{5x}{128b}\right)\frac{1}{X^4}+\frac{5}{128a^3b}\int\frac{\partial x}{X}$$

$$\int\frac{x^3\partial x}{X^5}=\left(-\frac{x^2}{6b}-\frac{a}{24b^2}\right)\frac{1}{X^4}$$

$$\int\frac{x^4\partial x}{X^5}=\left(\frac{3bx^7}{128a^2}+\frac{11x^5}{128a}-\frac{11x^3}{128b}-\frac{3ax}{128b^2}\right)\frac{1}{X^4}+\frac{3}{128a^2b^2}\int\frac{\partial x}{X}$$

$$\int\frac{x^5\partial x}{X^5}=\left(-\frac{x^4}{4b}-\frac{ax^2}{6b^2}-\frac{a^2}{24b^3}\right)\frac{1}{X^4}$$

$$\int\frac{x^6\partial x}{X^5}=\left(\frac{5x^7}{128a}-\frac{73x^5}{384b}-\frac{55ax^3}{384b}-\frac{5a^2x}{128b^3}\right)\frac{1}{X^4}+\frac{5}{128ab^3}\int\frac{\partial x}{X}$$

$$\int\frac{x^7\partial x}{X^5}=\left(-\frac{x^6}{2b}-\frac{3ax^4}{4b^2}-\frac{a^2x^2}{2b^3}-\frac{a^3}{8b^4}\right)\frac{1}{X^4}$$

$$\int\frac{x^8\partial x}{X^5}=\left(-\frac{93x^7}{128b}-\frac{511ax^5}{384b^2}-\frac{385a^2x^3}{384b^3}-\frac{35a^3x}{128b^4}\right)\frac{1}{X^4}+\frac{35}{128b^4}\int\frac{\partial x}{X}$$

$$\int\frac{x^9\partial x}{X^5}=\left(\frac{2ax^6}{b^2}+\frac{9a^2x^4}{2b^3}+\frac{11a^3x^2}{3b^4}+\frac{25a^4}{24b^5}\right)\frac{1}{X^4}+\frac{1}{2b^5}\log X$$

$$\int\frac{x^{10}\partial x}{X^5}=\left(\frac{x^9}{b}+\frac{837ax^7}{128b^2}+\frac{1533a^2x^5}{128b^3}+\frac{1155a^3x^3}{128b^4}+\frac{315a^4x}{128b^5}\right)\frac{1}{X^4}$$
$$-\frac{315a}{128b^5}\int\frac{\partial x}{X}$$

$$\int\frac{x^{11}\partial x}{X^5}=\left(\frac{x^{10}}{2b}-\frac{10a^2x^6}{b^3}-\frac{45a^3x^4}{2b^4}-\frac{55a^4x^2}{3b^5}-\frac{125a^5}{24b^6}\right)\frac{1}{X^4}$$
$$-\frac{5a}{2b^6}\log X$$

$$\int \frac{x^m \partial x}{(a+bx^2)^6}$$ Taf. XIX.

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X^6} = \left(\frac{63b^4x^9}{256a^5} + \frac{147b^3x^7}{128a^4} + \frac{21b^2x^5}{10a^3} + \frac{237bx^3}{128a^2} + \frac{193x}{256a}\right)\frac{1}{X^5} + \frac{63}{256a^5}\int \frac{\partial x}{X}$$

$$\int \frac{x\partial x}{X^6} = -\frac{1}{10bX^5}$$

$$\int \frac{x^2\partial x}{X^6} = \left(\frac{7b^3x^9}{256a^4} + \frac{49b^2x^7}{384a^3} + \frac{7bx^5}{30a^2} + \frac{79x^3}{384a} - \frac{7x}{256b}\right)\frac{1}{X^5} + \frac{7}{256a^4b}\int \frac{\partial x}{X}$$

$$\int \frac{x^3\partial x}{X^6} = \left(-\frac{x^2}{8b} - \frac{a}{40b^2}\right)\frac{1}{X^5}$$

$$\int \frac{x^4\partial x}{X^6} = \left(\frac{3b^2x^9}{256a^3} + \frac{7bx^7}{128a^2} + \frac{x^5}{10a} - \frac{7x^3}{128b} - \frac{3ax}{256b^2}\right)\frac{1}{X^5} + \frac{3}{256a^3b^2}\int \frac{\partial x}{X}$$

$$\int \frac{x^5\partial x}{X^6} = \left(-\frac{x^4}{6b} - \frac{ax^2}{12b^2} - \frac{a^2}{60b^3}\right)\frac{1}{X^5}$$

$$\int \frac{x^6\partial x}{X^6} = \left(\frac{3bx^9}{256a^2} + \frac{7x^7}{128a} - \frac{x^5}{10b} - \frac{7ax^3}{128b^2} - \frac{3a^2x}{256b^3}\right)\frac{1}{X^5} + \frac{3}{256a^2b^3}\int \frac{\partial x}{X}$$

$$\int \frac{x^7\partial x}{X^6} = \left(-\frac{x^6}{4b} - \frac{ax^4}{4b^2} - \frac{a^2x^2}{8b^3} - \frac{a^3}{40b^4}\right)\frac{1}{X^5}$$

$$\int \frac{x^8\partial x}{X^6} = \left(\frac{7x^9}{256a} - \frac{79x^7}{384b} - \frac{7ax^5}{30b^2} - \frac{49a^2x^3}{384b^3} - \frac{7a^3x}{256b^4}\right)\frac{1}{X^5} + \frac{7}{256ab^4}\int \frac{\partial x}{X}$$

$$\int \frac{x^9\partial x}{X^6} = \left(-\frac{x^8}{2b} - \frac{ax^6}{b^2} - \frac{a^2x^4}{b^3} - \frac{a^3x^2}{2b^4} - \frac{a^4}{10b^5}\right)\frac{1}{X^5}$$

Taf. XX.

$$\int \frac{\partial x}{x^m(a+bx^2)}$$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{xX} = \frac{1}{2a}\log\frac{x^2}{X} = \frac{1}{a}\log\frac{x}{\sqrt{X}} = -\frac{1}{2a}\log\frac{X}{x^2} = -\frac{1}{a}\log\frac{\sqrt{X}}{x}$$

$$\int \frac{\partial x}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{a^2x} + \frac{b^2}{a^2}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \frac{b^2}{a^2}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \frac{b^2}{a^3x} - \frac{b^3}{a^3}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{4a^2x^4} - \frac{b^2}{2a^3x^2} - \frac{b^3}{a^3}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X} = -\frac{1}{7ax^7} + \frac{b}{5a^2x^5} - \frac{b^2}{3a^3x^3} + \frac{b^3}{a^4x} + \frac{b^4}{a^4}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9X} = -\frac{1}{8ax^8} + \frac{b}{6a^2x^6} - \frac{b^2}{4a^3x^4} + \frac{b^3}{2a^4x^2} + \frac{b^4}{a^4}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{10}X} = -\frac{1}{9ax^9} + \frac{b}{7a^2x^7} - \frac{b^2}{5a^3x^5} + \frac{b^3}{3a^4x^3} - \frac{b^4}{a^5x} - \frac{b^5}{a^5}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^{11}X} = -\frac{1}{10ax^{10}} + \frac{b}{8a^2x^8} - \frac{b^2}{6a^3x^6} + \frac{b^3}{4a^4x^4} - \frac{b^4}{2a^5x^2} - \frac{b^5}{a^5}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{12}X} = -\frac{1}{11ax^{11}} + \frac{b}{9ax^9} - \frac{b^2}{7a^3x^7} + \frac{b^3}{5a^4x^5} - \frac{b^4}{3a^5x^3} + \frac{b^5}{a^6x} + \frac{b^6}{a^6}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^m(a+bx^2)^2}$$ Taf. XXI.

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{xX^2} = \frac{1}{2aX} + \frac{1}{a}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{3bx}{2a^2}\right)\frac{1}{X} - \frac{3b}{2a^2}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{b}{a^2}\right)\frac{1}{X} - \frac{2b}{a^2}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} + \frac{5b}{3a^2x} + \frac{5b^2x}{2a^3}\right)\frac{1}{X} + \frac{5b^2}{2a^3}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{3b}{4a^2x^2} + \frac{3b^2}{2a^3}\right)\frac{1}{X} + \frac{3b^2}{a^3}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{7b}{15a^2x^3} - \frac{7b^2}{3a^3x} - \frac{7b^3x}{2a^4}\right)\frac{1}{X} - \frac{7b^3}{2a^4}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{3a^2x^4} - \frac{b^2}{a^3x^2} - \frac{2b^3}{a^4}\right)\frac{1}{X} - \frac{4b^3}{a^4}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^2} = \left(-\frac{1}{7ax^7} + \frac{9b}{35a^2x^5} - \frac{3b^2}{5a^3x^3} + \frac{3b^3}{a^4x} + \frac{9b^4x}{2a^5}\right)\frac{1}{X}$$
$$+ \frac{9b^4}{2a^5}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9X^2} = \left(-\frac{1}{8ax^3} + \frac{5b}{24a^2x^6} - \frac{5b^2}{12a^3x^4} + \frac{5b^3}{4a^4x^2} + \frac{5b^3}{2a^5}\right)\frac{1}{X}$$
$$+ \frac{5b^4}{a^5}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{10}X^2} = \left(-\frac{1}{9ax^9} + \frac{11b}{63a\ x^7} - \frac{11b^2}{35a^3x^6} + \frac{11b^3}{15a^4x^3} - \frac{11b^4}{3a^5x}\right.$$
$$\left. - \frac{11b^5x}{2a^6}\right)\frac{1}{X} - \frac{11b^5}{2a^6}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^1\ X^2} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{20a^2x^8} - \frac{b^2}{4a^3x^6} + \frac{b^3}{2a^4x^4} - \frac{3b^4}{2a^5x^2}\right.$$
$$\left. - \frac{3b^5}{a^6}\right)\frac{1}{X} - \frac{6b^5}{a^6}\int \frac{\partial x}{xX}$$

Taf. XXII. $$\int \frac{\partial x}{x^m(a+bx^2)^3}$$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{xX^3} = \left(\frac{3}{4a} + \frac{bx^2}{2a^2}\right)\frac{1}{X^2} + \frac{1}{a^2}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^3} = \left(-\frac{1}{ax} - \frac{25bx}{8a^2} - \frac{15b^2x^3}{8a^3}\right)\frac{1}{X^2} - \frac{15b}{8a^3}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^3} = \left(-\frac{1}{2ax^2} - \frac{9b}{4a^2} - \frac{3b^2x^2}{2a^3}\right)\frac{1}{X^2} - \frac{3b}{a^3}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^3} = \left(-\frac{1}{3ax^3} + \frac{7b}{3a^2x} + \frac{175b^2x}{24a^3} + \frac{35b^3x^3}{8a^4}\right)\frac{1}{X^2} + \frac{35b^2}{8a^4}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^3} = \left(-\frac{1}{4ax^4} + \frac{b}{a^2x^2} + \frac{9b^2}{2a^3} + \frac{3b^3x^2}{a^4}\right)\frac{1}{X^2} + \frac{6b^2}{a^4}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^3} = \left(-\frac{1}{5ax^4} + \frac{3b}{5a^2x^3} - \frac{21b^2}{5a^3x} - \frac{105b^3x}{8a^4} - \frac{63b^4x^3}{8a^5}\right)\frac{1}{X^2}$$
$$- \frac{63b^3}{8a^5}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^3} = \left(-\frac{1}{6ax^6} + \frac{5b}{12a^2x^4} - \frac{5b^2}{3a^3x^2} - \frac{15b^3}{2a^4} - \frac{5b^4x^2}{a^5}\right)\frac{1}{X^2}$$
$$- \frac{10b^3}{a^5}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^3} = \left(-\frac{1}{7ax^7} + \frac{11b}{35a^2x^5} - \frac{33b^2}{35a^3x^3} + \frac{33b^3}{5a^4x} + \frac{165b^4x}{8a^5}\right.$$
$$\left. + \frac{99b^5x^3}{8a^6}\right)\frac{1}{X^2} + \frac{99b^4}{8a^6}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9X^3} = \left(-\frac{1}{8ax^8} + \frac{b}{4a^2x^6} - \frac{5b^2}{8a^3x^4} + \frac{5b^3}{2a^4x^2} + \frac{45b^4}{4a^5}\right.$$
$$\left. + \frac{15b^5x^2}{2a^6}\right)\frac{1}{X^2} + \frac{15b^4}{a^6}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^{10}X^3} = \left(-\frac{1}{9ax^9} + \frac{13b}{63a^2x^7} - \frac{143b^2}{315a^3x^5} + \frac{143b^3}{105a^4x^3} - \frac{143b^4}{15a^5x}\right.$$
$$\left. - \frac{715b^5x}{24a^6} - \frac{143b^6x^3}{8a^7}\right)\frac{1}{X^2} - \frac{143b^5}{8a^7}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^m(a+bx^2)^4}$$ *Taf. XXIII.*

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{xX^4} = \left(\frac{11}{12a} + \frac{5bx^2}{4a^2} + \frac{b^2x^4}{2a^3}\right)\frac{1}{X^3} + \frac{1}{a^3}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^4} = \left(-\frac{1}{ax} - \frac{77bx}{16a^2} - \frac{35b^2x^3}{6a^3} - \frac{35b^3x^5}{16a^4}\right)\frac{1}{X^3} - \frac{35b}{16a^4}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^4} = \left(-\frac{1}{2ax^2} - \frac{11b}{3a^2} - \frac{5b^2x^2}{a^3} - \frac{2b^3x^4}{a^4}\right)\frac{1}{X^3} - \frac{4b}{a^4}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^4} = \left(-\frac{1}{3ax^3} + \frac{3b}{a^2x} + \frac{231b^2x}{16a^3} + \frac{35b^3x^3}{2a^4} + \frac{105b^4x^5}{16a^5}\right)\frac{1}{X^3}$$
$$+ \frac{105b^2}{16a^5}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^4} = \left(-\frac{1}{4ax^4} + \frac{5b}{4a^2x^2} + \frac{55b^2}{6a^3} + \frac{25b^3x^2}{2a^4} + \frac{5b^4x^4}{a^5}\right)\frac{1}{X^3}$$
$$+ \frac{10b^2}{a^5}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^4} = \left(-\frac{1}{5ax^5} + \frac{11b}{15a^2x^3} - \frac{33b^2}{5a^3x} - \frac{2541b^3x}{80a^4} - \frac{77b^4x^3}{2a^5}\right.$$
$$\left. - \frac{231b^5x^5}{16a^6}\right)\frac{1}{X^3} - \frac{231b^3}{16a^6}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^4} = \left(-\frac{1}{6ax^6} + \frac{b}{2a^2x^4} - \frac{5b^2}{2a^3x^2} - \frac{55b^3}{3a^4} - \frac{25b^4x^2}{a^5}\right.$$
$$\left. - \frac{10b^5x^4}{a^6}\right)\frac{1}{X^3} - \frac{20b^2}{a^6}\int \frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^4} = \left(-\frac{1}{7ax^7} + \frac{13b}{35a^2x^5} - \frac{143b^2}{105a^3x^3} + \frac{429b^3}{35a^4x} + \frac{4719b^4x}{80a^5}\right.$$
$$\left. + \frac{143b^5x^3}{2a^6} + \frac{429b^6x^5}{16a^7}\right)\frac{1}{X^3} + \frac{429b^4}{16a^7}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^9X^4} = \left(-\frac{1}{8ax^8} + \frac{7b}{24a^2x^6} - \frac{7b^2}{8a^3x^4} + \frac{35b^3}{8a^4x^2} + \frac{385b^4}{12a^5}\right.$$
$$\left. + \frac{175b^5x^2}{4a^6} + \frac{35b^6x^4}{2a^7}\right)\frac{1}{X^3} + \frac{35b^3}{a^7}\int \frac{\partial x}{xX}$$

Taf. XXIV.

$$\int \frac{\partial x}{x^m (a + bx^2)^5}$$

$$\text{VZ.} \quad a + bx^2 = X$$

$$\int \frac{\partial x}{xX^5} = \left(\frac{25}{24a} + \frac{13bx^2}{6a^2} + \frac{7b^2x^4}{4a^3} + \frac{b^3x^6}{2a^4}\right)\frac{1}{X^4} + \frac{1}{a^4}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^2X^5} = \left(-\frac{1}{ax} - \frac{837bx}{128a^2} - \frac{1533b^2x^3}{128a^3} - \frac{1155b^3x^5}{128a^4}\right.$$

$$\left. - \frac{315b^4x^7}{128a^5}\right)\frac{1}{X^4} - \frac{315b}{128a^5}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^5} = \left(-\frac{1}{2ax^2} - \frac{125b}{24a^2} - \frac{65b^2x^2}{6a^3} - \frac{35b^3x^4}{4a^4} - \frac{5b^4x^6}{2a^5}\right)\frac{1}{X^4}$$

$$- \frac{5b}{a^5}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^4X^5} = \left(-\frac{1}{3ax^3} + \frac{11b}{3a^2x} + \frac{3069b^2x}{128a^3} + \frac{5621b^3x^3}{128a^4} + \frac{4235b^4x^5}{128a^5}\right.$$

$$\left. + \frac{1155b^5x^7}{128a^6}\right)\frac{1}{X^4} + \frac{1155b^2}{128a^6}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^5X^5} = \left(-\frac{1}{4ax^4} + \frac{3b}{2a^2x^2} + \frac{125b^2}{8a^3} + \frac{65b^3x^2}{2a^4} + \frac{105b^4x^4}{4a^5}\right.$$

$$\left. + \frac{15b^5x^6}{2a^6}\right)\frac{1}{X^4} + \frac{15b^2}{a^6}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^6X^5} = \left(-\frac{1}{5ax^5} + \frac{13b}{15a^2x^3} - \frac{143b^2}{15a^3x} - \frac{39897b^3x}{640a^4} - \frac{73073b^4x^3}{640a^5}\right.$$

$$\left. - \frac{11011b^5x^5}{128a^6} - \frac{3003b^6x^7}{128a^7}\right)\frac{1}{X^4} - \frac{3003b^3}{128a^7}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^5} = \left(-\frac{1}{6ax^6} + \frac{7b}{12a^2x^4} - \frac{7b^2}{2a^3x^2} - \frac{875b^3}{24a^4} - \frac{455b^4x^2}{6a^5}\right.$$

$$\left. - \frac{245b^5x^4}{4a^6} - \frac{35b^6x^6}{2a^7}\right)\frac{1}{X^4} - \frac{35b^3}{a^7}\int\frac{\partial x}{xX}$$

$$\int \frac{\partial x}{x^8X^5} = \left(-\frac{1}{7ax^7} + \frac{3b}{7a^2x^5} - \frac{13b^2}{7a^3x^3} + \frac{143b^3}{7a^4x} + \frac{119691b^4x}{896a^5}\right.$$

$$\left. + \frac{31317b^5x^3}{128a^6} + \frac{23595b^6x^5}{128a^7} + \frac{6435b^7x^7}{128a^8}\right)\frac{1}{X^4} + \frac{6435b^4}{128a^8}\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^m(a+bx^2)^6}.$$ *Taf. XXV.*

VZ. $a+bx^2=X$

$$\int\frac{\partial x}{xX^6}=\left(\frac{137}{120a}+\frac{77bx^2}{24a^2}+\frac{47b^2x^4}{12a^3}+\frac{9b^3x^6}{4a^4}+\frac{b^4x^8}{2a^5}\right)\frac{1}{X^5}+\frac{1}{a^5}\int\frac{\partial x}{xX}$$

$$\int\frac{\partial x}{x^2X^6}=\left(-\frac{1}{ax}-\frac{2123bx}{256a^2}-\frac{2607b^2x^3}{128a^3}-\frac{231b^3x^5}{10a^4}-\frac{1617b^4x^7}{128a^5}-\frac{693b^5x^9}{256a^6}\right)\frac{1}{X^5}-\frac{693b}{256a^6}\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^3X^6}=\left(-\frac{1}{2ax^2}-\frac{137b}{20a^2}-\frac{77b^2x^2}{4a^3}-\frac{47b^3x^4}{2a^4}-\frac{27b^4x^6}{2a^5}-\frac{3b^5x^8}{a^6}\right)\frac{1}{X^5}-\frac{6b}{a^6}\int\frac{\partial x}{xX}$$

$$\int\frac{\partial x}{x^4X^6}=\left(-\frac{1}{3ax^3}+\frac{13b}{3a^2x}+\frac{27599b^2x}{768a^3}+\frac{11297b^3x^3}{128a^4}+\frac{1001b^4x^5}{10a^5}+\frac{7007b^5x^7}{128a^6}+\frac{3003b^6x^9}{256a^7}\right)\frac{1}{X^5}+\frac{3003b^2}{256a^7}\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^5X^6}=\left(-\frac{1}{4ax^4}+\frac{7b}{4a^2x^2}+\frac{959b^2}{40a^3}+\frac{539b^3x^2}{8a^4}+\frac{329b^4x^4}{4a^5}+\frac{189b^5x^6}{4a^6}+\frac{21b^6x^8}{2a^7}\right)\frac{1}{X^5}+\frac{21b^2}{a^7}\int\frac{\partial x}{xX}$$

$$\int\frac{\partial x}{x^6X^6}=\left(-\frac{1}{5ax^5}+\frac{b}{a^2x^3}-\frac{13b^2}{a^3x}-\frac{27599b^3x}{256a^4}-\frac{33891b^4x^3}{128a^5}-\frac{3003b^5x^5}{10a^6}-\frac{21021b^6x^7}{128a^7}-\frac{9009b^7x^9}{256a^8}\right)\frac{1}{X^5}-\frac{9009b^3}{256a^8}\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^7X^6}=\left(-\frac{1}{6ax^6}+\frac{2b}{3a^2x^4}-\frac{14b^2}{3a^3x^2}-\frac{959b^3}{15a^4}-\frac{539b^4x^2}{3a^5}-\frac{658b^5x^4}{3a^6}-\frac{126b^6x^6}{a^7}-\frac{28b^7x^8}{a^8}\right)\frac{1}{X^5}-\frac{56b^3}{a^8}\int\frac{\partial x}{xX}$$

$$\int\frac{\partial x}{x^8X^6}=\left(-\frac{1}{7ax^7}+\frac{17b}{35a^2x^5}-\frac{17b^2}{7a^3x^3}+\frac{221b^3}{7a^4x}+\frac{469183b^4x}{1792a^5}+\frac{576147b^5x^3}{896a^6}+\frac{7293b^6x^5}{10a^7}+\frac{51051b^7x^8}{128a^8}+\frac{21879b^8x^9}{256a^9}\right)\frac{1}{X^5}+\frac{21879b^4}{256a^9}\int\frac{\partial x}{X}$$

Taf. XXVI. $\int \frac{\partial x}{(a+bx+cx^2)^n}$

VZ. $a+bx+cx^2=X,\quad 4ac-b^2=k$

$$\int \frac{\partial x}{X} = \int \frac{\partial x}{X}$$ (Man s. die folgende Seite.)

$$\int \frac{\partial x}{X^2} = \frac{2cx+b}{kX} + \frac{2c}{k}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^3} = \left(\frac{1}{2kX^2} + \frac{3c}{k^2X}\right)(2cx+b) + \frac{6c^2}{k^2}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^4} = \left(\frac{1}{3kX^3} + \frac{5c}{3k^2X^2} + \frac{10c^2}{k^3X}\right)(2cx+b) + \frac{20c^3}{k^3}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^5} = \left(\frac{1}{4kX^4} + \frac{7c}{6k^2X^3} + \frac{35c^2}{6k^3X^2} + \frac{35c^3}{k^4X}\right)(2cx+b) + \frac{70c^4}{k^4}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^6} = \left(\frac{1}{5kX^5} + \frac{9c}{10k^2X^4} + \frac{21c^2}{5k^3X^3} + \frac{21c^3}{k^4X^2} + \frac{126c^4}{k^5X}\right)(2cx+b) + \frac{252c^5}{k^5}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^7} = \left(\frac{1}{6kX^6} + \frac{11c}{15k^2X^5} + \frac{33c^2}{10k^3X^4} + \frac{77c^3}{5k^4X^3} + \frac{77c^4}{k^5X^2} + \frac{462c^5}{k^6X}\right)\times (2cx+b) + \frac{924c^6}{k^6}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^8} = \left(\frac{1}{7kX^7} + \frac{13c}{21k^2X^6} + \frac{286c^2}{105k^3X^5} + \frac{858c^3}{70k^4X^4} + \frac{286c^4}{5k^5X^3} + \frac{286c^5}{k^6X^2} + \frac{1716c^6}{k^7X}\right)(2cx+b) + \frac{3432c^7}{k^7}\int \frac{\partial x}{X}$$

$$\int \frac{\partial x}{X^9} = \left(\frac{1}{8kX^8} + \frac{15c}{28k^2X^7} + \frac{65c^2}{28k^3X^6} + \frac{143c^3}{14k^4X^5} + \frac{1287c^4}{28k^5X^4} + \frac{429c^5}{2k^6X^3} + \frac{2145c^6}{2k^7X^2} + \frac{6435c^7}{k^8X}\right)(2cx+b) + \frac{12870c^8}{k^8}\int \frac{\partial x}{X}$$

Anmerkung zur vorhergehenden Tafel.

Es ist im Allgemeinen, wenn X seine Bedeutung auf der vorigen Seite hält,

$$\int\frac{\partial x}{X}=\frac{2}{\sqrt{(4ac-b^2)}}\,\text{Arc Tang}\,\frac{2cx+b}{\sqrt{(4ac-b^2)}}$$
$$=\frac{1}{\sqrt{(b^2-4ac)}}\log\frac{2cx+b-\sqrt{(b^2-4ac)}}{2cx+b+\sqrt{(b^2-4ac)}}.$$

ie erste Form wird reell, wenn $4ac-b^2$ positiv, die zweite wird es, enn $4ac-b^2$ negativ ist. Hieraus ergeben sich zwey Fälle:

I. $4ac-b^2$ *positiv.* (VZ. $4ac-b^2=k$.)

$$\int\frac{\partial x}{X}=\frac{2}{\sqrt{k}}\,\text{Arc Tang}\,\frac{2cx+b}{\sqrt{k}}=\frac{2}{\sqrt{k}}\,\text{Arc Cot}\,\frac{\sqrt{k}}{2cx+b}=\frac{2}{\sqrt{k}}\,\text{Arc Sec}\,\frac{2\sqrt{cX}}{\sqrt{k}}$$
$$=\frac{2}{\sqrt{k}}\,\text{Arc Cosce}\,\frac{2\sqrt{cX}}{2cx+b}=\frac{2}{\sqrt{k}}\,\text{Arc Cos}\,\frac{\sqrt{k}}{2\sqrt{cX}}=\frac{2}{\sqrt{k}}\,\text{Arc Sin}\,\frac{2cx+b}{2\sqrt{cX}}$$
$$=\frac{1}{\sqrt{k}}\,\text{Arc Sin}\,\frac{(2cx+b)\sqrt{k}}{2cX}=\frac{1}{\sqrt{k}}\,\text{Arc Cos}\left(\frac{k}{2cX}-1\right)$$
$$=\frac{1}{\sqrt{k}}\,\text{Arc Sin vers}\,\frac{(2cx+b)^2}{2cX}.$$

ıd wenn $\int\frac{\partial x}{X}$ für $x=0$ verschwinden soll,

$$\int\frac{\partial x}{X}=\frac{2}{\sqrt{k}}\,\text{Arc Tang}\,\frac{x\sqrt{k}}{2a+bx}=\frac{2}{\sqrt{k}}\,\text{Arc Cot}\,\frac{2a+bx}{x\sqrt{k}}=\frac{2}{\sqrt{k}}\,\text{Arc Sec}\,\frac{2\sqrt{aX}}{2a+bx}$$
$$=\frac{2}{\sqrt{k}}\,\text{Arc Cosec}\,\frac{2\sqrt{aX}}{x\sqrt{k}}=\frac{2}{\sqrt{k}}\,\text{Arc Sin}\,\frac{x\sqrt{k}}{2\sqrt{aX}}=\frac{2}{\sqrt{k}}\,\text{Arc Cos}\,\frac{2a+bx}{2\sqrt{aX}}$$
$$=\frac{1}{\sqrt{k}}\,\text{Arc Sin}\,\frac{(2ax+bx^2)\sqrt{k}}{2aX}=\frac{1}{\sqrt{k}}\,\text{Arc Sin vers}\,\frac{kx^2}{2aX}.$$

II. $4ac-b^2$ *negativ.* (VZ. $b^2-4ac=k'$.)

$$\int\frac{\partial x}{X}=\frac{1}{\sqrt{k'}}\log\frac{2cx+b-\sqrt{k'}}{2cx+b+\sqrt{k'}}=\frac{2}{\sqrt{k'}}\log\frac{2cx+b-\sqrt{k'}}{2\sqrt{cX}}$$

d wenn das Integral für $x=0$ verschwinden soll,

$$\int\frac{\partial x}{X}=\frac{1}{\sqrt{k'}}\log\frac{(b+\sqrt{k'})(2cx+b-\sqrt{k'})}{(b-\sqrt{k'})(2cx+b+\sqrt{k'})}.$$

In beiden Arten von Integralen kann $\sqrt{k}$ und $\sqrt{k'}$ sowohl positiv als gativ angenommen werden.

Taf. XXVII. $\int \frac{x^m \partial x}{a + bx + cx^2}$

VZ. $a + bx + cx^2 = X$

$$\int \frac{\partial x}{X} = \int \frac{\partial x}{X}$$ [Man s. die vorhergehende Seite.]

$$\int \frac{x \partial x}{X} = \frac{1}{2c} \log X - \frac{b}{2c} \int \frac{\partial x}{X}$$

$$\int \frac{x^2 \partial x}{X} = \frac{x}{c} - \frac{b}{2c^2} \log X + \left(\frac{b^2}{2c^2} - \frac{a}{c}\right) \int \frac{\partial x}{X}$$

$$\int \frac{x^3 \partial x}{X} = \frac{x^2}{2c} - \frac{bx}{c^2} + \left(\frac{b^2}{2c^3} - \frac{a}{2c^2}\right) \log X - \left(\frac{b^3}{2c^3} - \frac{3ab}{2c^2}\right) \int \frac{\partial x}{X}$$

$$\int \frac{x^4 \partial x}{X} = \frac{x^3}{3c} - \frac{bx^2}{2c^2} + \left(\frac{b^2}{c^3} - \frac{a}{c^2}\right) x - \left(\frac{b^3}{2c^4} - \frac{ab}{c^3}\right) \log X + \left(\frac{b^4}{2c^4} - \frac{2ab^2}{c^3} + \frac{a^2}{c^2}\right) \int \frac{\partial x}{X}$$

$$\int \frac{x^5 \partial x}{X} = \frac{x^4}{4c} - \frac{b}{c} \int \frac{x^4 \partial x}{X} - \frac{a}{c} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^6 \partial x}{X} = \frac{x^5}{5c} - \frac{bx^4}{4c^2} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right) \int \frac{x^4 \partial x}{X} + \frac{ab}{c^2} \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^6}{6c} - \frac{bx^5}{5c^2} + \left(\frac{b^2}{4c^3} - \frac{a}{4c^2}\right) x^4 - \left(\frac{b^2}{c^3} - \frac{2ab}{c^2}\right) \int \frac{x^4 \partial x}{X} - \left(\frac{ab^2}{c^3} - \frac{a^2}{c^2}\right) \int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^7}{7c} - \frac{b}{c} \int \frac{x^7 \partial x}{X} - \frac{a}{c} \int \frac{x^6 \partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^8}{8c} - \frac{bx^7}{7c^2} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right) \int \frac{x^7 \partial x}{X} + \frac{ab}{c^2} \int \frac{x^6 \partial x}{X}$$

$$\int\frac{x^m\partial x}{(a+bx+cx^2)^2}$$ *Taf. XXVIII.*

VZ. $a+bx+cx^2=X,\ 4ac-b^2=k$

$$\frac{\partial x}{X^2}=\frac{2cx+b}{kX}+\frac{2c}{k}\int\frac{\partial x}{X}$$

$$\frac{{}^{:}\partial x}{X^2}=-\frac{1}{2cX}-\frac{b}{2c}\int\frac{\partial x}{X^2}$$

$$\frac{{}^{2}\partial x}{X^2}=-\frac{x}{cX}+\frac{a}{c}\int\frac{\partial x}{X^2}$$

$$\frac{{}^{3}\partial x}{X^2}=\left(\frac{bx}{c^2}+\frac{a}{2c^2}\right)\frac{1}{X}+\frac{1}{2c^2}\log X-\frac{ab}{2c^2}\int\frac{\partial x}{X^2}-\frac{b}{2c^2}\int\frac{\partial x}{X}$$

$$\frac{{}^{4}\partial x}{X^2}=\frac{x^3}{cX}-\frac{2b}{c}\int\frac{x^3\partial x}{X^2}-\frac{3a}{c}\int\frac{x^2\partial x}{X^2}$$

$$\frac{{}^{5}\partial x}{X^2}=\left(\frac{x^4}{2c}-\frac{3bx^3}{2c^2}\right)\frac{1}{X}+\left(\frac{3b^2}{c^2}-\frac{2a}{c}\right)\int\frac{x^3\partial x}{X^2}+\frac{9ab}{2c^2}\int\frac{x^2\partial x}{X^2}$$

$$\frac{{}^{6}\partial x}{X^2}=\left[\frac{x^5}{3c}-\frac{2bx^4}{3c^2}+\left(\frac{2b^2}{c^3}-\frac{5a}{3c^2}\right)x^3\right]\frac{1}{X}-\left(\frac{4b^3}{c^3}-\frac{6ab}{c^2}\right)\times$$
$$\int\frac{x^3\partial x}{X^2}-\left(\frac{6ab^2}{c^3}-\frac{5a^2}{c^2}\right)\int\frac{x^2\partial x}{X^2}$$

$$\frac{{}^{7}\partial x}{X^2}=\left[\frac{x^6}{4c}-\frac{5bx^5}{12c^2}+\left(\frac{5b^2}{6c^3}-\frac{3a}{4c^2}\right)x^4-\left(\frac{5b^3}{2c^4}-\frac{13ab}{3c^3}\right)x^3\right]\frac{1}{X}$$
$$+\left(\frac{5b^4}{c^4}-\frac{12ab^2}{c^3}+\frac{3a^2}{c^2}\right)\int\frac{x^3\partial x}{X^2}+\left(\frac{15ab^3}{2c^4}-\frac{13a^2b}{c^3}\right)\int\frac{x^2\partial x}{X^2}$$

$$\frac{{}^{8}\partial x}{X^2}=\frac{x^7}{5cX}-\frac{6b}{5c}\int\frac{x^7\partial x}{X^2}-\frac{7a}{5c}\int\frac{x^6\partial x}{X^2}$$

Taf. **XXIX.** $\displaystyle\int \frac{x^m \partial x}{(a+bx+cx^2)^3}$

VZ. $a+bx+cx^2 = X,\quad 4ac-b^2 = k$

$$\int \frac{\partial x}{X^3} = \left(\frac{1}{2kX^2} + \frac{3c}{k^2 X}\right)(2cx+b) + \frac{6c^2}{k^2}\int \frac{\partial x}{X}$$

$$\int \frac{x\partial x}{X^3} = -\frac{1}{4cX^2} - \frac{b}{2c}\int \frac{\partial x}{X^3}$$

$$\int \frac{x^2\partial x}{X^3} = \left(-\frac{x}{3c} + \frac{b}{12c^2}\right)\frac{1}{X^2} + \left(\frac{b^2}{6c^2} + \frac{a}{3c}\right)\int \frac{\partial x}{X^3}$$

$$\int \frac{x^3\partial x}{X^3} = \left(-\frac{x^2}{2c} - \frac{a}{4c^2}\right)\frac{1}{X^2} - \frac{ab}{2c^2}\int \frac{\partial x}{X^3}$$

$$\int \frac{x^4\partial x}{X^3} = \left(-\frac{x^3}{c} - \frac{bx^2}{2c^2} - \frac{ax}{c^2}\right)\frac{1}{X^2} + \frac{a^2}{c^2}\int \frac{\partial x}{X^3}$$

$$\int \frac{x^5\partial x}{X^3} = \frac{1}{c}\int \frac{x^3\partial x}{X^2} - \frac{a}{c}\int \frac{x^3\partial x}{X^3} - \frac{b}{c}\int \frac{x^4\partial x}{X^3}$$

$$\int \frac{x^6\partial x}{X^3} = \frac{x^5}{cX^2} - \frac{3b}{c}\int \frac{x^5\partial x}{X^3} - \frac{5a}{c}\int \frac{x^4\partial x}{X^3}$$

$$\int \frac{x^7\partial x}{X^3} = \left(\frac{x^6}{2c} - \frac{2bx^5}{c^2}\right)\frac{1}{X^2} + \left(\frac{6b^2}{c^2} - \frac{3a}{c}\right)\int \frac{x^5\partial x}{X^3} + \frac{10ab}{c^2}\int \frac{x^4\partial x}{X^3}$$

$$\int \frac{x^8\partial x}{X^3} = \left[\frac{x^7}{3c} - \frac{5bx^6}{6c^2} + \left(\frac{10b^2}{3c^3} - \frac{7a}{3c^2}\right)x^5\right]\frac{1}{X^2}$$
$$-\left(\frac{10b^3}{c^3} - \frac{12ab}{c^3}\right)\int \frac{x^5\partial x}{X^3} - \left(\frac{50ab^2}{3c^3} - \frac{35a^2}{3c^2}\right)\int \frac{x^4\partial x}{X^3}$$

$$\int \frac{x^9\partial x}{X^3} = \frac{x^8}{4cX^2} - \frac{5b}{2c}\int \frac{x^8\partial x}{X^3} - \frac{2a}{c}\int \frac{x^7\partial x}{X^3}$$

$$\int \frac{x^m \partial x}{(a+bx+cx^2)^4}$$ *Taf.* XXX.

VZ. $a+bx+cx^2 = X,\ 4ac-b^2 = k$

$$\int \frac{\partial x}{X^4} = \left(\frac{1}{3kX^3}+\frac{5c}{3k^2X^2}+\frac{10c^2}{k^3X}\right)(2cx+b)+\frac{20c^3}{k^3}\int\frac{\partial x}{X}$$

$$\int \frac{x\partial x}{X^4} = -\frac{1}{6cX^3}-\frac{b}{2c}\int\frac{\partial x}{X^4}$$

$$\int \frac{x^2\partial x}{X^4} = \left(-\frac{x}{5c}+\frac{b}{15c^2}\right)\frac{1}{X^3}+\left(\frac{b^2}{5c^2}+\frac{a}{5c}\right)\int\frac{\partial x}{X^4}$$

$$\int \frac{x^3\partial x}{X^4} = \left[-\frac{x^2}{4c}+\frac{bx}{20c^2}-\left(\frac{b^2}{60c^3}+\frac{a}{12c^2}\right)\right]\frac{1}{X^3} - \left(\frac{b^3}{20c^3}+\frac{3ab}{10c^2}\right)\int\frac{\partial x}{X^4}$$

$$\int \frac{x^4\partial x}{X^4} = \left(-\frac{x^3}{3c}-\frac{ax}{5c^2}+\frac{ab}{15c^3}\right)\frac{1}{X^3}+\left(\frac{ab^2}{5c^3}+\frac{a^2}{5c^2}\right)\int\frac{\partial x}{X^4}$$

$$\int \frac{x^5\partial x}{X^4} = \left(-\frac{x^4}{2c}-\frac{bx^3}{6c^2}-\frac{ax^2}{2c^2}-\frac{a^2}{6c^3}\right)\frac{1}{X^3}-\frac{a^2b}{2c^3}\int\frac{\partial x}{X^4}$$

$$\int \frac{x^6\partial x}{X^4} = \left[-\frac{x^5}{c}-\frac{bx^4}{c^2}-\left(\frac{b^2}{3c^3}+\frac{5a}{3c^2}\right)x^3-\frac{abx^2}{c^3}-\frac{a^2x}{c^3}\right]\frac{1}{X^3} + \frac{a^3}{c^3}\int\frac{\partial x}{X^4}$$

$$\int \frac{x^7\partial x}{X^4} = \frac{1}{c}\int\frac{x^5\partial x}{X^3}-\frac{a}{c}\int\frac{x^5\partial x}{X^4}-\frac{b}{c}\int\frac{x^6\partial x}{X^4}$$

$$\int \frac{x^8\partial x}{X^4} = \frac{x^7}{cX^3}-\frac{4b}{c}\int\frac{x^7\partial x}{X^4}-\frac{7a}{c}\int\frac{x^6\partial x}{X^4}$$

$$\int \frac{x^9\partial x}{X^4} = \left(\frac{x^8}{2c}-\frac{5bx^7}{2c^2}\right)\frac{1}{X^3}+\left(\frac{10b^2}{c^2}-\frac{4a}{c}\right)\int\frac{x^7\partial x}{X^4}+\frac{35ab}{2c^2}\int\frac{x^6\partial x}{X^4}$$

Taf. XXXI. $\int \frac{x^m \partial x}{(a+bx+cx^2)^5}$

VZ. $a+bx+cx^2=X,\ 4ac-b\ =k$

$$\int\frac{\partial x}{X^5}=\left(\frac{1}{4kX^4}+\frac{7c}{6k^2X^3}+\frac{35c^2}{4k^3X^2}+\frac{35c^3}{k^4X}\right)(2cx+b)+\frac{70c^4}{k^4}\int\frac{\partial x}{X}$$

$$\int\frac{x\partial x}{X^5}=-\frac{1}{8cX^4}-\frac{b}{2c}\int\frac{\partial x}{X^5}$$

$$\int\frac{x^2\partial x}{X^5}=\left(-\frac{x}{7c}+\frac{3b}{56c^2}\right)\frac{1}{X^4}+\left(\frac{3b^2}{14c^2}+\frac{a}{7c}\right)\int\frac{\partial x}{X^5}$$

$$\int\frac{x^3\partial x}{X^5}=\left(-\frac{x^2}{6c}+\frac{bx}{21c^2}-\frac{b^2}{56c^3}-\frac{a}{24c^2}\right)\frac{1}{X^4}-\left(\frac{b^3}{14c^3}+\frac{3ab}{14c^2}\right)\int\frac{\partial x}{X^5}$$

$$\int\frac{x^4\partial x}{X^5}=\left[-\frac{x^3}{5c}+\frac{bx^2}{30c^2}-\left(\frac{b^2}{105c^3}+\frac{3a}{35c^2}\right)x+\frac{b^3}{280c^4}+\frac{17ab}{4.0c^3}\right]\frac{1}{X^4}$$
$$+\left(\frac{b^4}{70c^4}+\frac{6ab^2}{35c^3}+\frac{3a^2}{35c^2}\right)\int\frac{\partial x}{X^5}$$

$$\int\frac{x^5\partial x}{X^5}=\left(-\frac{x^4}{4c}-\frac{ax^2}{6c^2}+\frac{abx}{21c^3}-\frac{ab^2}{56c^4}-\frac{a^2}{24c^3}\right)\frac{1}{X^4}$$
$$-\left(\frac{ab^3}{14c^4}+\frac{3a^2b}{14c^3}\right)\int\frac{\partial x}{X^5}$$

$$\int\frac{x^6\partial x}{X^5}=-\frac{x^5}{3cX^4}+\frac{b}{3c}\int\frac{x^5\partial x}{X^5}+\frac{5a}{3c}\int\frac{x^4\partial x}{X^5}$$

$$\int\frac{x^7\partial x}{X^5}=\left(-\frac{x^6}{2c}-\frac{bx^5}{3c^2}\right)\frac{1}{X^4}+\left(\frac{b^2}{3c^2}+\frac{3a}{c}\right)\int\frac{x^5\partial x}{X^5}+\frac{5ab}{3c^2}\int\frac{x^4\partial x}{X^5}$$

$$\int\frac{x^8\partial x}{X^5}=\left[-\frac{x^7}{c}-\frac{3bx^6}{2c^2}-\left(\frac{b^2}{c^3}+\frac{7a}{3c^2}\right)x^5\right]\frac{1}{X^4}+\left(\frac{b^3}{c^3}+\frac{17ab}{3c^2}\right)\times$$
$$\int\frac{x^5\partial x}{X^5}+\left(\frac{5ab^2}{c^3}+\frac{35a^2}{3c^2}\right)\int\frac{x^4\partial x}{X^5}$$

$$\int \frac{x^m \partial x}{(a+bx+cx^2)^6}$$

Taf. XXXII.

VZ. $a+bx+cx^2 = X, \; 4ac - b^2 = k$

$$\frac{c}{6} = \left(\frac{1}{5kX^5} + \frac{9c}{10k^2X^4} + \frac{21c^2}{5k^3X^3} + \frac{21c^3}{k^4X^2} + \frac{126c^4}{k^5X}\right)(2cx+b) + \frac{252c^5}{k^5}\int\frac{\partial x}{X}$$

$$\frac{)x}{6} = -\frac{1}{10cX^5} - \frac{b}{2c}\int\frac{\partial x}{X^6}$$

$$\frac{\partial x}{6} = \left(-\frac{x}{9c} + \frac{2b}{45c^2}\right)\frac{1}{X^5} + \left(\frac{2b^2}{9c^2} + \frac{a}{9c}\right)\int\frac{\partial x}{X^6}$$

$$\frac{\partial x}{6} = \left(-\frac{x^2}{8c} + \frac{bx}{24c^2} - \frac{b^2}{60c^3} - \frac{a}{40c^2}\right)\frac{1}{X^5} - \left(\frac{b^3}{12c^3} + \frac{ab}{6c^2}\right)\int\frac{\partial x}{X^6}$$

$$\frac{\partial x}{6} = \left[-\frac{x^3}{7c} + \frac{bx^2}{28c^2} - \left(\frac{b^2}{84c^3} + \frac{a}{21c^2}\right)x + \frac{b^3}{210c^4} + \frac{11ab}{420c^3}\right]\frac{1}{X^5} + \left(\frac{b^4}{42c^4} + \frac{ab^2}{7c^3} + \frac{a^2}{21c^2}\right)\int\frac{\partial x}{X^6}$$

$$\frac{\partial x}{6} = -\frac{x^4}{6cX^5} - \frac{b}{6c}\int\frac{x^4\partial x}{X^6} + \frac{2a}{3c}\int\frac{x^3\partial x}{X^6}$$

$$\frac{\partial x}{6} = -\frac{x^5}{5cX^5} + \frac{a}{c}\int\frac{x^4\partial x}{X^6}$$

$$\frac{\partial x}{6} = \left(-\frac{x^6}{4c} - \frac{bx^5}{20c^2} - \frac{ax^4}{4c^2}\right)\frac{1}{X^5} + \frac{a^2}{c^2}\int\frac{x^3\partial x}{X^6}$$

$$\frac{\partial x}{6} = \left[-\frac{x^7}{3c} - \frac{bx^6}{6c^2} - \left(\frac{b^2}{30c^3} + \frac{7a}{15c^2}\right)x^5 - \frac{abx^4}{6c^3}\right]\frac{1}{X^5} + \frac{7a^2}{3c^2}\int\frac{x^4\partial x}{X^6} + \frac{2a^2b}{5c^3}\int\frac{x^3\partial x}{X^6}$$

Taf. XXXIII. $\int \frac{\partial x}{x^m(a+bx+cx^2)}$

VZ. $a+bx+cx^2 = X$

$$\int\frac{\partial x}{xX}=\frac{1}{2a}\log\frac{x^2}{X}-\frac{b}{2a}\int\frac{\partial x}{X}\ ^{*)}$$

$$\int\frac{\partial x}{x^2X}=-\frac{1}{ax}-\frac{b}{2a^2}\log\frac{x^2}{X}+\left(\frac{b^2}{2a^2}-\frac{c}{a}\right)\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^3X}=-\frac{1}{2ax^2}+\frac{b}{a^2x}+\left(\frac{b^2}{2a^3}-\frac{c}{2a^2}\right)\log\frac{x^2}{X}-\left(\frac{b^3}{2a^3}-\frac{3bc}{2a^2}\right)\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^4X}=-\frac{1}{3ax^3}+\frac{b}{2a^2x^2}-\left(\frac{b^2}{a^3}-\frac{c}{a^2}\right)\frac{1}{x}-\left(\frac{b^3}{2a^4}-\frac{bc}{a^3}\right)\log\frac{x^2}{X}$$
$$+\left(\frac{b^4}{2a^4}-\frac{2b^2c}{a^3}+\frac{c^2}{a^2}\right)\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^5X}=-\frac{1}{4ax^4}-\frac{b}{a}\int\frac{\partial x}{x^4X}-\frac{c}{a}\int\frac{\partial x}{x^3X}$$

$$\int\frac{\partial x}{x^6X}=-\frac{1}{5ax^5}+\frac{b}{4a^2x^4}+\left(\frac{b^2}{a^2}-\frac{c}{a}\right)\int\frac{\partial x}{x^4X}+\frac{bc}{a^2}\int\frac{\partial x}{x^3X}$$

$$\int\frac{\partial x}{x^7X}=-\frac{1}{6ax^6}+\frac{b}{5a^2x^5}-\left(\frac{b^2}{4a^3}-\frac{c}{4a^2}\right)\frac{1}{x^4}-\left(\frac{b^3}{a^3}-\frac{2bc}{a^2}\right)\int\frac{\partial x}{x^4X}$$
$$-\left(\frac{b^2c}{a^3}-\frac{c^2}{a^2}\right)\int\frac{\partial x}{x^3X}$$

$$\int\frac{\partial x}{x^8X}=-\frac{1}{7ax^7}+\frac{b}{6a^2x^6}-\left(\frac{b^2}{5a^3}-\frac{c}{5a^2}\right)\frac{1}{x^5}+\left(\frac{b^3}{4a^4}-\frac{bc}{2a^3}\right)\frac{1}{x^4}$$
$$+\left(\frac{b^4}{a^4}-\frac{3b^2c}{a^3}+\frac{c^2}{a^2}\right)\int\frac{\partial x}{x^4X}+\left(\frac{b^3c}{a^4}-\frac{2bc^2}{a^3}\right)\int\frac{\partial x}{x^3X}$$

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*) Das Integral $\int\frac{\partial x}{xX}$ kann für $x=0$ nicht verschwinden, weil alsdann $\log\frac{x^2}{X}=\log 0=-\infty$ wird. Uebrigens ist $\log\frac{x^2}{X}=-\log\frac{X}{x^2}$.

$$\int \frac{\partial x}{x^n(a+bx+cx^2)^2}$$ **Taf. XXXIV.**

VZ. $a + bx + cx^2 = X$

$$\frac{x}{X^2} = \frac{1}{2aX} + \frac{1}{2a^2}\log\frac{x^2}{X} - \frac{b}{2a}\int\frac{\partial x}{X^2} - \frac{b}{2a^2}\int\frac{\partial x}{X}$$

$$\frac{x}{X^2} = \left(-\frac{1}{ax} - \frac{b}{a^2}\right)\frac{1}{X} - \frac{b}{a^3}\log\frac{x^2}{X} + \left(\frac{b^2}{a^2} - \frac{3c}{a}\right)\int\frac{\partial x}{X^2}$$
$$+ \frac{b^2}{a^3}\int\frac{\partial x}{X}$$

$$\frac{x}{X^2} = \left(-\frac{1}{2ax^2} + \frac{3b}{2a^2x} + \frac{3b^2}{2a^3} - \frac{c}{a^2}\right)\frac{1}{X} + \left(\frac{3b^2}{2a^4} - \frac{c}{a^3}\right)\log\frac{x^2}{X}$$
$$- \left(\frac{3b^3}{2a^3} - \frac{11bc}{2a^2}\right)\int\frac{\partial x}{X^2} - \left(\frac{3b^3}{2a^4} - \frac{bc}{a^3}\right)\int\frac{\partial x}{X}$$

$$\frac{x}{X^2} = \left[-\frac{1}{3ax^3} + \frac{2b}{3a^2x^2} - \left(\frac{2b^2}{a^3} - \frac{5c}{3a^2}\right)\frac{1}{x} - \frac{2b^3}{a^4} + \frac{3bc}{a^3}\right]\frac{1}{X}$$
$$- \left(\frac{2b^3}{a^5} - \frac{3bc}{a^4}\right)\log\frac{x^2}{X} + \left(\frac{2b^4}{a^4} - \frac{9b^2c}{a^3} + \frac{5c^2}{a^2}\right)\int\frac{\partial x}{X^2}$$
$$+ \left(\frac{2b^4}{a^5} - \frac{3b^2c}{a^4}\right)\int\frac{\partial x}{X}$$

$$\frac{x}{X^2} = -\frac{1}{4ax^4X} - \frac{5b}{4a}\int\frac{\partial x}{x^4X^2} - \frac{3c}{2a}\int\frac{\partial x}{x^3X^2}$$

$$\frac{x}{X^2} = \left(-\frac{1}{5ax^5} + \frac{3b}{10a^2x^4}\right)\frac{1}{X} + \left(\frac{3b^2}{2a^2} - \frac{7c}{5a}\right)\int\frac{\partial x}{x^4X^2}$$
$$+ \frac{9bc}{5a^2}\int\frac{\partial x}{x^3X^2}$$

$$\frac{x}{X^2} = \left[-\frac{1}{6ax^6} + \frac{7b}{30a^2x^5} - \left(\frac{7b^2}{20a^3} - \frac{c}{3a^2}\right)\frac{1}{x^4}\right]\frac{1}{X}$$
$$- \left(\frac{7b^3}{4a^3} - \frac{33bc}{10a^2}\right)\int\frac{\partial x}{x^4X^2} - \left(\frac{21b^2c}{10a^3} - \frac{2c^2}{a^2}\right)\int\frac{\partial x}{x^3X^2}$$

$$\frac{x}{X^2} = -\frac{1}{7ax^7X} - \frac{8b}{7a}\int\frac{\partial x}{x^7X^2} - \frac{9c}{7a}\int\frac{\partial x}{x^6X^2}$$

Taf. XXXV. $\int \frac{\partial x}{x^m(a+bx+cx^2)^3}$

VZ. $a+bx+cx^2 = X$

$$\int \frac{\partial x}{xX^3} = \frac{1}{4aX^2} + \frac{1}{2a^2X} + \frac{1}{2a^3}\log\frac{x^2}{X} - \frac{b}{2a}\int\frac{\partial x}{X^3} - \frac{b}{2a^2}\int\frac{\partial x}{X^2} - \frac{b}{2a^3}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^2X^3} = -\frac{1}{axX^2} - \frac{3b}{a}\int\frac{\partial x}{xX^3} - \frac{5c}{a}\int\frac{\partial x}{X^3}$$

$$\int \frac{\partial x}{x^3X^3} = \left(-\frac{1}{2ax^2} + \frac{2b}{a^2x}\right)\frac{1}{X^2} + \left(\frac{6b^2}{a^2} - \frac{3c}{a}\right)\int\frac{\partial x}{xX^3} + \frac{10bc}{a^2}\int\frac{\partial x}{X^3}$$

$$\int \frac{\partial x}{x^4X^3} = \left[-\frac{1}{3ax^3} + \frac{5b}{6a^2x^2} - \left(\frac{10b^2}{3a^3} - \frac{7c}{3a^2}\right)\frac{1}{x}\right]\frac{1}{X^2} - \left(\frac{10b^3}{a^3} - \frac{12bc}{a^2}\right)\int\frac{\partial x}{xX^3} - \left(\frac{50b^2c}{3a^3} - \frac{35c^2}{3a^2}\right)\int\frac{\partial x}{X^3}$$

$$\int \frac{\partial x}{x^5X^3} = -\frac{1}{4ax^4X^2} - \frac{3b}{2a}\int\frac{\partial x}{x^4X^3} - \frac{2c}{a}\int\frac{\partial x}{x^3X^3}$$

$$\int \frac{\partial x}{x^6X^3} = \left(-\frac{1}{5ax^5} + \frac{7b}{20a^2x^4}\right)\frac{1}{X^2} + \left(\frac{21b^2}{10a^2} - \frac{9c}{5a}\right)\int\frac{\partial x}{x^4X^3} + \frac{14bc}{5a^2}\int\frac{\partial x}{x^3X^3}$$

$$\int \frac{\partial x}{x^7X^3} = \left[-\frac{1}{6ax^6} + \frac{4b}{15a^2x^5} - \left(\frac{7b^2}{15a^3} - \frac{5c}{12a^2}\right)\right]\frac{1}{X^2} - \left(\frac{14b^3}{5a^3} - \frac{49bc}{10a^2}\right)\int\frac{\partial x}{x^4X^3} - \left(\frac{56b^2c}{15a^3} - \frac{10c^2}{3a^2}\right)\int\frac{\partial x}{x^3X^3}$$

$$\int \frac{\partial x}{x^8X^3} = -\frac{1}{7ax^7X^2} - \frac{9b}{7a}\int\frac{\partial x}{x^7X^3} - \frac{11c}{7a}\int\frac{\partial x}{x^6X^3}$$

$$\int \frac{\partial x}{x^m(a+bx+cx^2)^4}$$ *Taf. XXXVI.*

VZ. $a + bx + cx^2 = X$

$$\ldots X^4 = \frac{1}{6aX^3} + \frac{1}{4a^2X^2} + \frac{1}{2a^3X} + \frac{1}{2a^4}\log\frac{x^2}{X} - \frac{b}{2a}\int\frac{\partial x}{X^4}$$
$$-\frac{b}{2a^2}\int\frac{\partial x}{X^3} - \frac{b}{2a^3}\int\frac{\partial x}{X^2} - \frac{b}{2a^4}\int\frac{\partial x}{X^4}$$

$$\ldots X^4 = -\frac{1}{axX^3} - \frac{4b}{a}\int\frac{\partial x}{xX^4} - \frac{7c}{a}\int\frac{\partial x}{X^4}$$

$$\ldots X^4 = \left(-\frac{1}{2ax^2} + \frac{5b}{2a^2x}\right)\frac{1}{X^3} + \left(\frac{10b^2}{a^2} - \frac{4c}{a}\right)\int\frac{\partial x}{xX^4}$$
$$+\frac{35bc}{2a^2}\int\frac{\partial x}{X^4}$$

$$\ldots X^4 = \left[-\frac{1}{3ax^3} + \frac{b}{a^2x^2} - \left(\frac{5b^2}{a^3} - \frac{3c}{a^2}\right)\frac{1}{x}\right]\frac{1}{X^3}$$
$$-\left(\frac{20b^3}{a^3} - \frac{20bc}{a^2}\right)\int\frac{\partial x}{xX^4} - \left(\frac{35b^2c}{a^3} - \frac{21c^2}{a^2}\right)\int\frac{\partial x}{X^4}$$

$$\ldots X^4 = -\frac{1}{4ax^4X^3} - \frac{7b}{4a}\int\frac{\partial x}{x^4X^4} - \frac{5c}{2a}\int\frac{\partial x}{x^3X^4}$$

$$\ldots X^4 = \left(-\frac{1}{5ax^5} + \frac{2b}{5a^2x^4}\right)\frac{1}{X^3} + \left(\frac{14b^2}{5a^2} - \frac{11c}{5a}\right)\int\frac{\partial x}{x^4X^4}$$
$$+\frac{4bc}{a^2}\int\frac{\partial x}{x^3X^4}$$

$$\ldots X^4 = \left[-\frac{1}{6ax^6} + \frac{3b}{10a^2x^5} - \left(\frac{3b^2}{5a^3} - \frac{c}{2a^2}\right)\frac{1}{x^4}\right]\frac{1}{X^3}$$
$$-\left(\frac{21b^3}{5a^3} - \frac{34bc}{5a^2}\right)\int\frac{\partial x}{x^4X^4} - \left(\frac{6b^2c}{a^3} - \frac{5c^2}{a^2}\right)\int\frac{\partial x}{x^3X^4}$$

$$\ldots X^4 = -\frac{1}{7ax^7X^3} - \frac{10b}{7a}\int\frac{\partial x}{x^7X^4} - \frac{13c}{7a}\int\frac{\partial x}{x^6X^4}$$

Taf. XXXVII. $\displaystyle\int\frac{\partial x}{x^m(a+bx+cx^2)^5}$

VZ. $a+bx+cx^2=X$

$$\int\frac{\partial x}{xX^5}=\frac{1}{8aX^4}+\frac{1}{6a^2X^3}+\frac{1}{4a^3X^2}+\frac{1}{2a^4X}+\frac{1}{2a^5}\log\frac{x^2}{X}-\frac{b}{2a}\int\frac{\partial x}{X^5}$$
$$-\frac{b}{2a^2}\int\frac{\partial x}{X^4}-\frac{b}{2a^3}\int\frac{\partial x}{X^3}-\frac{b}{2a^4}\int\frac{\partial x}{X^2}-\frac{b}{2a^5}\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^2X^5}=-\frac{1}{axX^4}-\frac{5b}{a}\int\frac{\partial x}{xX^5}-\frac{9c}{a}\int\frac{\partial x}{X^5}$$

$$\int\frac{\partial x}{x^3X^5}=\left(-\frac{1}{2ax^2}+\frac{3b}{a^2x}\right)\frac{1}{X^4}+\left(\frac{15b^2}{a^2}-\frac{5c}{a}\right)\int\frac{\partial x}{xX^5}+\frac{27bc}{a^2}\int\frac{\partial x}{X^5}$$

$$\int\frac{\partial x}{x^4X^5}=\left[-\frac{1}{3ax^3}+\frac{7b}{6a^2x^2}-\left(\frac{7b^2}{a^3}-\frac{11c}{3a^2}\right)\frac{1}{x}\right]\frac{1}{X^4}$$
$$-\left(\frac{35b^3}{a^3}-\frac{30bc}{a^2}\right)\int\frac{\partial x}{xX^5}-\left(\frac{63b^2c}{a^3}-\frac{33c^2}{a^2}\right)\int\frac{\partial x}{X^5}$$

$$\int\frac{\partial x}{x^5X^5}=-\frac{1}{4ax^4X^4}-\frac{2b}{a}\int\frac{\partial x}{x^4X^5}-\frac{3c}{a}\int\frac{\partial x}{x^3X^5}$$

$$\int\frac{\partial x}{x^6X^5}=\left(-\frac{1}{5ax^5}+\frac{9b}{20a^2x^4}\right)\frac{1}{X^4}+\left(\frac{18b^2}{5a^2}-\frac{13c}{5a}\right)\int\frac{\partial x}{x^4X^5}$$
$$+\frac{39c^2}{5a^2}\int\frac{\partial x}{x^3X^5}$$

$$\int\frac{\partial x}{x^7X^5}=\left[-\frac{1}{6ax^6}+\frac{b}{5a^2x^5}-\left(\frac{3b^2}{4a^3}-\frac{7c}{12a^2}\right)\frac{1}{x^4}\right]\frac{1}{X^4}$$
$$-\left(\frac{6b^3}{a^3}-\frac{9bc}{a^2}\right)\int\frac{\partial x}{x^4X^5}-\left(\frac{13c^3}{a^3}-\frac{7c^2}{a^2}\right)\int\frac{\partial x}{x^3X^5}$$

$$\int\frac{\partial x}{x^8X^5}=-\frac{1}{7ax^7}-\frac{11b}{7a}\int\frac{\partial x}{x^7X^5}-\frac{15c}{7a}\int\frac{\partial x}{x^6X^5}$$

$$\int \frac{\partial x}{x^m(a+bx+cx^2)^6}$$ *Taf. XXXVIII.*

VZ. $a+bx+cx^2=X$

$$\int\frac{\partial x}{xX^6}=\frac{1}{10aX^5}+\frac{1}{8a^2X^4}+\frac{1}{6a^3X^3}+\frac{1}{4a^4X^2}+\frac{1}{2a^5X}+\frac{1}{2a^6}\log\frac{x^2}{X}$$
$$-\frac{b}{2a}\int\frac{\partial x}{X^6}-\frac{b}{2a^2}\int\frac{\partial x}{X^5}-\frac{b}{2a^3}\int\frac{\partial x}{X^4}-\frac{b}{2a^4}\int\frac{\partial x}{X^3}$$
$$-\frac{b}{2a^5}\int\frac{\partial x}{X^2}-\frac{b}{2a^6}\int\frac{\partial x}{X}$$

$$\int\frac{\partial x}{x^2X^6}=-\frac{1}{axX^5}-\frac{6b}{a}\int\frac{\partial x}{xX^6}-\frac{11c}{a}\int\frac{\partial x}{X^6}$$

$$\int\frac{\partial x}{x^3X^6}=\left(-\frac{1}{2ax^2}+\frac{7b}{2a^2x}\right)\frac{1}{X^5}+\left(\frac{21b^2}{a^2}-\frac{6c}{a}\right)\int\frac{\partial x}{xX^6}+\frac{77bc}{a^2}\int\frac{\partial x}{X^6}$$

$$\int\frac{\partial x}{x^4X^6}=\left[-\frac{1}{3ax^3}+\frac{4b}{3a^2x^2}-\left(\frac{28b^2}{3a^3}-\frac{13c}{3a^2}\right)\frac{1}{x}\right]\frac{1}{X^5}$$
$$-\left(\frac{56b^3}{a^3}-\frac{42bc}{a^2}\right)\int\frac{\partial x}{xX^6}-\left(\frac{616b^2c}{3a^3}-\frac{143c^2}{3a^2}\right)\int\frac{\partial x}{X^6}$$

$$\int\frac{\partial x}{x^5X^6}=-\frac{1}{4ax^4X^5}-\frac{9b}{4a}\int\frac{\partial x}{x^4X^6}-\frac{7c}{2a}\int\frac{\partial x}{x^3X^6}$$

$$\int\frac{\partial x}{x^6X^6}=\left(-\frac{1}{5ax^5}+\frac{b}{2a^2x^4}\right)\frac{1}{X^5}+\left(\frac{9b^2}{2a^2}-\frac{3c}{a}\right)\int\frac{\partial x}{x^4X^6}+\frac{7bc}{a^2}\int\frac{\partial x}{x^3X^6}$$

$$\int\frac{\partial x}{x^7X^6}=\left[-\frac{1}{6ax^6}+\frac{11b}{30a^2x^5}-\left(\frac{11b^2}{12a^3}-\frac{2c}{3a^2}\right)\frac{1}{x^4}\right]\frac{1}{X^5}$$
$$-\left(\frac{33b^3}{4a^3}-\frac{23bc}{2a^2}\right)\int\frac{\partial x}{x^4X^6}-\left(\frac{77b^2c}{6a^3}-\frac{28c^2}{3a^2}\right)\int\frac{\partial x}{x^3X^6}$$

$$\int\frac{\partial x}{x^8X^6}=-\frac{1}{7ax^7X^5}-\frac{12b}{7a}\int\frac{\partial x}{x^7X^6}-\frac{17c}{7a}\int\frac{\partial x}{x^6X^6}$$

Taf. XLI. $\int \frac{\partial x}{x^m(a+bx^3)}, \int \frac{\partial x}{x^m(a+bx^3)^2}$

VZ. $a+bx^3 = X$

$$\int \frac{\partial x}{xX} = \frac{\log x}{a} - \frac{\log X}{3a} = \frac{1}{3a}\log\frac{x^3}{X} = -\frac{1}{3a}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int\frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{3a^2}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{a^2x} + \frac{b^2}{a^2}\int\frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{2a^2x^2} + \frac{b^2}{a^2}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{3a^2x^3} - \frac{b^2}{3a^3}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{xX^2} = \frac{1}{3aX} - \frac{1}{3a^2}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{4bx^2}{3a^2}\right)\frac{1}{X} - \frac{4b}{3a^2}\int\frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{5bx}{6a^2}\right)\frac{1}{X} - \frac{5b}{3a^2}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} - \frac{2b}{3a^2}\right)\frac{1}{X} + \frac{2b}{3a^3}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{7b}{4a^2x} + \frac{7b^2x^2}{3a^3}\right)\frac{1}{X} + \frac{7b^2}{3a^3}\int\frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{4b}{5a^2x^2} + \frac{4b^2x}{3a^3}\right)\frac{1}{X} + \frac{8b^2}{3a^3}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{2a^2x^3} + \frac{b^2}{a^3}\right)\frac{1}{X} - \frac{b^2}{a^4}\log\frac{X}{x^3}$$

$$\int \frac{x^m \partial x}{(a+bx^3)^2}, \quad \int \frac{x^m \partial x}{(a+bx^3)^3}$$ **Taf. XL.**

VZ. $a + bx^3 = X$

$$\frac{\partial x}{X^2} = \frac{x}{3aX} + \frac{2}{3a}\int\frac{\partial x}{X}$$

$$\frac{\partial x}{X^2} = \frac{x^2}{3aX} + \frac{1}{3a}\int\frac{x\partial x}{X}$$

$$\frac{{}^{2}\partial x}{X^2} = -\frac{1}{3bX}$$

$$\frac{{}^{3}\partial x}{X^2} = -\frac{x}{3bX} + \frac{1}{3b}\int\frac{\partial x}{X}$$

$$\frac{{}^{4}\partial x}{X^2} = -\frac{x^2}{3bX} + \frac{2}{3b}\int\frac{x\partial x}{X}$$

$$\frac{{}^{5}\partial x}{X^2} = \frac{a}{3b^2X} + \frac{1}{3b^2}\log X$$

$$\frac{{}^{6}\partial x}{X^2} = \left(\frac{x^4}{b} + \frac{4ax}{3b^2}\right)\frac{1}{X} - \frac{4a}{3b^2}\int\frac{\partial x}{X}$$

$$\frac{x}{X^3} = \left(\frac{5bx^4}{18a^2} + \frac{4x}{9a}\right)\frac{1}{X^2} + \frac{5}{9a^2}\int\frac{\partial x}{X}$$

$$\frac{\partial x}{X^3} = \left(\frac{2bx^5}{9a^2} + \frac{7x^2}{18a}\right)\frac{1}{X^2} + \frac{2}{9a^2}\int\frac{x\partial x}{X}$$

$$\frac{{}^{2}\partial x}{X^3} = -\frac{1}{6bX^2}$$

$$\frac{{}^{3}\partial x}{X^3} = \left(\frac{x^4}{18a} - \frac{x}{9b}\right)\frac{1}{X^2} + \frac{1}{9ab}\int\frac{\partial x}{X}$$

$$\frac{{}^{4}\partial x}{X^3} = \left(\frac{x^5}{9a} - \frac{x^2}{18b}\right)\frac{1}{X^2} + \frac{1}{9ab}\int\frac{x\partial x}{X}$$

$$\frac{{}^{5}\partial x}{X^3} = \frac{x^6}{6aX^2}$$

$$\frac{{}^{6}\partial x}{X^3} = \left(-\frac{7x^4}{18b} - \frac{2ax}{9b^2}\right)\frac{1}{X^2} + \frac{2}{9b^2}\int\frac{\partial x}{X}$$

Taf. XLI. $\int \frac{\partial x}{x^m(a+bx^3)}$, $\int \frac{\partial x}{x^m(a+bx^3)^2}$

VZ. $a+bx^3 = X$

$$\int \frac{\partial x}{xX} = \frac{\log x}{a} - \frac{\log X}{3a} = \frac{1}{3a}\log\frac{x^3}{X} = -\frac{1}{3a}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int\frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{3a^2}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{a^2x} + \frac{b^2}{a^2}\int\frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{2a^2x^2} + \frac{b^2}{a^2}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X} = -\frac{1}{6ax^6} + \frac{b}{3a^2x^3} - \frac{b^2}{3a^3}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{xX^2} = \frac{1}{3aX} - \frac{1}{3a^2}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{x^2X^2} = \left(-\frac{1}{ax} - \frac{4bx^2}{3a^2}\right)\frac{1}{X} - \frac{4b}{3a^2}\int\frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^3X^2} = \left(-\frac{1}{2ax^2} - \frac{5bx}{6a^2}\right)\frac{1}{X} - \frac{5b}{3a^2}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^4X^2} = \left(-\frac{1}{3ax^3} - \frac{2b}{3a^2}\right)\frac{1}{X} + \frac{2b}{3a^3}\log\frac{X}{x^3}$$

$$\int \frac{\partial x}{x^5X^2} = \left(-\frac{1}{4ax^4} + \frac{7b}{4a^2x} + \frac{7b^2x^2}{3a^3}\right)\frac{1}{X} + \frac{7b^2}{3a^3}\int\frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^6X^2} = \left(-\frac{1}{5ax^5} + \frac{4b}{5a^2x^2} + \frac{4b^2x}{3a^3}\right)\frac{1}{X} + \frac{8b^2}{3a^3}\int\frac{\partial x}{X}$$

$$\int \frac{\partial x}{x^7X^2} = \left(-\frac{1}{6ax^6} + \frac{b}{2a^2x^3} + \frac{b^2}{a^3}\right)\frac{1}{X} - \frac{b^2}{a^4}\log\frac{X}{x^3}$$

$$\int \frac{x^m \partial x}{a + bx^4}$$ **Taf. XLII. a.**

(*a* und *b* dieselben Zeichen)

VZ. $a + bx^4 = X, \quad \sqrt[4]{\frac{a}{b}} = k$

$$\frac{x}{X} = \frac{1}{4bk^3\sqrt{2}}\left(\log\frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + 2\,\mathrm{Arc\,Tang}\,\frac{kx\sqrt{2}}{k^2 - x^2}\right)$$

$$\frac{\partial x}{X} = -\frac{1}{2\sqrt{ab}}\,\mathrm{Arc\,Tang}\,\frac{\sqrt{a}}{x^2\sqrt{b}}$$

$$\frac{\partial x}{X} = \frac{1}{4bk\sqrt{2}}\left(-\log\frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + 2\,\mathrm{Arc\,Tang}\,\frac{kx\sqrt{2}}{k^2 - x^2}\right)$$

$$\frac{\partial x}{X} = \frac{1}{4b}\log X$$

$$\frac{\partial x}{X} = \frac{x}{b} - \frac{a}{b}\int\frac{\partial x}{X}$$

$$\frac{\partial x}{X} = \frac{x^2}{2b} - \frac{a}{b}\int\frac{x\partial x}{X}$$

$$\frac{\partial x}{X} = \frac{x^3}{3b} - \frac{a}{b}\int\frac{x^2\partial x}{X}$$

$$\frac{\partial x}{X} = \frac{x^4}{4b} - \frac{a}{b}\int\frac{x^3\partial x}{X}$$

$$\frac{\partial x}{X} = \frac{x^5}{5b} - \frac{ax}{b^2} + \frac{a^2}{b^2}\int\frac{\partial x}{X}$$

$$\frac{\partial x}{X} = \frac{x^6}{6b} - \frac{ax^2}{2b} + \frac{a^2}{b^2}\int\frac{x\partial x}{X}$$

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s ist $\log\frac{x^2 + kx\sqrt{2} + k^2}{x^2 - kx\sqrt{2} + k^2} + \text{Const.} = 2\log\frac{x^2 + kx\sqrt{2} + k^2}{\sqrt{X}} + \text{Const.}$

rc Tang $\frac{kx\sqrt{2}}{k^2 - x^2} = \mathrm{Arc\,Sec}\,\frac{\sqrt{X}}{\sqrt{a} - x^2\sqrt{b}} = \mathrm{Arc\,Cos}\,\frac{\sqrt{a} - x^2\sqrt{b}}{\sqrt{X}}.$

Taf. XLII. b.

$$\int \frac{x^m \partial x}{a + bx^4}$$

(*a* und *b* verschiedene Zeichen)

VZ. $a + bx^4 = X,\ \sqrt[4]{-\frac{a}{b}} = k$

$$\int \frac{\partial x}{X} = -\frac{1}{4bk^3}\left(\log \frac{x+k}{x-k} + 2 \text{ Arc Tang } \frac{x}{k}\right)$$

$$\int \frac{x\partial x}{X} = -\frac{1}{4bk^2} \log \frac{x^2 + k^2}{x^2 - k^2} \ ^{*)}$$

$$\int \frac{x^2 \partial x}{X} = -\frac{1}{4bk}\left(\log \frac{x+k}{x-k} - 2 \text{ Arc Tang } \frac{x}{k}\right)$$

Die übrigen Integrale wie in Tafel XLII. a.

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*) $\log \frac{x^2 + k^2}{x^2 - k^2} + \text{Const.} = \log \frac{k^2 + x^2}{k^2 - x^2} + \text{Const.}$,

und eben so

$$\log \frac{x + k}{x - k} + \text{Const.} = \log \frac{k + x}{k - x} + \text{Const.}$$

$$\int\frac{x^m\partial x}{(a+bx^4)^2},\quad \int\frac{x^m\partial x}{(a+bx^4)^3}$$ *Taf. XLIII.*

VZ. $a + bx^4 = X$

$$\frac{c}{2} = \frac{x}{4aX} + \frac{3}{4a}\int\frac{\partial x}{X}$$

$$\frac{)x}{2} = \frac{x^2}{4aX} + \frac{1}{2a}\int\frac{x\partial x}{X}$$

$$\frac{\partial x}{2} = \frac{x^3}{4aX} + \frac{1}{4a}\int\frac{x^2\partial x}{X}$$

$$\frac{\partial x}{2} = -\frac{1}{4bX}$$

$$\frac{\partial x}{2} = -\frac{x}{4bX} + \frac{1}{4b}\int\frac{\partial x}{X}$$

$$\frac{\partial x}{2} = -\frac{x^2}{4bX} + \frac{1}{2b}\int\frac{x\partial x}{X}$$

$$\frac{\partial x}{2} = -\frac{x^3}{4bX} + \frac{3}{4b}\int\frac{x^2\partial x}{X}$$

$$\frac{c}{3} = \left(\frac{7bx^5}{32a^2} + \frac{11x}{32a}\right)\frac{1}{X^2} + \frac{21}{32a^2}\int\frac{\partial x}{X}$$

$$\frac{x}{3} = \left(\frac{3bx^6}{16a^2} + \frac{5x^2}{16a}\right)\frac{1}{X^2} + \frac{3}{8a^2}\int\frac{x\partial x}{X}$$

$$\frac{)x}{3} = \left(\frac{5bx^7}{32a^2} + \frac{9x^3}{32a}\right)\frac{1}{X^2} + \frac{5}{32a^2}\int\frac{x^2\partial x}{X}$$

$$\frac{)x}{3} = -\frac{1}{8bX^2}$$

$$\frac{)x}{3} = \left(\frac{x^5}{32a} - \frac{3x}{32b}\right)\frac{1}{X^2} + \frac{3}{32ab}\int\frac{\partial x}{X}$$

$$\frac{\partial x}{3} = \left(\frac{x^6}{16a} - \frac{x^2}{16b}\right)\frac{1}{X^2} + \frac{1}{8ab}\int\frac{x\partial x}{X}$$

$$\frac{'x}{3} = \left(\frac{3x^7}{32a} - \frac{x^3}{32ab}\right)\frac{1}{X^2} + \frac{3}{32ab}\int\frac{x^2\partial x}{X}$$

Taf. XLV.

$$\int \frac{x^m \partial x}{a + bx^5}$$

(a und b positiv oder negativ)

$$\text{VZ.}\quad a + bx^5 = X, \quad \sqrt[5]{\frac{a}{b}} = k$$

$$x^2 - 2kx \operatorname{Cos} 36^\circ + k^2 = Y$$

$$x^2 + 2kx \operatorname{Cos} 72^\circ + k^2 = Y'$$

$$\frac{x \operatorname{Sin} 36^\circ}{k - x \operatorname{Cos} 36^\circ} = Z, \quad \frac{x \operatorname{Sin} 72^\circ}{k + x \operatorname{Cos} 72^\circ} = Z'$$

$$\frac{\partial x}{X} = \frac{1}{5bk^4}\left\{\begin{array}{c} -\operatorname{Cos} 36^\circ \log Y + 2 \operatorname{Sin} 36^\circ \operatorname{Arc Tang} Z \\ + \operatorname{Cos} 72^\circ \log Y' + 2 \operatorname{Sin} 72^\circ \operatorname{Arc Tang} Z' \\ + \log (x + k) \end{array}\right\}$$

$$\frac{{}^{:}\partial x}{X} = \frac{1}{5bk^3}\left\{\begin{array}{c} -\operatorname{Cos} 72^\circ \log Y + 2 \operatorname{Sin} 72^\circ \operatorname{Arc Tang} Z \\ + \operatorname{Cos} 36^\circ \log Y' - 2 \operatorname{Sin} 36^\circ \operatorname{Arc Tang} Z' \\ - \log (x + k) \end{array}\right\}$$

$$\frac{{}^{2}\partial x}{X} = \frac{1}{5bk^2}\left\{\begin{array}{c} \operatorname{Cos} 72^\circ \log Y + 2 \operatorname{Sin} 72^\circ \operatorname{Arc Tang} Z \\ - \operatorname{Cos} 36^\circ \log Y' - 2 \operatorname{Sin} 36^\circ \operatorname{Arc Tang} Z' \\ + \log (x + k) \end{array}\right\}$$

$$\frac{{}^{3}\partial x}{X} = \frac{1}{5bk}\left\{\begin{array}{c} \operatorname{Cos} 36^\circ \log Y + 2 \operatorname{Sin} 36^\circ \operatorname{Arc Tang} Z \\ - \operatorname{Cos} 72^\circ \log Y' + 2 \operatorname{Sin} 72^\circ \operatorname{Arc Tang} Z' \\ - \log (x + k) \end{array}\right\}$$

$$\frac{{}^{4}\partial x}{X} = \frac{1}{5b} \log X$$

$$\frac{{}^{5}\partial x}{X} = \frac{x}{b} - \frac{a}{b}\int \frac{\partial x}{X}$$

$$\frac{{}^{6}\partial x}{X} = \frac{x^2}{2b} - \frac{a}{b}\int \frac{x\partial x}{X}$$

$$\frac{{}^{7}\partial x}{X} = \frac{x^3}{3b} - \frac{a}{b}\int \frac{x^2\partial x}{X}$$

$$\frac{{}^{8}\partial x}{X} = \frac{x^4}{4b} - \frac{a}{b}\int \frac{x^3\partial x}{X}$$

$$\frac{{}^{9}\partial x}{X} = \frac{x^5}{5b} - \frac{a}{b}\int \frac{x^4\partial x}{X}$$

Taf. XLVII. b. $\int \frac{x^m \partial x}{a + bx^6}$

(a und b verschiedene Zeichen)

VZ. $a + bx^6 = X$, $\sqrt[6]{-\frac{a}{b}} = k$

$$\int \frac{\partial x}{X} = \frac{-1}{6bk^5}\left(\tfrac{1}{2}\log\frac{(x+k)^2(x^2+kx+k^2)}{(x-k)^2(x^2-kx+k^2)} + \sqrt{3}.\,\mathrm{Arc\,Tang}\frac{kx\sqrt{3}}{k^2-x^2}\right)$$

$$\int \frac{x\partial x}{X} = \frac{-1}{6bk^4}\left(\tfrac{1}{2}\log\frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} + \sqrt{3}.\,\mathrm{Arc\,Tang}\frac{x^2\sqrt{3}}{2k^2+x^2}\right)$$

$$\int \frac{x^2\partial x}{X} = \frac{-1}{6bk^3}\log\frac{x^3+k^3}{x^3-k^3}$$

$$\int \frac{x^3\partial x}{X} = \frac{-1}{6bk^2}\left(\tfrac{1}{2}\log\frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} - \sqrt{3}.\,\mathrm{Arc\,Tang}\frac{x^2\sqrt{3}}{2k^2+x^2}\right)$$

$$\int \frac{x^4\partial x}{X} = \frac{-1}{6bk}\left(\tfrac{1}{2}\log\frac{(x+k)^2(x^2+kx+k^2)}{(x-k)^2(x^2-kx+k^2)} - \sqrt{3}.\,\mathrm{Arc\,Tang}\frac{kx\sqrt{3}}{k^2-x^2}\right)$$

Die übrigen Integrale wie in Taf. XLVII. a.

Diese Integrale verschwinden sämmtlich für $x = 0$. Auch ist

$$\log\frac{x^4+k^2x^2+k^4}{(x^2-k^2)^2} + \mathrm{Const.} = \log\frac{X}{(x^2-k^2)^3} = 3\log\frac{\sqrt[3]{X}}{x^2-k^2}.$$

$$\int \frac{x^m \partial x}{a + bx^6}$$ *Taf. XLVII. a.*

(*a* und *b* dieselben Zeichen)

VZ. $a + bx^6 = X, \quad \sqrt[6]{\frac{a}{b}} = k$

$$\frac{x}{X} = \frac{1}{6bk^5}\left(\frac{\sqrt{3}}{2}\log\frac{x^2+kx\sqrt{3}+k^2}{x^2-kx\sqrt{3}+k^2} + \text{Arc Tang}\,\frac{3kx(k^2-x^2)}{x^4-4k^2x^2+k^4}\right)$$

$$\frac{\partial x}{X} = \frac{1}{6bk^4}\left(\tfrac{1}{2}\log\frac{(x^2+k^2)^2}{x^4-k^2x^2+k^4} + \sqrt{3}\,.\,\text{Arc Tang}\,\frac{x^2\sqrt{3}}{2k^2-x^2}\right)$$

$$\frac{\partial x}{X} = \frac{1}{3\sqrt{ab}}\,\text{Arc Tang}\,x^3\sqrt{\frac{b}{a}}$$

$$\frac{\partial x}{X} = \frac{1}{6bk^2}\left(\tfrac{1}{2}\log\frac{x^4-k^2x^2+k^4}{(x^2+k^2)^2} + \sqrt{3}\,.\,\text{Arc Tang}\,\frac{x^2\sqrt{3}}{2k^2-x^2}\right)$$

$$\frac{\partial x}{X} = \frac{1}{6bk}\left(\frac{\sqrt{3}}{2}\log\frac{x^2-kx\sqrt{3}+k^2}{x^2+kx\sqrt{3}+k^2} + \text{Arc Tang}\,\frac{3kx(k^2-x^2)}{x^4-4k^2x^2+k^4}\right)$$

$$\frac{\partial x}{X} = \frac{1}{6b}\log X$$

$$\frac{\partial x}{X} = \frac{x}{b} - \frac{a}{b}\int\frac{\partial x}{X}$$

$$\frac{\partial x}{X} = \frac{x^2}{2b} - \frac{a}{b}\int\frac{x\partial x}{X}$$

$$\frac{\partial x}{X} = \frac{x^3}{3b} - \frac{a}{b}\int\frac{x^2\partial x}{X}$$

$$\frac{\partial x}{X} = \frac{x^4}{4b} - \frac{a}{b}\int\frac{x^3\partial x}{X}$$

Diese Integrale verschwinden sämmtlich für $x = 0$. Auch ist

$$\frac{-k^2x^2+k^4}{(x^2+k^2)^2} + \text{Const.} = \log\frac{X}{(x^2+k^2)^3} = 3\log\frac{\sqrt[3]{X}}{x^2+k^2}.$$

Taf. XLIX. a.

$$\int \frac{x^m \partial x}{a + bx^2 + cx^4}$$

($b^2 - 4ac$ eine positive Gröſse)

$$a + bx^2 + cx^4 = X$$

$$\tfrac{1}{2}b - \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = f,\quad \tfrac{1}{2}b + \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = g$$

$$\sqrt{(b^2 - 4ac)} = g - f = h$$

$$\int \frac{\partial x}{X} = \frac{c}{h}\left[\int \frac{\partial x}{cx^2 + f} - \int \frac{\partial x}{cx^2 + g}\right]$$

$$\int \frac{x\partial x}{X} = \frac{1}{2h} \log \frac{cx^2 + f}{cx^2 + g}$$

$$\int \frac{x^2 \partial x}{X} = \frac{g}{h}\int \frac{\partial x}{cx^2 + g} - \frac{f}{h}\int \frac{\partial x}{cx^2 + f}$$

$$\int \frac{x^3 \partial x}{X} = \frac{1}{2ch}\left[g \log (cx^2 + g) - f \log (cx^2 + f)\right]$$

$$\int \frac{x^4 \partial x}{X} = \frac{x}{c} - \frac{a}{c}\int \frac{\partial x}{X} - \frac{b}{c}\int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^5 \partial x}{X} = \frac{x^2}{2c} - \frac{a}{c}\int \frac{x\partial x}{X} - \frac{b}{c}\int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^6 \partial x}{X} = \frac{x^3}{3c} - \frac{bx}{c^2} + \frac{ab}{c^2}\int \frac{\partial x}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^4}{4c} - \frac{bx^2}{2c^2} + \frac{ab}{c^2}\int \frac{x\partial x}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^5}{5c} - \frac{bx^3}{3c^2} + \left(\frac{b^2}{c^3} - \frac{a}{c^2}\right)x - \left(\frac{ab^2}{c^3} - \frac{a^2}{c^2}\right)\int \frac{\partial x}{X}$$
$$- \left(\frac{b^3}{c^3} - \frac{2ab}{c^2}\right)\int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^6}{6c} - \frac{bx^4}{4c^2} + \left(\frac{b^2}{2c^3} - \frac{a}{2c^2}\right)x^2 - \left(\frac{ab^2}{c^3} - \frac{a^2}{c^2}\right)\int \frac{x\partial x}{X}$$
$$- \left(\frac{b^3}{c^3} - \frac{2ab}{c^2}\right)\int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^{10} \partial x}{X} = \frac{x^7}{7c} - \frac{a}{c}\int \frac{x^6 \partial x}{X} - \frac{b}{c}\int \frac{x^8 \partial x}{X}$$

$$\int\frac{x^m\partial x}{(a+bx^6)^2},\quad \int\frac{\partial x}{x^m(a+bx^6)}\qquad \textit{Taf. XLVIII.}$$

VZ. $a+bx^6=X$

$$\frac{x}{{}^2} = \frac{x}{6aX}+\frac{5}{6a}\int\frac{\partial x}{X}$$

$$\frac{x}{{}^2} = \frac{x^2}{6aX}+\frac{2}{3a}\int\frac{x\partial x}{X}$$

$$\frac{\partial x}{{}^2} = \frac{x^3}{6aX}+\frac{1}{2a}\int\frac{x^2\partial x}{X}$$

$$\frac{\partial x}{{}^2} = \frac{x^4}{6aX}+\frac{1}{3a}\int\frac{x^3\partial x}{X}$$

$$\frac{\partial x}{{}^2} = \frac{x^5}{6aX}+\frac{1}{6a}\int\frac{x^4\partial x}{X}$$

$$\frac{\partial x}{{}^2} = -\frac{1}{6bX}$$

$$\frac{\partial x}{{}^2} = -\frac{x}{6bX}+\frac{1}{6b}\int\frac{\partial x}{X}$$

$$\frac{\partial x}{{}^2} = -\frac{x^2}{6bX}+\frac{1}{3b}\int\frac{x\partial x}{X}$$

$$\frac{x}{X} = \frac{\log x}{a}-\frac{\log X}{6a}=\frac{1}{6a}\log\frac{x^6}{X}=-\frac{1}{6a}\log\frac{X}{x^6}$$

$$\frac{x}{X} = -\frac{1}{ax}-\frac{b}{a}\int\frac{x^4\partial x}{X}$$

$$\frac{x}{X} = -\frac{1}{2ax^2}-\frac{b}{a}\int\frac{x^3\partial x}{X}$$

$$\frac{x}{X} = -\frac{1}{3ax^3}-\frac{b}{a}\int\frac{x^2\partial x}{X}$$

$$\frac{x}{X} = -\frac{1}{4ax^4}-\frac{b}{a}\int\frac{x\partial x}{X}$$

$$\frac{x}{X} = -\frac{1}{5ax^5}-\frac{b}{a}\int\frac{\partial x}{X}$$

$$\frac{x}{X} = -\frac{1}{6ax^6}-\frac{b}{a}\int\frac{\partial x}{xX}$$

Taf. XLIX. *a.*

$$\int \frac{x^m \partial x}{a + bx^2 + cx^4}$$

($b^2 - 4ac$ eine positive Gröſse)

$$a + bx^2 + cx^4 = X$$

$$\tfrac{1}{2}b - \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = f,\ \tfrac{1}{2}b + \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = g$$

$$\sqrt{(b^2 - 4ac)} = g - f = h$$

$$\int \frac{\partial x}{X} = \frac{c}{h}\left[\int \frac{\partial x}{cx^2 + f} - \int \frac{\partial x}{cx^2 + g}\right]$$

$$\int \frac{x \partial x}{X} = \frac{1}{2h} \log \frac{cx^2 + f}{cx^2 + g}$$

$$\int \frac{x^2 \partial x}{X} = \frac{g}{h}\int \frac{\partial x}{cx^2 + g} - \frac{f}{h}\int \frac{\partial x}{cx^2 + f}$$

$$\int \frac{x^3 \partial x}{X} = \frac{1}{2ch}\left[g \log (cx^2 + g) - f \log (cx^2 + f)\right]$$

$$\int \frac{x^4 \partial x}{X} = \frac{x}{c} - \frac{a}{c}\int \frac{\partial x}{X} - \frac{b}{c}\int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^5 \partial x}{X} = \frac{x^2}{2c} - \frac{a}{c}\int \frac{x \partial x}{X} - \frac{b}{c}\int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^6 \partial x}{X} = \frac{x^3}{3c} - \frac{bx}{c^2} + \frac{ab}{c^2}\int \frac{\partial x}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^7 \partial x}{X} = \frac{x^4}{4c} - \frac{bx^2}{2c^2} + \frac{ab}{c^2}\int \frac{x \partial x}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^8 \partial x}{X} = \frac{x^5}{5c} - \frac{bx^3}{3c^2} + \left(\frac{b^2}{c^3} - \frac{a}{c^2}\right)x - \left(\frac{ab^2}{c^3} - \frac{a^2}{c^2}\right)\int \frac{\partial x}{X}$$
$$- \left(\frac{b^3}{c^3} - \frac{2ab}{c^2}\right)\int \frac{x^2 \partial x}{X}$$

$$\int \frac{x^9 \partial x}{X} = \frac{x^6}{6c} - \frac{bx^4}{4c^2} + \left(\frac{b^2}{2c^3} - \frac{a}{2c^2}\right)x^2 - \left(\frac{ab^2}{c^3} - \frac{a^2}{c^2}\right)\int \frac{x \partial x}{X}$$
$$- \left(\frac{b^3}{c^3} - \frac{2ab}{c^2}\right)\int \frac{x^3 \partial x}{X}$$

$$\int \frac{x^{10} \partial x}{X} = \frac{x^7}{7c} - \frac{a}{c}\int \frac{x^6 \partial x}{X} - \frac{b}{c}\int \frac{x^8 \partial x}{X}$$

$$\int \frac{x^m \partial x}{a + bx^2 + cx^4}$$ *Taf. XLIX. b.*

($b^2 - 4ac$ eine negative Gröſse)

VZ. $a + bx^2 + cx^4 = X$, $\sqrt[4]{\frac{a}{c}} = f$

α ein Winkel, dessen Cosinus $= -\frac{b}{2\sqrt{ac}}$.

$$\frac{\partial x}{X} = \frac{1}{4cf^3 \operatorname{Sin}\alpha} \left\{ \begin{matrix} \operatorname{Sin}\frac{\alpha}{2} \log \dfrac{x^2 + 2fx \operatorname{Cos}\frac{\alpha}{2} + f^2}{x^2 - 2fx \operatorname{Cos}\frac{\alpha}{2} + f^2} \\ + 2 \operatorname{Cos}\frac{\alpha}{2} \operatorname{Arc\,Tang} \dfrac{2fx \operatorname{Sin}\frac{\alpha}{2}}{f^2 - x^2} \end{matrix} \right\}$$

$$\frac{x \partial x}{X} = \frac{1}{2cf^2 \operatorname{Sin}\alpha} \operatorname{Arc\,Tang} \frac{f^2 \operatorname{Sin}\alpha}{f^2 \operatorname{Cos}\alpha - x^2}$$

$$\frac{x^2 \partial x}{X} = \frac{1}{4cf \operatorname{Sin}\alpha} \left\{ \begin{matrix} \operatorname{Sin}\frac{\alpha}{2} \log \dfrac{x^2 - 2fx \operatorname{Cos}\frac{\alpha}{2} + f^2}{x^2 + 2fx \operatorname{Cos}\frac{\alpha}{2} + f^2} \\ + 2 \operatorname{Cos}\frac{\alpha}{2} \operatorname{Arc\,Tang} \dfrac{2fx \operatorname{Sin}\frac{\alpha}{2}}{f^2 - x^2} \end{matrix} \right\}$$

$$\frac{x^3 \partial x}{X} = \frac{1}{4c \operatorname{Sin}\alpha} \left\{ \begin{matrix} \operatorname{Sin}\alpha \log (x^4 - 2f^2x^2 \operatorname{Sin}\alpha + f^4) \\ + 2 \operatorname{Cos}\alpha \operatorname{Arc\,Tang} \dfrac{2fx \operatorname{Sin}\frac{\alpha}{2}}{f^2 - x^2} \end{matrix} \right\}$$

Die übrigen Integrale wie in Tafel XLIX. a.

Ein Winkel, dessen Cosinus $= -\frac{b}{2\sqrt{ac}}$, läſst sich immer finden; denn diese Gröſse positiv, so ist der Winkel spitz, ist sie negativ, so ist der nkel stumpf. Gröſser als 1 kann sie nicht werden, weil sonst $b^2 - 4ac$ ht negativ seyn könnte.

Taf. LII. $$\int \frac{\partial x}{x^m(a+bx^2+cx^4)^2}$$

VZ. $a+bx^2+cx^4=X,\quad 2a(b^2-4ac)=k$

$$\int\frac{\partial x}{xX^2}=\frac{bcx^2+b^2-2ac}{kX}+\frac{\log x}{a^2}+\left(\frac{2bc}{k}-\frac{b}{a^2}\right)\int\frac{x\partial x}{X}-\frac{bc}{a^2}\int\frac{x^3\partial x}{X}$$

$$\int\frac{\partial x}{x^2X^2}=-\frac{1}{axX}-\frac{3b}{a}\int\frac{\partial x}{X^2}-\frac{5c}{a}\int\frac{x^2\partial x}{X^2}$$

$$\int\frac{\partial x}{x^3X^2}=-\frac{1}{2ax^2X}-\frac{2b}{a}\int\frac{\partial x}{xX^2}-\frac{3c}{a}\int\frac{x\partial x}{X^2}$$

$$\int\frac{\partial x}{x^4X^2}=\left(-\frac{1}{3ax^3}+\frac{5b}{3a^2x}\right)\frac{1}{X}+\left(\frac{5b^2}{a^2}-\frac{7c}{3a}\right)\int\frac{\partial x}{X^2}+\frac{25c^2}{3a^2}\int\frac{x^2\partial x}{X^2}$$

$$\int\frac{\partial x}{x^5X^2}=\left(-\frac{1}{4ax^4}+\frac{3b}{4a^2x^2}\right)\frac{1}{X}+\left(\frac{3b^2}{a^2}-\frac{2c}{a}\right)\int\frac{\partial x}{xX^2}+\frac{9bc}{2a^2}\int\frac{x\partial x}{X^2}$$

$$\int\frac{\partial x}{x^6X^2}=-\frac{1}{5ax^5X}-\frac{7b}{5a}\int\frac{\partial x}{x^4X^2}-\frac{9c}{5a}\int\frac{\partial x}{x^2X^2}$$

$$\int\frac{\partial x}{x^7X^2}=-\frac{1}{6ax^6X}-\frac{4b}{3a}\int\frac{\partial x}{x^5X^2}-\frac{5c}{3a}\int\frac{\partial x}{x^3X^2}$$

$$\int\frac{\partial x}{x^8X^2}=\left(-\frac{1}{7ax^7}+\frac{9b}{35a^2x^5}\right)\frac{1}{X}+\left(\frac{9b^2}{5a^2}-\frac{11c}{7a}\right)\int\frac{\partial x}{x^4X^2}+\frac{81bc}{35a^2}\int\frac{\partial x}{x^2X^2}$$

$$\int\frac{\partial x}{x^9X^2}=\left(-\frac{1}{8ax^8}+\frac{5b}{24a^2x^6}\right)\frac{1}{X}+\left(\frac{5b^2}{3a^2}-\frac{3c}{2a}\right)\int\frac{\partial x}{x^5X^2}+\frac{25bc}{12a^2}\int\frac{\partial x}{x^3X^2}$$

$$\int \frac{\partial x}{x^m(a+bx^2+cx^4)}$$ **Taf. LI.**

VZ. $a + bx^2 + cx^4 = X$

$$\int \frac{\partial x}{xX} = \frac{\log x}{a} - \frac{b}{a}\int \frac{x\partial x}{X} - \frac{c}{a}\int \frac{x^3\partial x}{X}$$

$$\int \frac{\partial x}{x^2X} = -\frac{1}{ax} - \frac{b}{a}\int \frac{\partial x}{X} - \frac{c}{a}\int \frac{x^2\partial x}{X}$$

$$\int \frac{\partial x}{x^3X} = -\frac{1}{2ax^2} - \frac{b}{a}\int \frac{\partial x}{xX} - \frac{c}{a}\int \frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^4X} = -\frac{1}{3ax^3} + \frac{b}{a^2x} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int \frac{\partial x}{X} + \frac{bc}{a^2}\int \frac{x^2\partial x}{X}$$

$$\int \frac{\partial x}{x^5X} = -\frac{1}{4ax^4} + \frac{b}{2a^2x^2} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int \frac{\partial x}{xX} + \frac{bc}{a^2}\int \frac{x\partial x}{X}$$

$$\int \frac{\partial x}{x^6X} = -\frac{1}{5ax^5} + \frac{b}{3a^2x^3} - \left(\frac{b^2}{a^3} - \frac{c}{a^2}\right)\frac{1}{x} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right)\int \frac{\partial x}{X}$$
$$- \left(\frac{b^2c}{a^3} - \frac{c^2}{a^2}\right)\int \frac{x^2\partial x}{X}$$

$$\int \frac{\partial x}{x^7X} = -\frac{1}{6ax^6} - \frac{b}{a}\int \frac{\partial x}{x^5X} - \frac{c}{a}\int \frac{\partial x}{x^3X}$$

$$\int \frac{\partial x}{x^8X} = -\frac{1}{7ax^7} - \frac{b}{a}\int \frac{\partial x}{x^6X} - \frac{c}{a}\int \frac{\partial x}{x^4X}$$

$$\int \frac{\partial x}{x^9X} = -\frac{1}{8ax^8} + \frac{b}{6a^2x^6} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int \frac{\partial x}{x^5X} + \frac{bc}{a^2}\int \frac{\partial x}{x^3X}$$

$$\int \frac{\partial x}{x^{10}X} = -\frac{1}{9ax^9} + \frac{b}{7a^2x^7} + \left(\frac{b^2}{a^2} - \frac{c}{a}\right)\int \frac{\partial x}{x^6X} + \frac{bc}{a^2}\int \frac{\partial x}{x^4X}$$

$$\int \frac{\partial x}{x^{11}X} = -\frac{1}{10ax^{10}} + \frac{b}{8a^2x^8} - \left(\frac{b^2}{6a^3} - \frac{c}{6a^2}\right)\frac{1}{x^6} - \left(\frac{b^3}{a^3} - \frac{2bc}{a^2}\right)\int \frac{\partial x}{x^5X}$$
$$- \left(\frac{b^2c}{a^3} - \frac{c^2}{a^2}\right)\int \frac{\partial x}{x^3X}$$

Taf. LIII. b. $\int \frac{x^m \partial x}{a + bx^3 + cx^6}$

($b^2 - 4ac$ eine negative Gröfse)

VZ. $a + bx^3 + cx^6 = X$, $\sqrt[6]{\frac{a}{c}} = f$

α ein Winkel, dessen Cosinus $= -\frac{b}{2\sqrt{ac}}$

$$\frac{\alpha}{3} = \varphi', \ 120^\circ + \frac{\alpha}{3} = \varphi'', \ 240^\circ + \frac{\alpha}{3} = \varphi'''$$

$$x^2 - 2fx \operatorname{Cos} \varphi' + f^2 = Y'$$

$$x^2 - 2fx \operatorname{Cos} \varphi'' + f^2 = Y''$$

$$x^2 - 2fx \operatorname{Cos} \varphi''' + f^2 = Y'''$$

$$\frac{x \operatorname{Sin} \varphi'}{f - x \operatorname{Cos} \varphi'} = Z', \quad \frac{x \operatorname{Sin} \varphi''}{f - x \operatorname{Cos} \varphi''} = Z'',$$

$$\frac{x \operatorname{Sin} \varphi'''}{f - x \operatorname{Cos} \varphi'''} = Z'''$$

$$\int \frac{\partial x}{X} = \frac{1}{6cf^5 \operatorname{Sin} \alpha} \left\{ \begin{array}{l} - \operatorname{Sin} 2\varphi' \log Y' + 2 \operatorname{Cos} 2\varphi' \operatorname{Arc Tang} Z' \\ - \operatorname{Sin} 2\varphi'' \log Y'' + 2 \operatorname{Cos} 2\varphi'' \operatorname{Arc Tang} Z'' \\ - \operatorname{Sin} 2\varphi''' \log Y''' + 2 \operatorname{Cos} 2\varphi''' \operatorname{Arc Tang} Z''' \end{array} \right\}$$

$$\int \frac{x \partial x}{X} = \frac{1}{6cf^4 \operatorname{Sin} \alpha} \left\{ \begin{array}{l} - \operatorname{Sin} \varphi' \log Y' + 2 \operatorname{Cos} \varphi' \operatorname{Arc Tang} Z' \\ - \operatorname{Sin} \varphi'' \log Y'' + 2 \operatorname{Cos} \varphi'' \operatorname{Arc Tang} Z'' \\ - \operatorname{Sin} \varphi''' \log Y''' + 2 \operatorname{Cos} \varphi''' \operatorname{Arc Tang} Z''' \end{array} \right\}$$

$$\int \frac{x^2 \partial x}{X} = \frac{1}{3cf^3 \operatorname{Sin} \alpha} \operatorname{Arc Tang} \frac{x^3 \operatorname{Sin} \alpha}{f^3 - x^3 \operatorname{Cos} \alpha}$$

$$\int \frac{x^3 \partial x}{X} = \frac{1}{6cf^2 \operatorname{Sin} \alpha} \left\{ \begin{array}{l} \operatorname{Sin} \varphi' \log Y' + 2 \operatorname{Cos} \varphi' \operatorname{Arc Tang} Z' \\ + \operatorname{Sin} \varphi'' \log Y'' + 2 \operatorname{Cos} \varphi'' \operatorname{Arc Tang} Z'' \\ + \operatorname{Sin} \varphi''' \log Y''' + 2 \operatorname{Cos} \varphi''' \operatorname{Arc Tang} Z''' \end{array} \right\}$$

$$\int \frac{x^4 \partial x}{X} = \frac{1}{6cf \operatorname{Sin} \alpha} \left\{ \begin{array}{l} \operatorname{Sin} 2\varphi' \log Y' + 2 \operatorname{Cos} 2\varphi' \operatorname{Arc Tang} Z' \\ + \operatorname{Sin} 2\varphi'' \log Y'' + 2 \operatorname{Cos} 2\varphi'' \operatorname{Arc Tang} Z'' \\ + \operatorname{Sin} 2\varphi''' \log Y''' + 2 \operatorname{Cos} 2\varphi''' \operatorname{Arc Tang} Z''' \end{array} \right\}$$

$$\int \frac{x^5 \partial x}{X} = \frac{1}{6c \operatorname{Sin} \alpha} \left\{ \begin{array}{l} \operatorname{Sin} 3\varphi' \log Y' + 2 \operatorname{Cos} 3\varphi' \operatorname{Arc Tang} Z' \\ + \operatorname{Sin} 3\varphi'' \log Y'' + 2 \operatorname{Cos} 3\varphi'' \operatorname{Arc Tang} Z'' \\ + \operatorname{Sin} 3\varphi''' \log Y''' + 2 \operatorname{Cos} 3\varphi''' \operatorname{Arc Tang} Z''' \end{array} \right\}$$

$$\int \frac{x^m \partial x}{a + bx^3 + cx^6}.$$ *Taf. LIII. a.*

($b^2 - 4ac$ eine positive Gröſse)

$$a + bx^3 + cx^6 = X$$
$$\tfrac{1}{2}b - \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = f,\ \tfrac{1}{2}b + \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = g$$
$$\sqrt{(b^2 - 4ac)} = g - f = h$$

$$= \frac{c}{h}\left[\int \frac{\partial x}{cx^3 + f} - \int \frac{\partial x}{cx^3 + g}\right]$$

$$= \frac{c}{h}\left[\int \frac{x\partial x}{cx^3 + f} - \int \frac{x\partial x}{cx^3 + g}\right]$$

$$= \frac{1}{3h} \log \frac{cx^3 + f}{cx^3 + g}$$

$$= \frac{g}{h}\int \frac{\partial x}{cx^3 + g} - \frac{f}{h}\int \frac{\partial x}{cx^3 + f}$$

$$= \frac{g}{h}\int \frac{x\partial x}{cx^3 + g} - \frac{f}{h}\int \frac{x\partial x}{cx^3 + f}$$

$$= \frac{g}{3h} \log (cx^3 + g) - \frac{f}{3h} \log (cx^3 + f)$$

$$= \frac{x}{c} - \frac{a}{c}\int \frac{\partial x}{X} - \frac{b}{c}\int \frac{x^3 \partial x}{X}$$

$$= \frac{x^2}{2c} - \frac{a}{c}\int \frac{x\partial x}{X} - \frac{b}{c}\int \frac{x^4 \partial x}{X}$$

$$= \frac{x^3}{3c} - \frac{a}{c}\int \frac{x^2 \partial x}{X} - \frac{b}{c}\int \frac{x^5 \partial x}{X}$$

$$= \frac{x^4}{4c} - \frac{bx}{c^2} + \frac{ab}{c^2}\int \frac{\partial x}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^3 \partial x}{X}$$

$$= \frac{x^5}{5c} - \frac{bx^2}{2c^2} + \frac{ab}{c^2}\int \frac{x\partial x}{X} + \left(\frac{b^2}{c^2} - \frac{a}{c}\right)\int \frac{x^4 \partial x}{X}$$

Taf. LV.

$$\int \frac{x^m \partial x}{X}$$

(*X* ein Product von binomischen und trinomischen Factoren.)

$$\int \frac{\partial x}{(x+f)(x+g)} = \frac{1}{g-f} \log \frac{x+f}{x+g}$$

$$\int \frac{x\partial x}{(x+f)(x+g)} = \frac{1}{g-f} [g \log (x+g) - f \log (x+f)]$$

$$\int \frac{\partial x}{(x+f)(x+g)^2} = \frac{1}{(g-f)(x+g)} + \frac{1}{(g-f)^2} \log \frac{x+f}{x+g}$$

$$\int \frac{x\partial x}{(x+f)(x+g)^2} = \frac{-g}{(g-f)(x+g)} - \frac{f}{(g-f)^2} \log \frac{x+f}{x+g}$$

$$\int \frac{x^2 \partial x}{(x+f)(x+g)^2} = \frac{g^2}{(g-f)(x+g)} + \frac{f^2}{(g-f)^2} \log (x+f) + \frac{g^2 - 2fg}{(g-f)^2} \log (x+g)$$

$$\int \frac{\partial x}{(x+f)^2 (x+g)^2} = \frac{-1}{(g-f)^2} \left(\frac{1}{x+f} + \frac{1}{x+g} \right) - \frac{2}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x\partial x}{(x+f)^2 (x+g)^2} = \frac{1}{(g-f)^2} \left(\frac{f}{x+f} + \frac{g}{x+g} \right) + \frac{f+g}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x^2 \partial x}{(x+f)^2 (x+g)^2} = \frac{-1}{(g-f)^2} \left(\frac{f^2}{x+f} + \frac{g^2}{x+g} \right) - \frac{2fg}{(g-f)^3} \log \frac{x+f}{x+g}$$

$$\int \frac{x^3 \partial x}{(x+f)^2 (x+g)^2} = \frac{1}{(g-f)^2} \left(\frac{f^3}{x+f} + \frac{g^3}{x+g} \right) + \frac{f^2 (3g-f)}{(g-f)^3} \log (x+f) + \frac{g^2 (g-3f)}{(g-f)^3} \log (x+g)$$

$$\int \frac{\partial x}{(x+f)(x+g)(x+h)} = \frac{1}{(g-f)(h-f)} \log (x+f) + \frac{1}{(f-g)(h-g)} \log (x+g) + \frac{1}{(f-h)(g-h)} \log (x+h)$$

$$\int \frac{x\partial x}{(x+f)(x+g)(x+h)} = -\frac{f}{(g-f)(h-g)} \log (x+f) - \frac{g}{(f-g)(h-g)} \log (x+g) - \frac{h}{(f-h)(g-h)} \log (x+h)$$

$$\int\frac{x^m\partial x}{(a+bx^3+cx^6)^2},\ \int\frac{\partial x}{x^m(a+bx^3+cx^6)}$$ *Taf. LIV.*

VZ. $a+bx^3+cx^6=X,\ 3a(b^2-4ac)=k$

$$\int\frac{\partial x}{X^2}=[bcx^4+(b^2-2ac)x]\frac{1}{kX}+\frac{2b^2-10ac}{k}\int\frac{\partial x}{X}+\frac{2bc}{k}\int\frac{x^3\partial x}{X}$$

$$\int\frac{x\partial x}{X^2}=[bcx^5+(b^2-2ac)x^2]\frac{1}{kX}+\frac{b^2-8ac}{k}\int\frac{x\partial x}{X}+\frac{bc}{k}\int\frac{x^4\partial x}{X}$$

$$\int\frac{x^2\partial x}{X^2}=[bcx^6+(b^2-2ac)x^3]\frac{1}{kX}-\frac{6ac}{k}\int\frac{x^2\partial x}{X}$$

$$\int\frac{x^3\partial x}{X^2}=[bcx^6+(b^2-2ac)x^4]\frac{1}{kX}-\frac{b^2+4ac}{k}\int\frac{x^3\partial x}{X}-\frac{bc}{k}\int\frac{x^6\partial x}{X}$$

$$\int\frac{\partial x}{xX}=\frac{\log x}{a}-\frac{b}{a}\int\frac{x^2\partial x}{X}-\frac{c}{a}\int\frac{x^5\partial x}{X}$$

$$\int\frac{\partial x}{x^2X}=-\frac{1}{ax}-\frac{b}{a}\int\frac{x\partial x}{X}-\frac{c}{a}\int\frac{x^4\partial x}{X}$$

$$\int\frac{\partial x}{x^3X}=-\frac{1}{2ax^2}-\frac{b}{a}\int\frac{\partial x}{X}-\frac{c}{a}\int\frac{x^3\partial x}{X}$$

$$\int\frac{\partial x}{x^4X}=-\frac{1}{3ax^3}-\frac{b}{a}\int\frac{\partial x}{xX}-\frac{c}{a}\int\frac{x^2\partial x}{X}$$

$$\int\frac{\partial x}{x^5X}=-\frac{1}{4ax^4}+\frac{b}{a^2x}+\left(\frac{b^2}{a^2}-\frac{c}{a}\right)\int\frac{x\partial x}{X}+\frac{bc}{a^2}\int\frac{x^4\partial x}{X}$$

$$\int\frac{\partial x}{x^6X}=-\frac{1}{5ax^5}+\frac{b}{2a^2x^2}+\left(\frac{b^2}{a^2}-\frac{c}{a}\right)\int\frac{\partial x}{X}+\frac{bc}{a^2}\int\frac{x^3\partial x}{X}$$

$$\int\frac{\partial x}{x^7X}=-\frac{1}{6ax^6}+\frac{b}{3a^2x^3}+\left(\frac{b^2}{a^2}-\frac{c}{a}\right)\int\frac{\partial x}{xX}+\frac{bc}{a^2}\int\frac{x^2\partial x}{X}$$

$$\int\frac{\partial x}{x^8X}=-\frac{1}{7ax^7}+\frac{b}{4a^2x^4}-\left(\frac{b^2}{a^3}-\frac{c}{a^2}\right)\frac{1}{x}-\left(\frac{b^3}{a^3}-\frac{2bc}{a^2}\right)\int\frac{x\partial x}{X}$$
$$-\left(\frac{b^2c}{a^3}-\frac{c^2}{a^2}\right)\int\frac{x^4\partial x}{X}$$

Taf. LV.

$$\int \frac{x^m \partial x}{X}$$

(*X* ein Product von binomischen und trinomischen Factoren)

$$\int \frac{x^3 \partial x}{(x+f)^2(x^2+a)} = \frac{f^2(f^2+3a)}{(f^2+a)^2}\log(x+f) - \frac{a(f^2-a)}{2(f^2+a)^2}\log(x^2+a)$$
$$-\frac{2a^2 f}{(f^2+a)^2}\int \frac{\partial x}{x^2+a} + \frac{f^3}{(f^2+a)(x+f)}$$

$$\int \frac{\partial x}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times$$
$$\left[\tfrac{1}{2}\log\frac{(x+f)^2}{x^2+ax+b} + (f-\tfrac{1}{2}a)\int \frac{\partial x}{x^2+ax+b}\right]$$

$$\int \frac{x\partial x}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times$$
$$\left[-\tfrac{1}{2}f\log\frac{(x+f)^2}{x^2+ax+b} + (b-\tfrac{1}{2}af)\int \frac{\partial x}{x^2+ax+b}\right]$$

$$\int \frac{x^2\partial x}{(x+f)(x^2+ax+b)} = \frac{1}{f^2-af+b} \times$$
$$\left[\begin{array}{l} f^2\log(x+f) + \tfrac{1}{2}(b-af)\log(x^2+ax+b) \\ +\tfrac{1}{2}(a^2f-ab-2bf)\int \dfrac{\partial x}{x^2+ax+b} \end{array}\right]$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^n} = -\frac{1}{(n-1)bX^{n-1}}$$

$$\int \frac{x^m \partial x}{X} = \frac{x^m}{mb} - \frac{a}{b}\int \frac{x^{m-1}\partial x}{X}$$

$$\int \frac{x^m \partial x}{X} = \frac{x^m}{mb} - \frac{ax^{m-1}}{(m-1)b^2} + \frac{a^2x^{m-2}}{(m-2)b^3} - \frac{a^3x^{m-3}}{(m-3)b^4} + \text{etc.}$$
$$\pm \frac{a^{i-1}x^{m-i+1}}{(m-i+1)b^i} \mp \frac{a^i}{b^i}\int \frac{x^{m-i}\partial x}{X}$$

$$\int \frac{x^m \partial x}{X} = \frac{x^m}{mb} - \frac{ax^{m-1}}{(m-1)b^2} + \frac{a^2x^{m-2}}{(m-2)b^3} - \frac{a^3x^{m-3}}{(m-3)b^4} + \text{etc.}$$
$$\pm \frac{a^{m-1}x}{b^m} \mp \frac{a^m}{b^{m+1}} \log X$$

$$\int \frac{x^m \partial x}{X^2} = \frac{x^m}{(m-1)bX} - \frac{ma}{(m-1)b}\int \frac{x^{m-1}\partial x}{X^2}$$

$$\int \frac{x^m \partial x}{X^2} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \text{etc.}$$
$$\pm Kx^{m-i+2} \mp Lx^{m-i+1})\frac{1}{X} \pm L(m-i+1)a\int \frac{x^{m-i}\partial x}{X^2}$$

$$A = \frac{1}{(m-1)b},\quad B = \frac{ma}{(m-2)b}A,\quad C = \frac{(m-1)a}{(m-3)b}B$$
$$D = \frac{(m-2)a}{(m-4)b}C,\quad E = \frac{(m-3)a}{(m-5)b}D,\ \text{etc.},\quad L = \frac{(m-i+2)a}{(m-i)b}K.$$

$$\int \frac{x^m \partial x}{X^3} = \frac{x^m}{(m-2)bX^2} - \frac{ma}{(m-2)b}\int \frac{x^{m-1}\partial x}{X^3}$$

$\int \frac{x^m \partial x}{X^3} = (Ax^m - Bx^{m-1}$ [illegible] $- Dx^{m-3} + Ex^{m-4} - \text{etc.}$

$\pm Kx^{m-}$ [illegible] $-i+1)a\int$ [illegible]

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$A = \frac{1}{(m-2)b},\ B = \frac{ma}{(m-3)b}A,\ C = \frac{(m-1)a}{(m-4)b}B,$$

$$D = \frac{(m-2)a}{(m-5)b}C,\ E = \frac{(m-3)a}{(m-6)b}D, \text{ etc.},\ L = \frac{(m-i+2)a}{(m-i-1)b}K.$$

$$\int \frac{x^m \partial x}{X^4} = \frac{x^m}{(m-3)bX^3} - \frac{ma}{(m-3)b}\int \frac{x^{m-1}\partial x}{X^4}$$

$$\int \frac{x^m \partial x}{X^4} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \text{etc.}$$

$$\pm Kx^{m-i+2} \mp Lx^{m-i+1})\frac{1}{X^3} \pm L(m-i+1)a\int \frac{x^{m-i}\partial x}{X^4}$$

$$A = \frac{1}{(m-3)b},\ B = \frac{ma}{(m-4)b}A,\ C = \frac{(m-1)a}{(m-5)b}B,$$

$$D = \frac{(m-2)a}{(m-6)b}C,\ E = \frac{(m-3)a}{(m-7)b}D, \text{ etc.},\ L = \frac{(m-i+2)a}{(m-i-2)b}K.$$

$$\int \frac{x^m \partial x}{X^5} = \frac{x^m}{(m-4)bX^4} - \frac{ma}{(m-4)b}\int \frac{x^{m-1}\partial x}{X^5}$$

$$\int \frac{x^m \partial x}{X^5} = (Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} + \text{etc.}$$

$$\pm Kx^{m-i+2} \mp Lx^{m-i+1})\frac{1}{X^4} \pm L(m-i+1)a\int \frac{x^{m-i}\partial x}{X^5}$$

$$A = \frac{1}{(m-4)b},\ B = \frac{ma}{(m-5)b}A,\ C = \frac{(m-1)a}{(m-6)b}B,$$

$$D = \frac{(m-2)a}{(m-7)b}C,\ E = \frac{(m-3)a}{(m-8)b}D, \text{ etc.},\ L = \frac{(m-i+2)a}{(m-i-3)b}K.$$

$$\int \frac{x^m \partial x}{X^6} = \frac{x^m}{(m-5)bX^5} - \frac{ma}{(m-5)b}\int \frac{x^{m-1}\partial x}{X^6}$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$\int \frac{x^m \partial x}{X^6} = \Big(Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + Ex^{m-4} - \text{etc.}$$

$$\pm Kx^{m-i+2} \mp Lx^{m-i+1}\Big)\frac{1}{X^5} \pm L(m-i+1)a\int \frac{x^{m-i}\partial x}{X^5}$$

$$A = \frac{1}{(m-5)b},\quad B = \frac{ma}{(m-6)b}A,\quad C = \frac{(m-1)a}{(m-7)b}B,$$

$$D = \frac{(m-2)a}{(m-8)b}C,\quad E = \frac{(m-3)a}{(m-9)b}D,\ \text{etc.},\quad L = \frac{(m-i+2)a}{(m-i-4)b}K.$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a}\int \frac{\partial x}{x^{m-1}X}$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} + \frac{b}{(m-2)a^2x^{m-2}} - \frac{b^2}{(m-3)a^3x^{m-3}}$$

$$+\frac{b^3}{(m-4)a^4x^{m-4}} - \text{etc.} \pm \frac{b^{i-1}}{(m-i)a^ix^{m-i}} + \frac{b^i}{a^i}\int \frac{\partial x}{x^{m-i}X}$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} + \frac{b}{(m-2)a^2x^{m-2}} - \frac{b^2}{(m-3)a^3x^{m-3}}$$

$$+\frac{b^3}{(m-4)a^4x^{m-3}} - \text{etc.} \pm \frac{b^{m-2}}{a^{m-1}x} \mp \frac{b^{m-1}}{a^m}\log\frac{X}{x}$$

$$\int \frac{\partial x}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{mb}{(m-1)a}\int \frac{\partial x}{x^{m-1}X^2}$$

$$\int \frac{\partial x}{x^m X^2} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.}$$

$$\pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\Big)\frac{1}{X} \mp L(m-i+1)b\int \frac{\partial x}{x^{m-i}X^2}$$

$$A = -\frac{1}{(m-1)a},\quad B = \frac{mb}{(m-2)a}A,\quad C = \frac{(m-1)b}{(m-3)a}B,$$

$$D = \frac{(m-2)b}{(m-4)a}C,\quad E = \frac{(m-3)b}{(m-5)a}D,\ \text{etc.},\quad L = \frac{(m-i+2)b}{(m-i)a}K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$\int \frac{\partial x}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+1)b}{(m-1)a}\int \frac{\partial x}{x^{m-1}X^3}$$

$$\int \frac{\partial x}{x^m X^3} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.}$$

$$\pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\Big)\frac{1}{X^2} \mp L(m-i+2)b\int \frac{\partial x}{x^{m-i}X^3}$$

$$A = -\frac{1}{(m-1)a},\quad B = \frac{(m+1)b}{(m-2)a}A,\quad C = \frac{mb}{(m-3)a}B,$$

$$D = \frac{(m-1)b}{(m-4)a}C,\quad E = \frac{(m-2)b}{(m-5)a}D, \text{ etc.},\quad L = \frac{(m-i+3)b}{(m-i)a}K.$$

$$\int \frac{\partial x}{x^m X^4} = -\frac{1}{(m-1)ax^{m-1}X^3} - \frac{(m+2)b}{(m-1)a}\int \frac{\partial x}{x^{m-1}X^4}$$

$$\int \frac{\partial x}{x^m X^4} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.}$$

$$\pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\Big)\frac{1}{X^3} \mp L(m-i+3)b\int \frac{\partial x}{x^{m-i}X^4}$$

$$A = -\frac{1}{(m-1)a},\quad B = \frac{(m+2)b}{(m-2)a}A,\quad C = \frac{(m+1)b}{(m-3)a}B,$$

$$D = \frac{mb}{(m-4)a}C,\quad E = \frac{(m-1)b}{(m-5)a}D, \text{ etc.},\quad L = \frac{(m-i+4)b}{(m-i)a}K.$$

$$\int \frac{\partial x}{x^m X^5} = -\frac{1}{(m-1)ax^{m-1}X^4} - \frac{(m+3)b}{(m-1)a}\int \frac{\partial x}{x^{m-1}X^5}$$

$$\int \frac{\partial x}{x^m X^5} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.}$$

$$\pm \frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\Big)\frac{1}{X^4} \mp L(m-i+4)b\int \frac{\partial x}{x^{m-i}X^5}$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+3)b}{(m-2)a}A,\ C = \frac{(m+2)b}{(m-3)a}B,$$

$$D = \frac{(m+1)b}{(m-4)a}C,\ E = \frac{mb}{(m-5)a}D, \text{ etc., } L = \frac{(m-i+5)b}{(m-i)a}K.$$

$$\int\frac{\partial x}{x^m X^6} = -\frac{1}{(m-1)ax^{m-1}X^5} - \frac{(m+4)b}{(m-1)a}\int\frac{\partial x}{x^{m-1}X^6}$$

$$\int\frac{\partial x}{x^m X^6} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \text{etc.}\right.$$

$$\left.\pm\frac{K}{x^{m-i+1}} \mp \frac{L}{x^{m-i}}\right)\frac{1}{X^5} \mp L(m-i+5)b\int\frac{\partial x}{x^{m-i}X^6}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+4)b}{(m-2)a}A,\ C = \frac{(m+3)b}{(m-3)a}B,$$

$$D = \frac{(m+2)b}{(m-4)a}C,\ E = \frac{(m+1)b}{(m-5)a}D, \text{ etc., } L = \frac{(m-i+6)b}{(m-i)a}K.$$

VZ. $a + bx^2 = X$

$$\int\frac{\partial x}{X^p} = \frac{x}{(p-1)2aX^{p-1}} + \frac{2p-3}{(p-1)2a}\int\frac{\partial x}{X^{p-1}}$$

$$\int\frac{\partial x}{X^p} = \left(\frac{A}{X^{p-1}} + \frac{B}{X^{p-2}} + \frac{C}{X^{p-3}} + \frac{D}{X^{p-4}} + \frac{E}{X^{p-5}} + \text{etc.}\right.$$

$$\left.+\frac{K}{X^{p-i+1}} + \frac{L}{X^{p-i}}\right)x + L(2p-2i-1)\int\frac{\partial x}{X^{p-i}}$$

$$A = \frac{1}{(p-1)2a},\ B = \frac{2p-3}{(p-2)2a}A,\ C = \frac{2p-5}{(p-3)2a}B,$$

$$D = \frac{2p-7}{(p-4)2a}C,\ E = \frac{2p-9}{(p-5)2a}D, \text{ etc., } L = \frac{2p-2i+1}{(p-i)2a}K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$\int \frac{x^m \partial x}{X} = \frac{x^{m-1}}{(m-1)b} - \frac{a}{b} \int \frac{x^{m-2} \partial x}{X}$$

$$\int \frac{x^m \partial x}{X} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.}$$
$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1} \pm L(m-2i+1)a \int \frac{x^{m-2i} \partial x}{X}$$

$$A = \frac{1}{(m-1)b}, \quad B = \frac{(m-1)a}{(m-3)b} A, \quad C = \frac{(m-3)a}{(m-5)b} B,$$
$$D = \frac{(m-5)a}{(m-7)b} C, \quad E = \frac{(m-7)a}{(m-9)b} D, \text{ etc.}, \quad L = \frac{(m-2i+3)a}{(m-2i+1)b} K.$$

$$\int \frac{x^m \partial x}{X^2} = \frac{x^{m-1} \partial x}{(m-3)bX} - \frac{(m-1)a}{(m-3)b} \int \frac{x^{m-2} \partial x}{X^2}$$

$$\int \frac{x^m \partial x}{X^2} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.}$$
$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) \frac{1}{X} \pm L(m-2i+1)a \int \frac{x^{m-2i} \partial x}{X^2}$$

$$A = \frac{1}{(m-3)b}, \quad B = \frac{(m-1)a}{(m-5)b} A, \quad C = \frac{(m-3)a}{(m-7)b} B,$$
$$D = \frac{(m-5)a}{(m-9)b} C, \quad E = \frac{(m-7)a}{(m-11)b} D, \text{ etc.}, \quad L = \frac{(m-2i+3)a}{(m-2i-1)b} K.$$

$$\int \frac{x^m \partial x}{X^3} = \frac{x^{m-1}}{(m-5)b} - \frac{(m-1)a}{(m-5)b} \int \frac{x^{m-2} \partial x}{X^3}$$

$$\int \frac{x^m \partial x}{X^3} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.}$$
$$\pm Kx^{m-2i+3} \mp Lx^{m-2i+1}) \frac{1}{X^2} \pm L(m-2i+1)a \int \frac{x^{m-2i} \partial x}{X^3}$$

$$A = \frac{1}{(m-5)b}, \quad B = \frac{(m-1)a}{(m-7)b} A, \quad C = \frac{(m-3)a}{(m-9)b} B,$$
$$D = \frac{(m-5)a}{(m-11)b} C, \quad E = \frac{(m-7)a}{(m-13)b} D, \text{ etc.}, \quad L = \frac{(m-2i+3)a}{(m-2i-3)b} K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$\int\frac{x^m\partial x}{X^4} = \frac{x^{m-1}}{(m-7)bX^3} - \frac{(m-1)a}{(m-7)b}\int\frac{x^{m-2}\partial x}{X^4}$$

$$\int\frac{x^m\partial x}{X^4} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.}$$

$$\pm Kx^{m-2i+5} \mp Lx^{m-2i+1})\frac{1}{X^3} \pm L(m-2i+1)a\int\frac{x^{m-2i}\partial x}{X^4}$$

$$A = \frac{1}{(m-7)b},\ B = \frac{(m-1)a}{(m-9)b}A,\ C = \frac{(m-3)a}{(m-11)b}B,$$

$$D = \frac{(m-5)a}{(m-13)b}C,\ E = \frac{(m-7)a}{(m-15)b}E, \text{etc.},\ L = \frac{(m-2i+3)a}{(m-2i-5)b}K.$$

$$\int\frac{x^m\partial x}{X^5} = \frac{x^{m-1}}{(m-9)bX^4} - \frac{(m-1)a}{(m-9)b}\int\frac{x^{m-2}\partial x}{X^5}$$

$$\int\frac{x^m\partial x}{X^5} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.}$$

$$\pm Kx^{m-2i+5} \mp Lx^{m-2i+1})\frac{1}{X^4} \pm L(m-2i+1)a\int\frac{x^{m-2i}\partial x}{X^5}$$

$$A = \frac{1}{(m-9)b},\ B = \frac{(m-1)a}{(m-11)b}A,\ C = \frac{(m-3)a}{(m-13)b}B,$$

$$D = \frac{(m-5)a}{(m-15)b}C,\ E = \frac{(m-7)a}{(m-17)b}D, \text{etc.},\ L = \frac{(m-2i+3)a}{(m-2i-7)b}K.$$

$$\int\frac{x^m\partial x}{X^6} = \frac{x^{m-1}}{(m-11)bX^5} - \frac{(m-1)a}{(m-11)b}\int\frac{x^{m-2}\partial x}{X^6}$$

$$\int\frac{x^m\partial x}{X^6} = (Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \text{etc.}$$

$$\pm Kx^{m-2i+5} \mp Lx^{m-2i+1})\frac{1}{X^5} \pm L(m-2i+1)a\int\frac{x^{m-2i}\partial x}{X^6}$$

$$A = \frac{1}{(m-11)b},\ B = \frac{(m-1)a}{(m-13)b}A,\ C = \frac{(m-3)a}{(m-15)b}B,$$

$$D = \frac{(m-5)a}{(m-17)b}C,\ E = \frac{(m-7)a}{(m-19)b}D, \text{etc.},\ L = \frac{(m-2i+3)a}{(m-2i-9)b}K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a}\int \frac{\partial x}{x^{m-2}X}$$

$$\int \frac{\partial x}{x^m X} = \frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.}$$
$$\pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}} \mp L(m-2i+1)b\int \frac{\partial x}{x^{m-2i}X}$$

$$A = -\frac{1}{(m-1)a},\quad B = \frac{(m-1)b}{(m-3)a}A,\quad C = \frac{(m-3)b}{(m-5)a}B,$$
$$D = \frac{(m-5)b}{(m-7)a}C,\quad E = \frac{(m-7)b}{(m-9)a}D,\ \text{etc.},\quad L = \frac{(m-2i+3)b}{(m-2i+1)a}K.$$

$$\int \frac{\partial x}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+1)b}{(m-1)a}\int \frac{\partial x}{x^{m-2}X^2}$$

$$\int \frac{\partial x}{x^m X^2} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.}$$
$$\pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\Big)\frac{1}{X} \mp L(m-2i+3)b\int \frac{\partial x}{x^{m-2i}X^2}$$

$$A = -\frac{1}{(m-1)a},\quad B = \frac{(m+1)b}{(m-3)a}A,\quad C = \frac{(m-1)b}{(m-5)a}B,$$
$$D = \frac{(m-3)b}{(m-7)a}C,\quad E = \frac{(m-5)b}{(m-9)a}D,\ \text{etc.},\quad L = \frac{(m-2i+5)b}{(m-2i+1)a}K.$$

$$\int \frac{\partial x}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+3)b}{(m-1)a}\int \frac{\partial x}{x^{m-2}X^3}$$

$$\int \frac{\partial x}{x^m X^3} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.}$$
$$\pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\Big)\frac{1}{X^2} \mp L(m-2i+5)b\int \frac{\partial x}{x^{m-2i}X^3}$$

$$A = -\frac{1}{(m-1)a},\quad B = \frac{(m+3)b}{(m-3)a}A,\quad C = \frac{(m+1)b}{(m-5)a}B,$$
$$D = \frac{(m-1)b}{(m-7)a}C,\quad E = \frac{(m-3)b}{(m-9)a}D,\ \text{etc.},\quad L = \frac{(m-2i+7)b}{(m-2i+1)a}K.$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{x^m X^4} = -\frac{1}{(m-1)ax^{m-1}X^3} - \frac{(m+5)b}{(m-1)a}\int \frac{\partial x}{x^{m-2}X^4}$$

$$\int \frac{\partial x}{x^m X^4} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.}\right.$$
$$\left. \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right)\frac{1}{X^3} \mp L(m-2i+7)b\int \frac{\partial x}{x^{m-2i}X^4}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+5)b}{(m-3)a}A,\ C = \frac{(m+3)b}{(m-5)a}B,$$
$$D = \frac{(m+1)b}{(m-7)a}C,\ E = \frac{(m-1)b}{(m-9)a}D,\ \text{etc.},\ L = \frac{(m-2i+9)b}{(m-2i+1)a}K.$$

$$\int \frac{\partial x}{x^m X^5} = -\frac{1}{(m-1)ax^{m-1}X^4} - \frac{(m+7)b}{(m-1)a}\int \frac{\partial x}{x^{m-2}X^5}$$

$$\int \frac{\partial x}{x^m X^5} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.}\right.$$
$$\left. \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right)\frac{1}{X^4} \mp L(m-2i+9)b\int \frac{\partial x}{x^{m-2i}X^5}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+7)b}{(m-3)a}A,\ C = \frac{(m+5)b}{(m-5)a}B,$$
$$D = \frac{(m+3)b}{(m-7)a}C,\ E = \frac{(m+1)b}{(m-9)a}D,\ \text{etc},\ L = \frac{(m-2i+11)b}{(m-2i+1)a}K.$$

$$\int \frac{\partial x}{x^m X^6} = -\frac{1}{(m-1)ax^{m-1}X^5} - \frac{(m+9)b}{(m-1)a}\int \frac{\partial x}{x^{m-2}X^6}$$

$$\int \frac{\partial x}{x^m X^6} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \text{etc.}\right.$$
$$\left. \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\right)\frac{1}{X^5} \mp L(m-2i+11)b\int \frac{\partial x}{x^{m-2i}X^6}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(m+9)b}{(m-3)a}A,\ C = \frac{(m+7)b}{(m-5)a}B,$$
$$D = \frac{(m+5)b}{(m-7)a}C,\ E = \frac{(m+3)b}{(m-9)a}D,\ \text{etc.},\ L = \frac{(m-2i+13)b}{(m-2i+1)a}K.$$

Tafel

einiger allgemeineren Formeln.

V. $a+bx+cx^2=X,\ 4ac-b^2=k$

$$\int\frac{\partial x}{X^p}=\frac{2cx+b}{(p-1)kX^{p-1}}+\frac{(2p-3)2c}{(p-1)k}\int\frac{\partial x}{X^{p-1}}$$

$$\int\frac{\partial x}{X^p}=\left(\frac{A}{X^{p-1}}+\frac{B}{X^{p-2}}+\frac{C}{X^{p-3}}+\text{etc.}+\frac{K}{X^{p-i+1}}+\frac{L}{X^{p-i}}\right)(2cx+b)$$
$$+L(4p-2i-2)c\int\frac{\partial x}{X^{p-i}}$$

$$A=\frac{1}{(p-1)k},\quad B=\frac{(4p-6)c}{(p-2)k}A,\quad C=\frac{(4p-10)c}{(p-3)k}B,$$
$$D=\frac{(4p-14)c}{(p-4)k}C,\quad E=\frac{(4p-18)c}{(p-5)k}D,\ \text{etc.},\quad L=\frac{(4p-4i+2)c}{(p-i)k}K.$$

$$\int\frac{x^m\partial x}{X}=\frac{x^{m-1}}{(m-1)c}-\frac{a}{c}\int\frac{x^{m-2}\partial x}{X}-\frac{b}{c}\int\frac{x^{m-1}\partial x}{X}$$

$$\int\frac{x^m\partial x}{X^2}=\frac{x^{m-1}}{(m-3)cX}-\frac{(m-1)a}{(m-3)c}\int\frac{x^{m-2}\partial x}{X^2}-\frac{(m-2)b}{(m-3)c}\int\frac{x^{m-1}\partial x}{X^2}$$

$$\int\frac{x^m\partial x}{X^3}=\frac{x^{m-1}}{(m-5)cX^2}-\frac{(m-1)a}{(m-5)c}\int\frac{x^{m-2}\partial x}{X^3}-\frac{(m-3)b}{(m-5)c}\int\frac{x^{m-1}\partial x}{X^3}$$

$$\int\frac{x^m\partial x}{X^4}=\frac{x^{m-1}}{(m-7)cX^3}-\frac{(m-1)a}{(m-7)c}\int\frac{x^{m-2}\partial x}{X^4}-\frac{(m-4)b}{(m-7)c}\int\frac{x^{m-1}\partial x}{X^4}$$

$$\int\frac{x^m\partial x}{X^5}=\frac{x^{m-1}}{(m-9)cX^4}-\frac{(m-1)a}{(m-9)c}\int\frac{x^{m-2}\partial x}{X^5}-\frac{(m-5)b}{(m-9)c}\int\frac{x^{m-1}\partial x}{X^5}$$

$$\int\frac{x^m\partial x}{X^6}=\frac{x^{m-1}}{(m-11)cX^5}-\frac{(m-1)a}{(m-11)c}\int\frac{x^{m-2}\partial x}{X^6}-\frac{(m-6)b}{(m-11)c}\int\frac{x^{m-1}\partial x}{X^6}$$

$$\int\frac{\partial x}{x^mX}=-\frac{1}{(m-1)ax^{m-1}}-\frac{b}{a}\int\frac{\partial x}{x^{m-1}X}-\frac{c}{a}\int\frac{\partial x}{x^{m-2}X}$$

$$\int\frac{\partial x}{x^mX^2}=-\frac{1}{(m-1)ax^{m-1}X}-\frac{mb}{(m-1)a}\int\frac{\partial x}{x^{m-1}X^2}-\frac{(m+1)c}{(m-1)a}\int\frac{\partial x}{x^{m-2}X^2}$$

$$\int\frac{\partial x}{x^mX^3}=-\frac{1}{(m-1)ax^{m-1}X^2}-\frac{(m+1)b}{(m-1)a}\int\frac{\partial x}{x^{m-1}X^3}-\frac{(m+3)c}{(m-1)a}\int\frac{\partial x}{x^{m-2}X^3}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a+bx+cx^2=X,\ 4ac-b^2=k$

$$\int\frac{\partial x}{x^m X^4}=-\frac{1}{(m-1)ax^{m-1}X^4}-\frac{(m+2)b}{(m-1)a}\int\frac{\partial x}{x^{m-1}X^4}-\frac{(m+5)c}{(m-1)a}\int\frac{\partial x}{x^{m-2}X^4}$$

$$\int\frac{\partial x}{x^m X^5}=-\frac{1}{(m-1)ax^{m-1}X^5}-\frac{(m+3)b}{(m-1)a}\int\frac{\partial x}{x^{m-1}X^5}-\frac{(m+7)c}{(m-1)a}\int\frac{\partial x}{x^{m-2}X^5}$$

$$\int\frac{\partial x}{x^m X^6}=-\frac{1}{(m-1)ax^{m-1}X^6}-\frac{(m+4)a}{(m-1)a}\int\frac{\partial x}{x^{m-1}X^6}-\frac{(m+9)c}{(m-1)a}\int\frac{\partial x}{x^{m-2}X^6}$$

VZ. $a+bx^3=X$

$$\int\frac{x^m\partial x}{X}=\frac{x^{m-2}}{(m-2)b}-\frac{a}{b}\int\frac{x^{m-3}\partial x}{X}$$

$$\int\frac{x^m\partial x}{X^2}=\frac{x^{m-2}}{(m-5)bX}-\frac{(m-2)a}{(m-5)b}\int\frac{x^{m-3}\partial x}{X^2}$$

$$\int\frac{x^m\partial x}{X^3}=\frac{x^{m-2}}{(m-8)bX^2}-\frac{(m-2)a}{(m-8)b}\int\frac{x^{m-3}\partial x}{X^3}$$

$$\int\frac{\partial x}{x^m X}=-\frac{1}{(m-1)ax^{m-1}}-\frac{b}{a}\int\frac{\partial x}{x^{m-3}X}$$

$$\int\frac{\partial x}{x^m X^2}=-\frac{1}{(m-1)ax^{m-1}X}-\frac{(m+2)b}{(m-1)a}\int\frac{\partial x}{x^{m-3}X^2}$$

$$\int\frac{\partial x}{x^m X^3}=-\frac{1}{(m-1)ax^{m-1}X^2}-\frac{(m+5)b}{(m-1)a}\int\frac{\partial x}{x^{m-3}X^3}$$

VZ. $a+bx^4=X$

$$\int\frac{x^m\partial x}{X}=\frac{x^{m-3}}{(m-3)b}-\frac{a}{b}\int\frac{x^{m-4}\partial x}{X}$$

$$\int\frac{x^m\partial x}{X}=\frac{x^{m-3}}{(m-7)bX}-\frac{(m-3)a}{(m-7)b}\int\frac{x^{m-4}\partial x}{X^2}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx^4 = X$

$$\int \frac{x^m \partial x}{X^3} = \frac{x^{m-3}}{(m-11)bX^2} - \frac{(m-3)a}{(m-11)b} \int \frac{x^{m-4}\partial x}{X^3}$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{\partial x}{x^{m-4}X}$$

$$\int \frac{\partial x}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+3)b}{(m-1)a} \int \frac{\partial x}{x^{m-4}X^2}$$

$$\int \frac{\partial x}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+7)b}{(m-1)a} \int \frac{\partial x}{x^{m-4}X^3}$$

VZ. $a + bx^5 = X$

$$\int \frac{x^m \partial x}{X} = \frac{x^{m-4}}{(m-4)b} - \frac{a}{b} \int \frac{x^{m-5}\partial x}{X}$$

$$\int \frac{x^m \partial x}{X^2} = \frac{x^{m-4}}{(m-9)b} - \frac{(m-4)a}{(m-9)b} \int \frac{x^{m-5}\partial x}{X^2}$$

$$\int \frac{x^m \partial x}{X^3} = \frac{x^{m-4}}{(m-14)b} - \frac{(m-4)a}{(m-14)b} \int \frac{x^{m-5}\partial x}{X^3}$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{\partial x}{x^{m-5}X}$$

$$\int \frac{\partial x}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+4)b}{(m-1)a} \int \frac{\partial x}{x^{m-5}X^2}$$

$$\int \frac{\partial x}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+9)b}{(m-1)a} \int \frac{\partial x}{x^{m-5}X^3}$$

VZ. $a + bx^6 = X$

$$\int \frac{x^m \partial x}{X} = \frac{x^{m-5}}{(m-5)b} - \frac{a}{b} \int \frac{x^{m-6}\partial x}{X}$$

$$\int \frac{x^m \partial x}{X^2} = \frac{x^{m-5}}{(m-11)bX} - \frac{(m-5)a}{(m-11)b} \int \frac{x^{m-6}\partial x}{X^2}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx^6 = X$

$$\int \frac{x^m \partial x}{X^3} = \frac{x^{m-5}}{(m-17)bX^2} - \frac{(m-5)a}{(m-17)b} \int \frac{x^{m-6}\partial x}{X^3}$$

$$\int \frac{\partial x}{x^m X} = -\frac{1}{(m-1)ax^{m-1}} - \frac{b}{a} \int \frac{\partial x}{x^{m-6}X}$$

$$\int \frac{\partial x}{x^m X^2} = -\frac{1}{(m-1)ax^{m-1}X} - \frac{(m+5)b}{(m-1)a} \int \frac{\partial x}{x^{m-6}X^2}$$

$$\int \frac{\partial x}{x^m X^3} = -\frac{1}{(m-1)ax^{m-1}X^2} - \frac{(m+11)b}{(m-1)a} \int \frac{\partial x}{x^{m-6}X^3}$$

VZ. $a+bx^2+cx^4=X,\ (p-1)(b^2-4ac)2a=k$

$$\int \frac{\partial x}{X^p} = \frac{bcx^3+(b^2-2ac)x}{kX^{p-1}} + \frac{(4p-7)bc}{k} \int \frac{x^2\partial x}{X^{p-1}}$$
$$+ \frac{2(p-1)(b^2-4ac)+2ac-b^2}{k} \int \frac{\partial x}{X^{p-1}}$$

$$\int \frac{x^m\partial x}{X^p} = \frac{bcx^{m+3}+(b^2-2ac)x^{m+1}}{kX^{p-1}} + \frac{(4p-m-5)bc}{k} \int \frac{x^{m+2}\partial x}{X^{p-1}}$$
$$+ \frac{2(p-1)(b^2-4ac)+(m+1)(2ac-b^2)}{k} \int \frac{x^m\partial x}{X^{p-1}}$$

$$\int \frac{x^m\partial x}{X^p} = \frac{x^{m-3}}{(m-4p+1)cX^{p-1}} - \frac{(m-3)a}{(m-4p+1)c} \int \frac{x^{m-4}\partial x}{X^p}$$
$$- \frac{(m-2p-1)b}{(m-4p+1)c} \int \frac{x^{m-2}\partial x}{X^p}$$

$$\int \frac{\partial x}{x^m X^p} = -\frac{1}{(m-1)ax^{m-1}X^{p-1}} - \frac{(m+2p-3)b}{(m-1)a} \int \frac{\partial x}{x^{m-2}X^p}$$
$$- \frac{m+4p-5}{(m-1)a} \int \frac{\partial x}{x^{m-4}X^p}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a+bx^3+cx^6=X$, $(p-1)(b^2-4ac)3a=k$

$$\int\frac{\partial x}{X^p}=\frac{bcx^4+(b^2-2ac)x}{kX^{p-1}}+\frac{(6p-10)bc}{k}\int\frac{x^3\partial x}{X^{p-1}}$$
$$+\frac{3(p-1)(b^2-4ac)+2ac-b^2}{k}\int\frac{\partial x}{X^{p-1}}$$

$$\int\frac{x^m\partial x}{X^p}=\frac{bcx^{m+4}+(b^2-2ac)x^{m+1}}{kX^{p-1}}+\frac{(6p-m-10)bc}{k}\int\frac{x^{m+3}\partial x}{X^{p-1}}$$
$$+\frac{3(p-1)(b^2-4ac)+(m+1)(2ac-b^2)}{k}\int\frac{x^m\partial x}{X^{p-1}}$$

$$\int\frac{x^m\partial x}{X^p}=\frac{x^{m-5}}{(m-6p+1)cX^{p-1}}-\frac{(m-5)a}{(m-6p+1)c}\int\frac{x^{m-6}\partial x}{X^p}$$
$$-\frac{(m-3p-2)a}{(m-6p+1)c}\int\frac{x^{m-3}\partial x}{X^p}$$

$$\int\frac{\partial x}{x^mX^p}=-\frac{1}{(m-1)ax^{m-1}X^{p-1}}-\frac{(m+3p-4)b}{(m-1)a}\int\frac{\partial x}{x^{m-3}X^p}$$
$$-\frac{(m+6p-7)c}{(m-1)a}\int\frac{\partial x}{x^{m-6}X^p}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx^n = X$, $\pi = 180°$

Es ist im Allgemeinen, wenn $m < n$, oder $= 0$,

$$\int \frac{x^m \partial x}{X} = U + V,$$

wo U eine blofs logarithmische Function oder $= 0$ ist, und V ein Aggregat von Gliedern der folgenden Form:

$$\frac{1}{nbk^{n-m-1}} \left\{ \begin{array}{l} \operatorname{Cos}(n-m-1)\varphi \log(x^2 - 2kx \operatorname{Cos}\varphi + k^2) \\ + 2 \operatorname{Sin}(n-m-1)\varphi \operatorname{Arc Tang} \dfrac{x \operatorname{Sin}\varphi}{k - x \operatorname{Cos}\varphi} \end{array} \right\}$$

Nähere Bestimmung.

1) *n eine ungerade Zahl; a und b beliebige Vorzeichen.* Man setze $k = \sqrt[n]{\frac{a}{b}}$; alsdann ist

$$U = \frac{1}{nb(-k)^{n-m-1}} \log(x + k)$$

und V ein Aggregat von Gliedern der obigen Form, welche sämmtlich erhalten werden, wenn man in diesem Ausdrucke für φ successive die $\frac{n-1}{2}$ Werthe $\frac{\pi}{n}, \frac{3\pi}{n}, \frac{5\pi}{n}, \frac{7\pi}{n}, \ldots \frac{(n-2)\pi}{n}$ setzt.

2) *n eine gerade Zahl; a und b verschiedene Vorzeichen.* Man setze $k = \sqrt[n]{-\frac{a}{b}}$; alsdann ist

$$U = \frac{1}{nbk^{n-m-1}} \log(x - k) + \frac{1}{nb(-k)^{n-m-1}} \log(x + k)$$

und V ein Aggregat von Gliedern der obigen Form, welche sämmtlich erhalten werden, wenn man in diesem Ausdrucke für φ successive die $\frac{n-2}{2}$ Werthe $\frac{2\pi}{n}, \frac{4\pi}{n}, \frac{6\pi}{n}, \ldots \frac{(n-2)\pi}{n}$ setzt.

3) *n eine gerade Zahl; a und b dieselben Vorzeichen.* Man setze $k = \sqrt[n]{\frac{a}{b}}$; alsdann ist $U = 0$, und V ein Aggregat von Gliedern, welche aus dem obigen Ausdrucke entstehen, wenn man darin für φ die $\frac{n}{2}$ Werthe $\frac{\pi}{n}, \frac{3\pi}{n}, \frac{5\pi}{n}, \frac{7\pi}{n}, \ldots \frac{(n-1)\pi}{n}$ setzt.

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^n + cx^{2n} = X, \quad \pi = 180^0$

Das Integral $\int \frac{x^m \partial x}{X}$ erhält, wenn reelle Ausdrücke gefordert werden, zwey verschiedene Formen, je nachdem $4ac - b^2$ eine positive oder eine negative Gröſse ist.

I. $4ac - b^2$ *eine positive Gröſse;* $m < 2n$.

Es sey $k = \sqrt[2n]{\frac{a}{c}}$, und α ein Winkel, dessen Cosinus $= -\frac{b}{2\sqrt{ac}}$, so ist $\int \frac{x^m \partial x}{X}$ ein Aggregat von n Gliedern der folgenden Form:

$$\frac{1}{2nck^{2n-m-1}\operatorname{Sin}\alpha}\left\{\begin{array}{l} -\operatorname{Sin}(n-m-1)\varphi \log(x^2 - 2kx\operatorname{Cos}\varphi + k^2) \\ +2\operatorname{Cos}(n-m-1)\varphi \operatorname{Arc\,Tang}\frac{x\operatorname{Sin}\varphi}{k - x\operatorname{Cos}\varphi} \end{array}\right\}$$

welche sämmtlich erhalten werden, wenn man in diesem Ausdrucke successive $\frac{\alpha}{n}$, $\frac{2\pi+\alpha}{n}$, $\frac{4\pi+\alpha}{n}$, $\frac{6\pi+\alpha}{n}$, $\frac{8\pi+\alpha}{n}$, ... $\frac{(2n-2)\pi+\alpha}{n}$ für φ setzt.

Wenn $m > 2n$, wird man das Integral $\int \frac{x^m \partial x}{X}$ auf andere reduciren, für welche $m < 2n$.

II. $4ac - b^2$ *eine negative Gröſse.*

Es sey

$$\tfrac{1}{2}b - \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = f$$
$$\tfrac{1}{2}b + \tfrac{1}{2}\sqrt{(b^2 - 4ac)} = g$$
$$\sqrt{(b^2 - 4ac)} = g - f = h$$

so ist

$$\int \frac{\partial x}{X} = \frac{c}{h}\left[\int \frac{\partial x}{cx^n + f} - \int \frac{\partial x}{cx^n + g}\right]$$
$$\int \frac{x^m \partial x}{X} = \frac{c}{h}\left[\int \frac{x^m \partial x}{cx^n + f} - \int \frac{x^m \partial x}{cx^n + g}\right]$$

Tafel

einiger allgemeineren Formeln.

VZ. $\begin{Bmatrix} Ax^h + Bx^{h-1} + Cx^{h-2} + Dx^{h-3} + \text{etc.} + Kx + L = U \\ ax^n + bx^{n-1} + cx^{n-2} + dx^{n-3} + \text{etc.} + kx + l = V \end{Bmatrix}$

Es sey

$$\frac{\partial V}{\partial x} = nax^{n-1} + (n-1)bx^{n-2} + (n-2)cx^{n-3} + (n-3)dx^{n-4} + \ldots\ldots + k = Z.$$

Es seyen ferner

$r', r'', r''', r'''', \ldots\ldots r^{n\prime}$ die n Wurzeln der Gleichung $V = 0$,

$U', U'', U''', U'''', \ldots\ldots U^{n\prime}$ die Werthe der Function U, wenn diese Wurzeln für x substituirt werden,

$Z', Z'', Z''', Z'''', \ldots\ldots Z^{n\prime}$ die Werthe der Function Z, wenn diese Wurzeln für x substituirt werden;

so ist im Allgemeinen, jedoch nur unter der Voraussetzung, daſs die Wurzeln r', r'', r''', etc., sämmtlich von einander verschieden seyen, und $h < n$:

$$\int \frac{U\partial x}{V} = \frac{U'}{Z'}\log(x-r') + \frac{U''}{Z''}\log(x-r'') + \frac{U'''}{Z'''}\log(x-r''') + \frac{U''''}{Z''''}\log(x-r'''') + \text{etc.} + \frac{U^{n\prime}}{Z^{n\prime}}\log(x-r^{n\prime})$$

$$\int \frac{x^m U\partial x}{V} = \frac{U'}{Z'}\int\frac{x^m\partial x}{x-r'} + \frac{U''}{Z''}\int\frac{x^m\partial x}{x-r''} + \frac{U'''}{Z'''}\int\frac{x^m\partial x}{x-r'''} + \frac{U''''}{Z''''}\int\frac{x^m\partial x}{x-r''''} + \text{etc.} + \frac{U^{n\prime}}{Z^{n\prime}}\int\frac{x^m\partial x}{x-r^{n\prime}}$$

$$\int \frac{U\partial x}{x^m V} = \frac{U'}{Z'}\int\frac{\partial x}{x^m(x-r')} + \frac{U''}{Z''}\int\frac{\partial x}{x^m(x-r'')} + \frac{U'''}{Z'''}\int\frac{\partial x}{x^m(x-r''')} + \frac{U''''}{Z''''}\int\frac{\partial x}{x^m(x-r'''')} + \text{etc.} + \frac{U^{n\prime}}{Z^{n\prime}}\int\frac{\partial x}{x^m(x-r^{n\prime})}$$

und diese Formeln werden reell, wenn die Wurzeln r', r'', r''' etc. reell sind.

Erinnerungen zu der vorhergehenden Tafel.

1) Die Formeln Seite 97 — 110 sind sämmtlich aus den allgemeinen Reductionsformeln Seite 19 — 23 abgeleitet. Bey der ersten Formel Seite 106 ist jedoch zu bemerken, daſs sie nicht unmittelbar in dieser Gestalt gefunden wird; denn wenn man in der Formel V, Seite 23, $-p$ für p und $m=1$, $n=1$ setzt, so erhält man eigentlich

$$\int\frac{\partial x}{X^p}=\frac{Ax+Bx^2}{KX^{p-1}}+\frac{C}{K}\int\frac{\partial x}{X^{p-1}}+\frac{D}{K}\int\frac{x\partial x}{X^{p-1}};$$

wo $A=2ac-b^2$, $B=-bc$, $C=(p-1)(4ac-b^2)+b^2-2ac$, $D=(-2p+4)bc$, $K=(p-1)(4ac-b^2)a$. Es ist aber

$$\int\frac{x\partial x}{X^{p-1}}=\frac{-1}{2c(p-2)X^{p-2}}-\frac{b}{2c}\int\frac{\partial x}{X^{p-1}}.$$

Substituirt man diesen Werth, so erhält man nach der gehörigen Reduction die Formel, welche Seite 106 angegeben worden.

2) Die Zerlegung der Brüche $\frac{x^m}{a+bx^n}$, $\frac{x^m}{a+bx^n+cx^{2n}}$, welche zur Integrirung der Differentiale $\frac{x^m\partial x}{a+bx^n}$, $\frac{x^m\partial x}{a+bx^n+cx^{2n}}$ erfordert wird, geschiehet am leichtesten nach der zweiten Methode des dritten Falles Seite 10. Es ist bekannt, daſs jeder trinomische Factor von $x^n+\frac{a}{b}$ und $x^{2n}+\frac{b}{c}x^n+\frac{a}{c}$ (wenn b^2-4ac negativ) die Form $x^2-2kx\operatorname{Cos}\varphi+k^2$ hat. Man darf also nur $\operatorname{Cos}\varphi+\operatorname{Sin}\varphi\sqrt{-1}$ für x setzen, und sich dabey erinnern, daſs im Allgemeinen $(\operatorname{Cos}\varphi+\operatorname{Sin}\varphi\sqrt{-1})^n=\operatorname{Cos}n\varphi+\operatorname{Sin}n\varphi\sqrt{-1}$. Man erhält auf diese Weise für den ersten Bruch partielle Brüche von der Form

$$\frac{2}{nbk^{n-m-2}}\cdot\frac{-k\operatorname{Cos}(n-m)\varphi+\operatorname{Cos}(n-m-1)\varphi\,.\,x}{x^2-2kx\operatorname{Cos}\varphi+k^2}$$

und für den zweiten Bruch

$$\frac{1}{nck^{2n-m-2}\operatorname{Sin}\alpha}\cdot\frac{k\operatorname{Sin}(n-m)\varphi-\operatorname{Sin}(n-m-1)\varphi\,.\,x}{x^2-2kx\operatorname{Cos}\varphi+k^2}$$

Integraltafeln

für

rrationale Differentiale.

(Es werden in diesem Abschnitte die Integrale der am häufigsten in der Ausübung vorkommenden irrationalen Differentiale gegeben. Man hat sich dabey gröſstentheils auf solche beschränkt, worin nur quadratische Wurzelgröſsen vorkommen, theils aus dem Grunde, weil die vollständige Integration bey den andern Wurzelgröſsen nur in wenigen Fällen ausführbar ist, theils auch deſshalb, weil sie gegenwärtig nur selten oder fast gar nicht gebraucht werden. Die allgemeineren Methoden und Formeln sind jedoch am Ende angeführt.)

$$\int \frac{x^m \partial x}{\sqrt{(a+bx)}}$$

Taf. I.

VZ. $a + bx = X$

$$\int \frac{\partial x}{\sqrt{X}} = \frac{2}{b}\sqrt{X}$$

$$\int \frac{x\partial x}{\sqrt{X}} = \left(\tfrac{1}{3}X - a\right)\frac{2\sqrt{X}}{b^2}$$

$$\int \frac{x^2\partial x}{\sqrt{X}} = \left(\tfrac{1}{5}X^2 - \tfrac{2}{3}aX + a^2\right)\frac{2\sqrt{X}}{b^3}$$

$$\int \frac{x^3\partial x}{\sqrt{X}} = \left(\tfrac{1}{7}X^3 - \tfrac{3}{5}aX^2 + a^2X - a^3\right)\frac{2\sqrt{X}}{b^4}$$

$$\int \frac{x^4\partial x}{\sqrt{X}} = \left(\tfrac{1}{9}X^4 - \tfrac{4}{7}aX^3 + \tfrac{6}{5}a^2X^2 - \tfrac{4}{3}a^3X + a^4\right)\frac{2\sqrt{X}}{b^5}$$

$$\int \frac{x^5\partial x}{\sqrt{X}} = \left(\tfrac{1}{11}X^5 - \tfrac{5}{9}aX^4 + \tfrac{10}{7}a^2X^3 - 2a^3X^2 + \tfrac{5}{3}a^4X - a^5\right)\frac{2\sqrt{X}}{b^6}$$

$$\int \frac{x^6\partial x}{\sqrt{X}} = \Big(\tfrac{1}{13}X^6 - \tfrac{6}{11}aX^5 + \tfrac{5}{3}a^2X^4 - \tfrac{20}{7}a^3X^3 + 3a^4X^2$$
$$- 2a^5X + a^6\Big)\frac{2\sqrt{X}}{b^7}$$

$$\int \frac{x^7\partial x}{\sqrt{X}} = \Big(\tfrac{1}{15}X^7 - \tfrac{7}{13}aX^6 + \tfrac{21}{11}a^2X^5 - \tfrac{35}{9}a^3X^4 + 5a^4X^3$$
$$- \tfrac{21}{5}a^5X^2 + \tfrac{7}{3}a^6X - a^7\Big)\frac{2\sqrt{X}}{b^8}$$

$$\int \frac{x^8\partial x}{\sqrt{X}} = \Big(\tfrac{1}{17}X^8 - \tfrac{8}{15}aX^7 + \tfrac{28}{13}a^2X^6 - \tfrac{56}{11}a^3X^5 + \tfrac{70}{9}a^4X^4$$
$$- 8a^5X^3 + \tfrac{28}{5}a^6X^2 - \tfrac{8}{3}a^7X + a^8\Big)\frac{2\sqrt{X}}{b^9}$$

$$\int \frac{x^9\partial x}{\sqrt{X}} = \Big(\tfrac{1}{19}X^9 - \tfrac{9}{17}aX^8 + \tfrac{12}{5}a^2X^7 - \tfrac{84}{13}a^3X^6 + \tfrac{126}{11}a^4X^5$$
$$- 14a^5X^4 + 12a^6X^3 - \tfrac{36}{5}a^7X^2 + 3a^8X - a^9\Big)\frac{2\sqrt{X}}{b^{10}}$$

$$\int \frac{x^{10}\partial x}{\sqrt{X}} = \Big(\tfrac{1}{21}X^{10} - \tfrac{10}{19}aX^9 + \tfrac{45}{17}a^2X^8 - 8a^3X^7 + \tfrac{210}{13}a^4X^6 - \tfrac{252}{11}a^5X^5$$
$$+ \tfrac{70}{3}a^6X^4 - \tfrac{120}{7}a^7X^3 + 9a^8X^2 - \tfrac{10}{3}a^9X + a^{10}\Big)\frac{2\sqrt{X}}{b^{11}}$$

Taf. II.

$$\int \frac{\partial x}{x^m \sqrt{(a+bx)}}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{x\sqrt{X}} = \int \frac{\partial x}{x\sqrt{X}}$$ (Man s. die folgende Seite.)

$$\int \frac{\partial x}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^3\sqrt{X}} = \left(-\frac{1}{2ax^2} + \frac{3b}{4a^2x}\right)\sqrt{X} + \frac{3b^2}{8a^2}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^4\sqrt{X}} = \left(-\frac{1}{3ax^3} + \frac{5b}{12a^2x^2} - \frac{5b^2}{8a^3x}\right)\sqrt{X} - \frac{5b^3}{16a^3}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^5\sqrt{X}} = \left(-\frac{1}{4ax^4} + \frac{7b}{24a^2x^3} - \frac{35b^2}{96a^3x^2} + \frac{35b^3}{64a^4x}\right)\sqrt{X} + \frac{35b^4}{128a^4}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^6\sqrt{X}} = \left(-\frac{1}{5ax^5} + \frac{9b}{40a^2x^4} - \frac{21b^2}{80a^3x^3} + \frac{21b^3}{64a^4x^2} - \frac{63b^4}{128a^5x}\right)\sqrt{X} - \frac{63b^5}{256a^5}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^7\sqrt{X}} = \left(-\frac{1}{6ax^6} + \frac{11b}{60a^2x^5} - \frac{33b^2}{160a^3x^4} + \frac{77b^3}{320a^4x^3} - \frac{77b^4}{256a^5x^2} + \frac{231b^5}{512a^6x}\right)\sqrt{X} + \frac{231b^6}{1024a^6}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^8\sqrt{X}} = \left(-\frac{1}{7ax^7} + \frac{13b}{84a^2x^6} - \frac{143b^2}{840a^3x^5} + \frac{429b^3}{2240a^4x^4} - \frac{143b^4}{640a^5x^3} + \frac{143b^5}{512a^6x^2} - \frac{429b^6}{1024a^7x}\right)\sqrt{X} - \frac{429b^7}{2048a^7}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^9\sqrt{X}} = \left(-\frac{1}{8ax^8} + \frac{15b}{112a^2x^7} - \frac{65b^2}{448a^3x^6} + \frac{143b^3}{896a^4x^5} - \frac{1287b^4}{7168a^5x^4} + \frac{429b^5}{2048a^6x^3} - \frac{2145b^6}{8192a^7x^2} + \frac{6435b^7}{16384a^8x}\right)\sqrt{X} + \frac{6435b^8}{32768a^8}\int \frac{\partial x}{x\sqrt{X}}$$

Anmerkung zu der vorhergehenden Tafel.

Es ist im Allgemeinen

$$\int \frac{\partial x}{x\sqrt{(a+bx)}} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{(a+bx)}+\sqrt{a}} + \text{Const.}$$

$$\text{oder} \int \frac{\partial x}{x\sqrt{(a+bx)}} = \frac{2}{\sqrt{-a}} \text{Arc Tang} \frac{\sqrt{(a+bx)}}{\sqrt{-a}} + \text{Const.}$$

Der erste Ausdruck wird reell, wenn a positiv, der zweite, wenn a negativ ist.

I. a positiv.

$$\int \frac{\partial x}{x\sqrt{(a+bx)}} + \text{Const.} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{(a+bx)}+\sqrt{a}}$$

$$= \frac{2}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{x}} = -\frac{2}{\sqrt{a}} \log \frac{\sqrt{x}}{\sqrt{(a+bx)}-\sqrt{a}},$$

und in diesen Ausdrücken kann $\sqrt{a}$ sowohl positiv als negativ genommen werden. Der Integralausdruck $\int \frac{\partial x}{x\sqrt{(a+bx)}}$ kann, wenn er endlich seyn soll, nicht von $x=0$ anfangen, und daher auch nicht für diesen Werth verschwinden.

II. a negativ.

$$\int \frac{\partial x}{x\sqrt{(bx-a)}} = \frac{2}{\sqrt{a}} \text{Arc Tang} \sqrt{\frac{bx-a}{a}} = \frac{2}{\sqrt{a}} \text{Arc Cot} \sqrt{\frac{a}{bx-a}}$$

$$= \frac{2}{\sqrt{a}} \text{Arc Sec} \sqrt{\frac{bx}{a}} = \frac{2}{\sqrt{a}} \text{Arc Cosec} \sqrt{\frac{bx}{bx-a}} = \frac{2}{\sqrt{a}} \text{Arc Cos} \sqrt{\frac{a}{bx}}$$

$$= \frac{2}{\sqrt{a}} \text{Arc Sin} \sqrt{\frac{bx-a}{bx}} = \frac{1}{\sqrt{a}} \text{Arc Cos} \frac{2a-bx}{bx}$$

$$= \frac{1}{\sqrt{a}} \text{Arc Sin vers} \frac{2(bx-a)}{bx}.$$

In dem Integralausdruck $\int \frac{\partial x}{x\sqrt{(bx-a)}}$ kann b nicht negativ seyn; auch kann sich derselbe erst von $x = \frac{a}{b}$ anfangen, und daher für keinen kleinern Werth von x verschwinden. Für diesen Werth des x verschwinden nun auch die hier angegebenen Integrale.

Taf. III.

$$\int \frac{x^m \partial x}{(a+bx)^{\frac{3}{2}}}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = -\frac{2}{b\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{3}{2}}} = \left(X + a\right)\frac{2}{b^2\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{3}X^2 - 2aX - a^2\right)\frac{2}{b^3\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{5}X^3 - aX^2 + 3a^2X + a^3\right)\frac{2}{b^4\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{7}X^4 - \tfrac{4}{5}aX^3 + 2a^2X^2 - 4a^3X - a^4\right)\frac{2}{b^5\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{9}X^5 - \tfrac{5}{7}aX^4 + 2a^2X^3 - \tfrac{10}{3}a^3X^2 + 5a^4X + a^5\right)\frac{2}{b^6\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{11}X^6 - \tfrac{2}{3}aX^5 + \tfrac{15}{7}a^2X^4 - 4a^3X^3 + 5a^4X^2 - 6a^5X - a^6\right)\frac{2}{b^7\sqrt{X}}$$

$$\begin{aligned}\int \frac{x^7\partial x}{X^{\frac{3}{2}}} = \Big(&\tfrac{1}{13}X^7 - \tfrac{7}{11}aX^6 + \tfrac{7}{3}a^2X^5 - 5a^3X^4 + 7a^4X^3 - 7a^5X^2 \\ &+ 7a^6X + a^7\Big)\frac{2}{b^8\sqrt{X}}\end{aligned}$$

$$\begin{aligned}\int \frac{x^8\partial x}{X^{\frac{3}{2}}} = \Big(&\tfrac{1}{15}X^8 - \tfrac{8}{13}aX^7 + \tfrac{28}{11}a^2X^6 - \tfrac{56}{9}a^3X^5 + 10a^4X^4 - \tfrac{56}{5}a^5X^3 \\ &+ \tfrac{28}{3}a^6X^2 - 8a^7X - a^8\Big)\frac{2}{b^9\sqrt{X}}\end{aligned}$$

$$\begin{aligned}\int \frac{x^9\partial x}{X^{\frac{3}{2}}} = \Big(&\tfrac{1}{17}X^9 - \tfrac{3}{5}aX^8 + \tfrac{36}{13}a^2X^7 - \tfrac{84}{11}a^3X^6 + 14a^4X^5 - 18a^5X^4 \\ &+ \tfrac{84}{5}a^6X^3 - 12a^7X^2 + 9a^8X + a^9\Big)\frac{2}{b^{10}\sqrt{X}}\end{aligned}$$

$$\begin{aligned}\int \frac{x^{10}\partial x}{X^{\frac{3}{2}}} = \Big(&\tfrac{1}{19}X^{10} - \tfrac{10}{17}aX^9 + 3a^2X^8 - \tfrac{120}{13}a^3X^7 + \tfrac{210}{11}a^4X^6 \\ &- 28a^5X^5 + 30a^6X^4 - 24a^7X^3 + 15a^8X^2 \\ &- 10a^9X - a^{10}\Big)\frac{2}{b^{11}\sqrt{X}}\end{aligned}$$

Anmerkung zu der vorhergehenden Tafel.

Es ist im Allgemeinen

$$\int \frac{\partial x}{x\sqrt{(a+bx)}} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{(a+bx)}+\sqrt{a}} + \text{Const.}$$

$$\text{oder } \int \frac{\partial x}{x\sqrt{(a+bx)}} = \frac{2}{\sqrt{-a}} \text{Arc Tang} \frac{\sqrt{(a+bx)}}{\sqrt{-a}} + \text{Const.}$$

Der erste Ausdruck wird reell, wenn a positiv, der zweite, wenn a negativ ist.

I. a positiv.

$$\int \frac{\partial x}{x\sqrt{(a+bx)}} + \text{Const.} = \frac{1}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{(a+bx)}+\sqrt{a}}$$

$$= \frac{2}{\sqrt{a}} \log \frac{\sqrt{(a+bx)}-\sqrt{a}}{\sqrt{x}} = -\frac{2}{\sqrt{a}} \log \frac{\sqrt{x}}{\sqrt{(a+bx)}-\sqrt{a}},$$

und in diesen Ausdrücken kann $\sqrt{a}$ sowohl positiv als negativ genommen werden. Der Integralausdruck $\int \frac{\partial x}{x\sqrt{(a+bx)}}$ kann, wenn er endlich seyn soll, nicht von $x = 0$ anfangen, und daher auch nicht für diesen Werth verschwinden.

II. a negativ.

$$\int \frac{\partial x}{x\sqrt{(bx-a)}} = \frac{2}{\sqrt{a}} \text{Arc Tang} \sqrt{\frac{bx-a}{a}} = \frac{2}{\sqrt{a}} \text{Arc Cot} \sqrt{\frac{a}{bx-a}}$$

$$= \frac{2}{\sqrt{a}} \text{Arc Sec} \sqrt{\frac{bx}{a}} = \frac{2}{\sqrt{a}} \text{Arc Cosec} \sqrt{\frac{bx}{bx-a}} = \frac{2}{\sqrt{a}} \text{Arc Cos} \sqrt{\frac{a}{bx}}$$

$$= \frac{2}{\sqrt{a}} \text{Arc Sin} \sqrt{\frac{bx-a}{bx}} = \frac{1}{\sqrt{a}} \text{Arc Cos} \frac{2a-bx}{bx}$$

$$= \frac{1}{\sqrt{a}} \text{Arc Sin vers} \frac{2(bx-a)}{bx}.$$

In dem Integralausdruck $\int \frac{\partial x}{x\sqrt{(bx-a)}}$ kann b nicht negativ seyn; auch kann sich derselbe erst von $x = \frac{a}{b}$ anfangen, und daher für keinen kleinern Werth von x verschwinden. Für diesen Werth des x verschwinden nun auch die hier angegebenen Integrale.

Taf. III.

$$\int \frac{x^m \partial x}{(a+bx)^{\frac{3}{2}}}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = -\frac{2}{b\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{3}{2}}} = \left(X + a\right)\frac{2}{b^2\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{3}X^2 - 2aX - a^2\right)\frac{2}{b^3\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{5}X^3 - aX^2 + 3a^2X + a^3\right)\frac{2}{b^4\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{7}X^4 - \tfrac{4}{5}aX^3 + 2a^2X^2 - 4a^3X - a^4\right)\frac{2}{b^5\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{9}X^5 - \tfrac{5}{7}aX^4 + 2a^2X^3 - \tfrac{10}{3}a^3X^2 + 5a^4X + a^5\right)\frac{2}{b^6\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{11}X^6 - \tfrac{2}{3}aX^5 + \tfrac{15}{7}a^2X^4 - 4a^3X^3 + 5a^4X^2 - 6a^5X - a^6\right)\frac{2}{b^7\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{13}X^7 - \tfrac{7}{11}aX^6 + \tfrac{7}{3}a^2X^5 - 5a^3X^4 + 7a^4X^3 - 7a^5X^2 \right.$$
$$\left. + 7a^6X + a^7\right)\frac{2}{b^8\sqrt{X}}$$

$$\int \frac{x^8\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{15}X^8 - \tfrac{8}{13}aX^7 + \tfrac{28}{11}a^2X^6 - \tfrac{56}{9}a^3X^5 + 10a^4X^4 - \tfrac{56}{5}a^5X^3 \right.$$
$$\left. + \tfrac{28}{3}a^6X^2 - 8a^7X - a^8\right)\frac{2}{b^9\sqrt{X}}$$

$$\int \frac{x^9\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{17}X^9 - \tfrac{3}{5}aX^8 + \tfrac{36}{13}a^2X^7 - \tfrac{84}{11}a^3X^6 + 14a^4X^5 - 18a^5X^4 \right.$$
$$\left. + \tfrac{84}{5}a^6X^3 - 12a^7X^2 + 9a^8X + a^9\right)\frac{2}{b^{10}\sqrt{X}}$$

$$\int \frac{x^{10}\partial x}{X^{\frac{3}{2}}} = \left(\tfrac{1}{19}X^{10} - \tfrac{10}{17}aX^9 + 3a^2X^8 - \tfrac{120}{13}a^3X^7 + \tfrac{210}{11}a^4X^6 \right.$$
$$- 28a^5X^5 + 30a^6X^4 - 24a^7X^3 + 15a^8X^2$$
$$\left. - 10a^9X - a^{10}\right)\frac{2}{b^{11}\sqrt{X}}$$

$$\int \frac{\partial x}{x^m(a+bx)^{\frac{3}{2}}}$$

Taf. IV.

VZ. $a + bx = X$

$$\int \frac{\partial x}{xX^{\frac{3}{2}}} = \frac{2}{aVX} + \frac{1}{a}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{ax} - \frac{3b}{a^2}\right)\frac{1}{VX} - \frac{3b}{2a^2}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x} + \frac{15b^2}{4a^3}\right)\frac{1}{VX} + \frac{15b^2}{8a^3}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{3ax^3} + \frac{7b}{12a^2x^2} - \frac{35b^2}{24a^3x} - \frac{35b^3}{8a^4}\right)\frac{1}{VX} - \frac{35b^3}{16a^4}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{4ax^4} + \frac{3b}{8a^2x^3} - \frac{21b^2}{32a^3x^2} + \frac{105b^3}{64a^4x} + \frac{315b^4}{64a^5}\right)\frac{1}{VX} + \frac{315b^4}{128a^5}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^5} + \frac{11b}{40a^2x^4} - \frac{33b^2}{80a^3x^3} + \frac{231b^3}{320a^4x^2} - \frac{231b^4}{128a^5x} - \frac{693b^5}{128a^6}\right)\frac{1}{VX} - \frac{693b^5}{256a^6}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^7X^{\frac{3}{2}}} = \left(-\frac{1}{6ax^6} + \frac{13b}{60a^2x^5} - \frac{143b^2}{480a^3x^4} + \frac{143b^3}{320a^4x^3} - \frac{1001b^4}{1280a^5x^2} + \frac{1001b^5}{512a^6x} + \frac{3003b^6}{512a^7}\right)\frac{1}{VX} + \frac{3003b^6}{1024a^7}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^8X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^7} + \frac{5b}{28a^2x^6} - \frac{13b^2}{56a^3x^5} + \frac{143b^3}{448a^4x^4} - \frac{429b^4}{896a^5x^3} + \frac{429b^5}{512a^6x^2} - \frac{2145b^6}{1024a^7x} - \frac{6435b^7}{1024a^8}\right)\frac{1}{VX} - \frac{6435b^7}{2048a^8}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{8ax^8} + \frac{17b}{112a^2x^7} - \frac{85b^2}{448a^3x^6} + \frac{221b^3}{896a^4x^5} - \frac{2431b^4}{7168a^5x^4} + \frac{7293b^5}{14336a^6x^3} - \frac{7293b^6}{8192a^7x^2} + \frac{36465b^7}{16384a^8x} + \frac{109395b^8}{16384a^9}\right)\frac{1}{VX} + \frac{109395b^8}{32768a^{10}}\int \frac{\partial x}{xVX}$$

Taf. V.

$$\int \frac{x^m \partial x}{(a+bx)^{\frac{5}{2}}}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^{\frac{5}{2}}} = -\frac{2}{3bX\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{5}{2}}} = \left(-X + \tfrac{1}{3}a\right)\frac{2}{b^2 X\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{5}{2}}} = \left(X^2 + 2aX - \tfrac{1}{3}a^2\right)\frac{2}{b^3 X\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{5}{2}}} = \left(\tfrac{1}{3}X^3 - 3aX^2 - 3a^2X + \tfrac{1}{3}a^3\right)\frac{2}{b^3 X\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{5}{2}}} = \left(\tfrac{1}{5}X^4 - \tfrac{4}{3}aX^3 + 6a^2X^2 + 4a^3X - \tfrac{1}{3}a^4\right)\frac{2}{b^4 X\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{5}{2}}} = \left(\tfrac{1}{7}X^5 - aX^4 + \tfrac{10}{3}a^2X^3 - 10a^3X^2 - 5a^4X + \tfrac{1}{3}a^5\right)\frac{2}{b^5 X\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{5}{2}}} = \left(\tfrac{1}{9}X^6 - \tfrac{6}{7}aX^5 + 3a^2X^4 - \tfrac{20}{3}a^3X^3 + 15a^4X^2 + 6a^5X - \tfrac{1}{3}a^6\right) \times \frac{2}{b^6 X\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X^{\frac{5}{2}}} = \left(\tfrac{1}{11}x^7 - \tfrac{7}{9}aX^6 + 3a^2X^5 - 7a^3X^4 + \tfrac{35}{3}a^4X^3 - 21a^5X^2 - 7a^6X + \tfrac{1}{3}a^7\right)\frac{2}{b^7 X\sqrt{X}}$$

$$\int \frac{x^8\partial x}{X^{\frac{5}{2}}} = \left(\tfrac{1}{13}X^8 - \tfrac{8}{11}aX^7 + \tfrac{28}{9}a^2X^6 - 8a^3X^5 + 14a^4X^4 - \tfrac{56}{5}a^5X^3 + 28a^6X^2 + 8a^7X - \tfrac{1}{3}a^8\right)\frac{2}{b^8 X\sqrt{X}}$$

$$\int \frac{x^9\partial x}{X^{\frac{5}{2}}} = \left(\tfrac{1}{15}X^9 - \tfrac{9}{13}aX^8 + \tfrac{36}{11}a^2X^7 - \tfrac{28}{3}a^3X^6 + 18a^4X^5 - \tfrac{126}{5}a^5X^4 + 28a^6X^3 - 36a^7X^2 - 9a^8X + \tfrac{1}{3}a^9\right)\frac{2}{b^9 X\sqrt{X}}$$

$$\int \frac{\partial x}{x^n (a+bx)^{\frac{5}{2}}}$$

Taf. VI.

VZ. $a + bx = X$

$$\int \frac{\partial x}{x X^{\frac{5}{2}}} = \left(\frac{8}{3a} + \frac{2bx}{a^2}\right)\frac{1}{XVX} + \frac{1}{a^2}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^2 X^{\frac{5}{2}}} = \left(-\frac{1}{ax} - \frac{20b}{3a^2} - \frac{5b^2x}{a^3}\right)\frac{1}{XVX} - \frac{5b}{2a^3}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^3 X^{\frac{5}{2}}} = \left(-\frac{1}{2ax^2} + \frac{7b}{4a^2x} + \frac{35b^2}{3a^3} + \frac{35b^3x}{4a^4}\right)\frac{1}{XVX} + \frac{35b^2}{8a^4}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^4 X^{\frac{5}{2}}} = \left(-\frac{1}{3ax^3} + \frac{3b}{4a^2x^2} - \frac{21b^2}{8a^3x} - \frac{35b^3}{2a^4} - \frac{105b^4x}{8a^5}\right)\frac{1}{XVX} - \frac{105b^3}{16a^5}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^5 X^{\frac{5}{2}}} = \left(-\frac{1}{4ax^4} + \frac{11b}{24a^2x^3} - \frac{33b^2}{32a^3x^2} + \frac{231b^3}{64a^4x} + \frac{385b^4}{16a^5} + \frac{1155b^5x}{64a^6}\right)\frac{1}{XVX} + \frac{1155b^4}{128a^6}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^6 X^{\frac{5}{2}}} = \left(-\frac{1}{5ax^5} + \frac{13b}{40a^2x^4} - \frac{143b^2}{240a^3x^3} + \frac{429b^3}{320a^4x^2} - \frac{3003b^4}{640a^5x} - \frac{1001b^5}{32a^6} - \frac{3003b^6x}{128a^7}\right)\frac{1}{XVX} - \frac{3003b^5}{256a^7}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^7 X^{\frac{5}{2}}} = \left(-\frac{1}{6ax^6} + \frac{b}{4a^2x^5} - \frac{13b^2}{32a^3x^4} + \frac{143b^3}{192a^4x^3} - \frac{429b^4}{256a^5x^2} + \frac{3003b^5}{512a^6x} + \frac{5005b^6}{128a^7} + \frac{15015b^7x}{512a^8}\right)\frac{1}{XVX} + \frac{15015b^6}{1024a^8}\int \frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^8 X^{\frac{5}{2}}} = \left(-\frac{1}{7ax^7} + \frac{17b}{84a^2x^6} - \frac{17b^2}{56a^3x^5} + \frac{221b^3}{448a^4x^4} - \frac{2431b^4}{2688a^5x^3} + \frac{7293b^5}{3584a^6x^2} - \frac{7293b^6}{1024a^7x} - \frac{12155b^7}{256a^8} - \frac{36465b^8x}{1024a^9}\right)\frac{1}{XVX} - \frac{36465b^7}{2048a^9}\int \frac{\partial x}{xVX}$$

Taf. VII.

$$\int \frac{x^m \partial x}{(a+bx)^{\frac{7}{2}}}$$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^{\frac{7}{2}}} = - \frac{2}{5bX^2\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{7}{2}}} = \left(-\tfrac{1}{3}X + \tfrac{1}{5}a\right)\frac{2}{b^2X^2\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{7}{2}}} = \left(-X^2 + \tfrac{2}{3}aX - \tfrac{1}{5}a^2\right)\frac{2}{b^3X^2\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{7}{2}}} = \left(X^3 + 3aX^2 - a^2X + \tfrac{1}{5}a^3\right)\frac{2}{b^4X^2\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{7}{2}}} = \left(\tfrac{1}{3}X^4 - 4aX^3 - 6a^2X^2 + \tfrac{4}{3}a^3X - \tfrac{1}{5}a^4\right)\frac{2}{b^5X^2\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{7}{2}}} = \left(\tfrac{1}{5}X^5 - \tfrac{5}{3}aX^4 + 10a^2X^3 + 10a^3X^2 - \tfrac{5}{3}a^4X + \tfrac{1}{5}a^5\right)\frac{2}{b^6X^2\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{7}{2}}} = \left(\tfrac{1}{7}X^6 - \tfrac{6}{5}aX^5 + 5a^2X^4 - 20a^3X^3 - 15a^4X^2 + 2a^5X - \tfrac{1}{5}a^6\right)\frac{2}{b^7X^2\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X^{\frac{7}{2}}} = \left(\tfrac{1}{9}X^7 - aX^6 + \tfrac{21}{5}a^2X^5 - \tfrac{35}{3}a^3X^4 + 35a^4X^3 + 21a^5X^2 - \tfrac{7}{3}a^6X + \tfrac{1}{5}a^7\right)\frac{2}{b^8X^2\sqrt{X}}$$

$$\int \frac{x^8\partial x}{X^{\frac{7}{2}}} = \left(\tfrac{1}{11}X^8 - \tfrac{8}{9}aX^7 + 4a^2X^6 - \tfrac{56}{5}a^3X^5 + \tfrac{70}{3}a^4X^4 - 56a^5X^3 - 28a^6X^2 + \tfrac{8}{3}a^7X - \tfrac{1}{5}a^8\right)\frac{2}{b^9X^2\sqrt{X}}$$

$$\int \frac{x^9\partial x}{X^{\frac{7}{2}}} = \left(\tfrac{1}{13}X^9 - \tfrac{9}{11}aX^8 + 4a^2X^7 - 12a^3X^6 + \tfrac{126}{5}a^4X^5 - 42a^5X^4 + 84a^6X^3 + 36a^7X^2 - 3a^8X + \tfrac{1}{5}a^9\right)\frac{2}{b^{10}X^2\sqrt{X}}$$

$$\int \frac{\partial x}{x^m (a+bx)^{\frac{7}{2}}}$$

Taf. VIII.

VZ. $a + bx = X$

$$\int \frac{\partial x}{x X^{\frac{7}{2}}} = \left(\frac{46}{15a} + \frac{14bx}{3a^2} + \frac{2b^2x^2}{a^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{1}{a^3}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2 X^{\frac{7}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{7b}{2a}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^3 X^{\frac{7}{2}}} = \left(-\frac{1}{2ax^2} + \frac{9b}{4a^2x}\right)\frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^4 X^{\frac{7}{2}}} = \left(-\frac{1}{3ax^3} + \frac{11b}{12a^2x^2} - \frac{33b^2}{8a^3x}\right)\frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^5 X^{\frac{7}{2}}} = \left(-\frac{1}{4ax^4} + \frac{13b}{24a^2x^3} - \frac{143b^2}{96a^3x^2} + \frac{429b^3}{64a^4x}\right)\frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^6 X^{\frac{7}{2}}} = \left(-\frac{1}{5ax^5} + \frac{3b}{8a^2x^4} - \frac{13b^2}{16a^3x^3} + \frac{143b^3}{64a^4x^2} - \frac{1287b^4}{128a^5x}\right)\frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^7 X^{\frac{7}{2}}} = \left(-\frac{1}{6ax^6} + \frac{17b}{60a^2x^5} - \frac{17b^2}{32a^3x^4} + \frac{221b^3}{192a^4x^3} - \frac{2431b^4}{768a^5x^2} + \frac{7293b^5}{512a^6x}\right)\frac{1}{X^2\sqrt{X}} + \frac{51051b^6}{1024a^6}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^8 X^{\frac{7}{2}}} = \left(-\frac{1}{7ax^7} + \frac{19b}{84a^2x^6} - \frac{323b^2}{840a^3x^5} + \frac{323b^3}{448a^4x^4} - \frac{4199b^4}{2688a^5x^3} + \frac{46189b^5}{10752a^6x^2} - \frac{138567b^6}{7168a^7x}\right)\frac{1}{X^2\sqrt{X}} - \frac{138567b^7}{2048a^7}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

Taf. IX. $\int \frac{x^m \partial x}{(a+bx)^{\frac{9}{2}}}, \int \frac{\partial x}{x^m (a+bx)^{\frac{9}{2}}}$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X^{\frac{9}{2}}} = -\frac{2}{7bX^3 \sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{9}{2}}} = \left(-\frac{1}{5}X + \frac{1}{7}a\right)\frac{2}{b^2 X^3 \sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{9}{2}}} = \left(-\frac{1}{3}X^2 + \frac{2}{5}aX - \frac{1}{7}a^2\right)\frac{2}{b^3 X^3 \sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{9}{2}}} = \left(-X^3 + aX^2 - \frac{3}{5}a^2 X + \frac{1}{7}a^3\right)\frac{2}{b^4 X^3 \sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{9}{2}}} = \left(X^4 + 4aX^3 - 2a^2X^2 + \frac{4}{5}a^3X - \frac{1}{7}a^4\right)\frac{2}{b^5 X^3 \sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{9}{2}}} = \left(\frac{1}{3}X^5 - 5aX^4 - 10a^2X^3 + \frac{10}{3}a^3X^2 - a^4X + \frac{1}{7}a^5\right)\frac{2}{b^6 X^3 \sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{9}{2}}} = \left(\frac{1}{5}X^6 - 2aX^5 + 15a^2X^4 + 20a^3X^3 - 5a^4X^2 + \frac{6}{5}a^5X - \frac{1}{7}a^6\right)\frac{2}{b^7 X^3 \sqrt{X}}$$

$$\int \frac{\partial x}{xX^{\frac{9}{2}}} = \left(\frac{352}{105a} + \frac{116bx}{15a^2} + \frac{20b^2x^2}{3a^3} + \frac{2b^3x^3}{a^4}\right)\frac{1}{X^3\sqrt{X}} + \frac{1}{a^4}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{9}{2}}} = -\frac{1}{axX^3\sqrt{X}} - \frac{9b}{2a}\int\frac{\partial x}{xX^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{9}{2}}} = \left(-\frac{1}{2ax^2} + \frac{11b}{4a^2x}\right)\frac{1}{X^3\sqrt{X}} + \frac{99b^2}{8a^2}\int\frac{\partial x}{xX^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{9}{2}}} = \left(-\frac{1}{3ax^3} + \frac{13b}{12a^2x^2} - \frac{143b^2}{24a^3x}\right)\frac{1}{X^3\sqrt{X}} - \frac{429b^3}{16a^3}\int\frac{\partial x}{xX^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{9}{2}}} = \left(-\frac{1}{4ax^4} + \frac{5b}{8a^2x^3} - \frac{65b^2}{32a^3x^2} + \frac{715b^3}{64a^4x}\right)\frac{1}{X^3\sqrt{X}} + \frac{6435b^4}{128a^4}\int\frac{\partial x}{xX^{\frac{9}{2}}}$$

$$\int x^m \partial x V(a+bx)$$ *Taf. X.*

VZ. $a + bx = X$

$$\int \partial x V X = \frac{2XVX}{3b}$$

$$\int x \partial x V X = \left(\tfrac{1}{5}X - \tfrac{1}{3}a\right)\frac{2XVX}{b^2}$$

$$\int x^2 \partial x V X = \left(\tfrac{1}{7}X^2 - \tfrac{2}{5}aX + \tfrac{1}{3}a^2\right)\frac{2XVX}{b^3}$$

$$\int x^3 \partial x V X = \left(\tfrac{1}{9}X^3 - \tfrac{3}{7}aX^2 + \tfrac{3}{5}a^2X - \tfrac{1}{3}a^3\right)\frac{2XVX}{b^4}$$

$$\int x^4 \partial x V X = \left(\tfrac{1}{11}X^4 - \tfrac{4}{9}aX^3 + \tfrac{6}{7}a^2X^2 - \tfrac{4}{5}a^3X + \tfrac{1}{3}a^4\right)\frac{2XVX}{b^5}$$

$$\int x^5 \partial x V X = \left(\tfrac{1}{13}X^5 - \tfrac{5}{11}aX^4 + \tfrac{10}{9}a^2X^3 - \tfrac{10}{7}a^3X^2 + a^4X - \tfrac{1}{3}a^5\right)\frac{2XVX}{b^6}$$

$$\int \partial^6 \partial x V X = \left(\tfrac{1}{15}X^6 - \tfrac{6}{13}aX^5 + \tfrac{15}{11}a^2X^4 - \tfrac{20}{9}a^3X^3 + \tfrac{15}{7}a^4X^2 - \tfrac{6}{5}a^5X + \tfrac{1}{3}a^6\right)\frac{2XVX}{b^7}$$

$$\int x^7 \partial x V X = \left(\tfrac{1}{17}X^7 - \tfrac{7}{15}aX^6 + \tfrac{21}{13}a^2X^5 - \tfrac{35}{11}a^3X^4 + \tfrac{35}{9}a^4X^3 - 3a^5X^2 + \tfrac{7}{5}a^6X - \tfrac{1}{3}a^7\right)\frac{2XVX}{b^8}$$

$$\int x^8 \partial x V X = \left(\tfrac{1}{19}X^8 - \tfrac{8}{17}aX^7 + \tfrac{28}{15}a^2X^6 - \tfrac{56}{13}a^3X^5 + \tfrac{70}{11}a^4X^4 - \tfrac{56}{9}a^5X^3 + 4a^6X^2 - \tfrac{8}{5}a^7X + \tfrac{1}{3}a^8\right)\frac{2XVX}{b^9}$$

$$\int x^9 \partial x V X = \left(\tfrac{1}{21}X^9 - \tfrac{9}{19}aX^8 + \tfrac{36}{17}a^2X^7 - \tfrac{28}{5}a^3X^6 + \tfrac{126}{13}a^4X^5 - \tfrac{126}{11}a^5X^4 + \tfrac{28}{3}a^6X^3 - \tfrac{36}{7}a^7X^2 + \tfrac{9}{5}a^8X - \tfrac{1}{3}a^9\right)\frac{2XVX}{b^{10}}$$

Taf. XI.

$$\int\frac{\partial x\sqrt{(a+bx)}}{x^m}$$

VZ. $a+bx=X$

$$\int\frac{\partial x\sqrt{X}}{x}=2\sqrt{X}+a\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x\sqrt{X}}{x^2}=-\frac{\sqrt{X}}{x}+\frac{b}{2}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x\sqrt{X}}{x^3}=-\frac{X\sqrt{X}}{2ax^2}+\frac{b\sqrt{X}}{4ax}-\frac{b^2}{8a}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x\sqrt{X}}{x^4}=\left(-\frac{1}{3ax^3}+\frac{b}{4a^2x^2}\right)X\sqrt{X}-\frac{b^2\sqrt{X}}{8a^2x}+\frac{b^3}{16a^2}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x\sqrt{X}}{x^5}=\left(-\frac{1}{4ax^4}+\frac{5b}{24a^2x^3}-\frac{5b^2}{32a^3x^2}\right)X\sqrt{X}+\frac{5b^3\sqrt{X}}{64a^3x}-\frac{5b^4}{128a^3}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x\sqrt{X}}{x^6}=\left(-\frac{1}{5ax^5}+\frac{7b}{40a^2x^4}-\frac{7b^2}{48a^3x^3}+\frac{7b^3}{64a^4x^2}\right)X\sqrt{X}-\frac{7b^4\sqrt{X}}{128a^4x}+\frac{7b^5}{256a^4}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x\sqrt{X}}{x^7}=\left(-\frac{1}{6ax^6}+\frac{3b}{20a^2x^5}-\frac{21b^2}{160a^3x^4}+\frac{7b^3}{64a^4x^3}-\frac{21b^4}{256a^5x^2}\right)X\sqrt{X}+\frac{21b^5\sqrt{X}}{512a^5x}-\frac{21b^6}{1024a^5}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x\sqrt{X}}{x^8}=-\frac{X\sqrt{X}}{7ax^7}-\frac{11b}{14a}\int\frac{\partial x\sqrt{X}}{x^7}$$

$$\int\frac{\partial x\sqrt{X}}{x^9}=\left(-\frac{1}{8ax^8}+\frac{13b}{112a^2x^7}\right)X\sqrt{X}+\frac{143b^2}{224a^2}\int\frac{\partial x\sqrt{X}}{x^7}$$

$$\int\frac{\partial x\sqrt{X}}{x^{10}}=\left(-\frac{1}{9ax^9}+\frac{5b}{48a^2x^8}-\frac{65b^2}{672a^3x^7}\right)X\sqrt{X}-\frac{715b^3}{1344a^3}\int\frac{\partial x\sqrt{X}}{x^7}$$

$$\int x^m \partial x (a+bx)^{\frac{3}{2}}$$

Taf. XII.

VZ. $a + bx = X$

$$\int \partial x X^{\frac{3}{2}} = \frac{2X^2\sqrt{X}}{5b}$$

$$\int x\partial x X^{\frac{3}{2}} = \left(\frac{1}{7}X - \frac{1}{5}a\right)\frac{2X^2\sqrt{X}}{b^2}$$

$$\int x^2\partial x X^{\frac{3}{2}} = \left(\frac{1}{9}X^2 - \frac{2}{7}aX + \frac{1}{5}a^2\right)\frac{2X^2\sqrt{X}}{b^3}$$

$$\int x^3\partial x X^{\frac{3}{2}} = \left(\frac{1}{11}X^3 - \frac{1}{3}aX^2 + \frac{3}{7}a^2X - \frac{1}{5}a^3\right)\frac{2X^2\sqrt{X}}{b^4}$$

$$\int x^4\partial x X^{\frac{3}{2}} = \left(\frac{1}{13}X^4 - \frac{4}{11}aX^3 + \frac{2}{3}a^2X^2 - \frac{4}{7}a^3X + \frac{1}{5}a^4\right)\frac{2X^2\sqrt{X}}{b^5}$$

$$\int x^5\partial x X^{\frac{3}{2}} = \left(\frac{1}{15}X^5 - \frac{5}{13}aX^4 + \frac{10}{11}a^2X^3 - \frac{10}{9}a^3X^2 + \frac{5}{7}a^4X - \frac{1}{5}a^5\right)\frac{2X^2\sqrt{X}}{b^6}$$

$$\int x^6\partial x X^{\frac{3}{2}} = \left(\frac{1}{17}X^6 - \frac{2}{5}a^2X^5 + \frac{15}{13}a^3X^4 - \frac{20}{11}a^4X^3 + \frac{5}{3}a^5X^2 - \frac{6}{7}a^6X + \frac{1}{5}a^7\right)\frac{2X^2\sqrt{X}}{b^7}$$

$$\int x^7\partial x X^{\frac{3}{2}} = \left(\frac{1}{19}X^7 - \frac{7}{17}aX^6 + \frac{7}{5}a^2X^5 - \frac{35}{13}a^3X^4 + \frac{35}{11}a^4X^3 - \frac{7}{3}a^5X^2 + a^6X - \frac{1}{5}a^7\right)\frac{2X^2\sqrt{X}}{b^8}$$

$$\int x^8\partial x X^{\frac{3}{2}} = \left(\frac{1}{21}X^8 - \frac{8}{19}aX^7 + \frac{28}{17}a^2X^6 - \frac{56}{15}a^3X^5 + \frac{70}{13}a^4X^4 - \frac{56}{11}a^5X^3 + \frac{28}{9}a^6X^2 - \frac{8}{7}a^7X + \frac{1}{5}a^8\right)\frac{2X^2\sqrt{X}}{b^9}$$

$$\int x^9\partial x X^{\frac{3}{2}} = \left(\frac{1}{23}X^9 - \frac{3}{7}aX^8 + \frac{36}{19}a^2X^7 - \frac{84}{17}a^3X^6 + \frac{42}{5}a^4X^5 - \frac{126}{13}a^5X^4 + \frac{84}{11}a^6X^3 - 4a^7X^2 + \frac{9}{7}a^8X - \frac{1}{5}a^9\right)\frac{2X^2\sqrt{X}}{b^{10}}$$

Taf. XIII.

$$\int\frac{\partial x(a+bx)^{\frac{3}{2}}}{x^m}$$

VZ. $a+bx=X$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x}=\left(\tfrac{1}{3}X+a\right)2\sqrt{X}+a^2\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^2}=-\frac{X^2\sqrt{X}}{ax}+\frac{3b}{2a}\int\frac{\partial xX^{\frac{3}{2}}}{x}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^3}=\left(-\frac{1}{2ax^2}-\frac{b}{4a^2x}\right)X^2\sqrt{X}+\frac{3b^2}{8a^2}\int\frac{\partial xX^{\frac{3}{2}}}{x}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^4}=\left(-\frac{1}{3ax^3}+\frac{b}{12a^2x^2}+\frac{b^2}{24a^3x}\right)X^2\sqrt{X}-\frac{b^3}{16a^3}\int\frac{\partial xX^{\frac{3}{2}}}{x}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^5}=\left(-\frac{1}{4ax^4}+\frac{b}{8a^2x^3}-\frac{b^2}{32a^3x^2}-\frac{b^3}{64a^4x}\right)X^2\sqrt{X}$$
$$+\frac{3b^4}{128a^4}\int\frac{\partial xX^{\frac{3}{2}}}{x}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^6}=\left(-\frac{1}{5ax^5}+\frac{b}{8a^2x^4}-\frac{b^2}{16a^3x^3}+\frac{b^3}{64a^4x^2}+\frac{b^4}{128a^5x}\right)X^2\sqrt{X}$$
$$-\frac{3b^5}{256a^5}\int\frac{\partial xX^{\frac{3}{2}}}{x}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^7}=\left(-\frac{1}{6ax^6}+\frac{7b}{60a^2x^5}-\frac{7b^2}{96a^3x^4}+\frac{7b^3}{192a^4x^3}-\frac{7b^4}{768a^5x^2}\right.$$
$$\left.-\frac{7b^5}{1536a^6x}\right)X^2\sqrt{X}+\frac{7b^6}{1024a^6}\int\frac{\partial xX^{\frac{3}{2}}}{x}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^8}=-\frac{X^2\sqrt{X}}{7ax^7}-\frac{9b}{14a}\int\frac{\partial xX^{\frac{3}{2}}}{x^7}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^9}=\left(-\frac{1}{8ax^8}+\frac{11b}{112a^2x^7}\right)X^2\sqrt{X}+\frac{99b^2}{224a^2}\int\frac{\partial xX^{\frac{3}{2}}}{x^7}$$

$$\int\frac{\partial xX^{\frac{3}{2}}}{x^{10}}=\left(-\frac{1}{9ax^9}+\frac{13b}{144a^2x^8}-\frac{143b^2}{2016a^3x^7}\right)X^2\sqrt{X}-\frac{143b^3}{448a^3}\int\frac{\partial xX^{\frac{3}{2}}}{x^7}$$

$$\int x^m \partial x (a+bx)^{\frac{5}{2}}$$ **Taf. XIV.**

VZ. $a + bx = X$

$$\int \partial x X^{\frac{5}{2}} = \frac{2X^3\sqrt{X}}{7b}$$

$$\int x\partial x X^{\frac{5}{2}} = \left(\frac{1}{9}X - \frac{1}{7}a\right)\frac{2X^3\sqrt{X}}{b^2}$$

$$\int x^2\partial x X^{\frac{5}{2}} = \left(\frac{1}{11}X^2 - \frac{2}{9}aX + \frac{1}{7}a^2\right)\frac{2X^3\sqrt{X}}{b^3}$$

$$\int x^3\partial x X^{\frac{5}{2}} = \left(\frac{1}{13}X^3 - \frac{3}{11}aX^2 + \frac{1}{3}a^2X - \frac{1}{7}a^3\right)\frac{2X^3\sqrt{X}}{b^4}$$

$$\int x^4\partial x X^{\frac{5}{2}} = \left(\frac{1}{15}X^4 - \frac{4}{13}aX^3 + \frac{6}{11}a^2X^2 - \frac{4}{9}a^3X + \frac{1}{7}a^4\right)\frac{2X^3\sqrt{X}}{b^5}$$

$$\int x^5\partial x X^{\frac{5}{2}} = \left(\frac{1}{17}X^5 - \frac{1}{3}aX^4 + \frac{10}{13}a^2X^3 - \frac{10}{11}a^3X^2 + \frac{5}{9}a^4X - \frac{1}{7}a^5\right)\frac{2X^3\sqrt{X}}{b^6}$$

$$\int x^6\partial x X^{\frac{5}{2}} = \left(\frac{1}{19}X^6 - \frac{6}{17}aX^5 + a^2X^4 - \frac{20}{13}a^3X^3 + \frac{15}{11}a^4X^2 - \frac{2}{3}a^5X + \frac{1}{7}a^6\right)\frac{2X^3\sqrt{X}}{b^7}$$

$$\int x^7\partial x X^{\frac{5}{2}} = \left(\frac{1}{21}X^7 - \frac{7}{19}aX^6 + \frac{21}{17}a^2X^5 - \frac{7}{3}a^3X^4 + \frac{35}{13}a^4X^3 - \frac{21}{11}a^5X^2 + \frac{7}{9}a^6X - \frac{1}{7}a^7\right)\frac{2X^3\sqrt{X}}{b^8}$$

$$\int x^8\partial x X^{\frac{5}{2}} = \left(\frac{1}{23}X^8 - \frac{8}{21}aX^7 + \frac{28}{19}a^2X^6 - \frac{56}{17}a^3X^5 + \frac{14}{3}a^4X^4 - \frac{56}{13}a^5X^3 + \frac{28}{11}a^6X^2 - \frac{8}{9}a^7X + \frac{1}{7}a^8\right)\frac{2X^3\sqrt{X}}{b^9}$$

$$\int x^9\partial x X^{\frac{5}{2}} = \left(\frac{1}{25}X^9 - \frac{9}{23}aX^8 + \frac{12}{7}a^2X^7 - \frac{84}{19}a^3X^6 + \frac{126}{17}a^4X^5 - \frac{42}{5}a^5X^4 + \frac{84}{13}a^6X^3 - \frac{36}{11}a^7X^2 + a^8X - \frac{1}{7}a^9\right)\frac{2X^3\sqrt{X}}{b^{10}}$$

Taf. XV.

$$\int \frac{\partial x (a+bx)^{\frac{5}{2}}}{x^m}$$

VZ. $a + bx = X$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x} = \left(\tfrac{1}{5}X^2 + \tfrac{1}{3}aX + a^2\right) 2\sqrt{X} + a^3 \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^2} = -\frac{X^3\sqrt{X}}{ax} + \frac{5b}{2a}\int \frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{3b}{4a^2x}\right) X^3\sqrt{X} + \frac{15b^2}{8a^2}\int \frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{b}{12a^2x^2} - \frac{b^2}{8a^3x}\right) X^3\sqrt{X} + \frac{5b^3}{16a^3}\int \frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{b}{24a^2x^3} + \frac{b^2}{96a^3x^2} + \frac{b^3}{64a^4x}\right) X^3\sqrt{X} - \frac{5b^4}{128a^4}\int \frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{3b}{40a^2x^4} - \frac{b^2}{80a^3x^3} - \frac{b^3}{320a^4x^2} - \frac{3b^4}{640a^5x}\right) X^3\sqrt{X} + \frac{3b^5}{256a^5}\int \frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{1}{12a^2x^7} - \frac{b^2}{32a^3x^4} + \frac{b^3}{192a^4x^3} + \frac{b^4}{768a^5x^2} + \frac{b^5}{512a^6x}\right) X^3\sqrt{X} - \frac{5b^6}{1024a^6}\int \frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^8} = -\frac{X^3\sqrt{X}}{7ax^7} - \frac{b}{2a}\int \frac{\partial x X^{\frac{5}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{9b}{112a^2x^7}\right) X^3\sqrt{X} + \frac{9b^2}{32a^2}\int \frac{\partial x X^{\frac{5}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{11b}{144a^2x^8} - \frac{11b^2}{224a^3x^7}\right) X^3\sqrt{X} - \frac{11b^3}{64a^3}\int \frac{\partial x X^{\frac{5}{2}}}{x^7}$$

$$\int x^m \partial x (a + bx)^{\frac{7}{2}}$$ *Taf. XVI.*

VZ. $a + bx = X$

$$\int \partial x X^{\frac{7}{2}} = \frac{2X^4\sqrt{X}}{9b}$$

$$\int x\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{11}X - \tfrac{1}{9}a\right)\frac{2X^4\sqrt{X}}{b^2}$$

$$\int x^2\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{13}X^2 - \tfrac{2}{11}aX + \tfrac{1}{9}a^2\right)\frac{2X^4\sqrt{X}}{b^3}$$

$$\int x^3\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{15}X^3 - \tfrac{3}{13}aX^2 + \tfrac{3}{11}a^2X - \tfrac{1}{9}a^3\right)\frac{2X^4\sqrt{X}}{b^4}$$

$$\int x^4\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{17}X^4 - \tfrac{4}{15}aX^3 + \tfrac{6}{13}a^2X^2 - \tfrac{4}{11}a^3X + \tfrac{1}{9}a^4\right)\frac{2X^4\sqrt{X}}{b^5}$$

$$\int x^5\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{19}X^5 - \tfrac{5}{17}aX^4 + \tfrac{2}{3}a^2X^3 - \tfrac{10}{13}a^3X^2 + \tfrac{5}{11}a^4X - \tfrac{1}{9}a^5\right)\frac{2X^4\sqrt{X}}{b^6}$$

$$\int x^6\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{21}X^6 - \tfrac{6}{19}aX^5 + \tfrac{15}{17}a^2X^4 - \tfrac{4}{3}a^3X^3 + \tfrac{15}{13}a^4X^2\right.$$
$$\left. - \tfrac{6}{11}a^5X + \tfrac{1}{9}a^6\right)\frac{2X^4\sqrt{X}}{b^7}$$

$$\int x^7\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{23}X^7 - \tfrac{1}{3}aX^6 + \tfrac{21}{19}a^2X^5 - \tfrac{35}{17}a^3X^4 + \tfrac{7}{3}a^4X^3\right.$$
$$\left. - \tfrac{21}{13}a^5X^2 + \tfrac{7}{11}a^6X - \tfrac{1}{9}a^7\right)\frac{2X^4\sqrt{X}}{b^8}$$

$$\int x^8\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{25}X^8 - \tfrac{8}{23}aX^7 + \tfrac{4}{3}a^2X^6 - \tfrac{56}{19}a^3X^5 + \tfrac{70}{17}a^4X^4\right.$$
$$\left. - \tfrac{56}{15}a^5X^3 + \tfrac{28}{13}a^6X^2 - \tfrac{8}{11}a^7X + \tfrac{1}{9}a^8\right)\frac{2X^4\sqrt{X}}{b^9}$$

$$\int x^9\partial x X^{\frac{7}{2}} = \left(\tfrac{1}{27}X^9 - \tfrac{9}{25}aX^8 + \tfrac{36}{23}a^2X^7 - 4a^3X^6 + \tfrac{126}{19}a^4X^5\right.$$
$$\left. - \tfrac{126}{17}a^5X^4 + \tfrac{28}{5}a^6X^3 - \tfrac{36}{13}a^7X^2 + \tfrac{9}{11}a^8X - \tfrac{1}{9}a^9\right)\frac{2X^4\sqrt{X}}{b^{10}}$$

Taf. XVII.

$$\int \frac{\partial x(a+bx)^{\frac{7}{2}}}{x^m}$$

VZ. $a + bx = X$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x} = \left(\tfrac{1}{7}X^3 + \tfrac{1}{5}aX^2 + \tfrac{1}{3}a^2 X + a^3\right)2\sqrt{X} + a^4 \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^2} = -\frac{X^4\sqrt{X}}{ax} + \frac{7b}{2a}\int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{5b}{4a^2 x}\right)X^4\sqrt{X} + \frac{35b^2}{8a^2}\int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{b}{4a^2x^2} - \frac{5b^2}{8a^3x}\right)X^4\sqrt{X} + \frac{35b^3}{16a^3}\int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^5} = \left(-\frac{1}{4ax^4} - \frac{b}{24a^2x^3} - \frac{b^2}{32a^3x^2} - \frac{5b^3}{64a^4x}\right)X^4\sqrt{X} + \frac{35b^4}{128a^4}\int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{b}{40a^2x^4} + \frac{b^2}{240a^3x^3} + \frac{b^3}{320a^4x^2} + \frac{b^4}{128a^5x}\right)X^4\sqrt{X} - \frac{7b^5}{256a^5}\int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{20a^2x^5} - \frac{b^2}{160a^3x^4} - \frac{b^3}{960a^4x^3} - \frac{b^4}{1280a^5x^2} - \frac{b^5}{512a^6x}\right)X^4\sqrt{X} + \frac{7b^6}{1024a^6}\int \frac{\partial x X^{\frac{7}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^8} = -\frac{X^4\sqrt{X}}{7ax^7} - \frac{5b}{14a}\int \frac{\partial x X^{\frac{7}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{b}{16a^2x^7}\right)X^4\sqrt{X} + \frac{5b^2}{32a^2}\int \frac{\partial x X^{\frac{7}{2}}}{x^7}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{b}{16a^2x^8} - \frac{b^2}{32a^3x^7}\right)X^4\sqrt{X} - \frac{5b^3}{64a^3}\int \frac{\partial x X^{\frac{7}{2}}}{x^7}$$

$$\int\frac{\partial x}{x^m\sqrt[3]{(a+bx)}},\quad\int\frac{\partial x}{x^m\sqrt[3]{(a+bx)^2}}$$ **Taf. XX.**

VZ. $a+bx=X$

$$\int\frac{\partial x}{x\sqrt[3]{X}}=\frac{1}{\sqrt[3]{a}}\left[\frac{3}{2}\log\frac{\sqrt[3]{X}-\sqrt[3]{a}}{\sqrt[3]{x}}+\sqrt{3}.\,\text{Arc Tang}\frac{\sqrt{3}.\sqrt[3]{X}}{\sqrt[3]{X}+2\sqrt[3]{a}}\right]$$

$$\int\frac{\partial x}{x^2\sqrt[3]{X}}=-\frac{\sqrt[3]{X^2}}{ax}-\frac{b}{3a}\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\int\frac{\partial x}{x^3\sqrt[3]{X}}=\left(-\frac{1}{2ax^2}+\frac{2b}{3a^2x}\right)\sqrt[3]{X^2}+\frac{2b^2}{9a^2}\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\int\frac{\partial x}{x^4\sqrt[3]{X}}=\left(-\frac{1}{3ax^3}+\frac{7b}{18a^2x^2}-\frac{14b^2}{27a^3x}\right)\sqrt[3]{X^2}-\frac{14b^3}{81a^3}\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\int\frac{\partial x}{x^5\sqrt[3]{X}}=\left(-\frac{1}{4ax^4}+\frac{5b}{18a^2x^3}-\frac{35b^2}{108a^3x^2}+\frac{35b^3}{81a^4x}\right)\sqrt[3]{X^2}+\frac{35b^4}{243a^4}\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\int\frac{\partial x}{x\sqrt[3]{X^2}}=\frac{1}{\sqrt[3]{a^2}}\left[\frac{3}{2}\log\frac{\sqrt[3]{X}-\sqrt[3]{a}}{\sqrt[3]{x}}-\sqrt{3}.\,\text{Arc Tang}\frac{\sqrt{3}.\sqrt[3]{X}}{\sqrt[3]{X}+2\sqrt[3]{a}}\right]$$

$$\int\frac{\partial x}{x^2\sqrt[3]{X^2}}=-\frac{\sqrt[3]{X}}{ax}-\frac{2b}{3a}\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

$$\int\frac{\partial x}{x^3\sqrt[3]{X^2}}=\left(-\frac{1}{2ax^2}+\frac{5b}{6a^2x}\right)\sqrt[3]{X}+\frac{5b^2}{9a^2}\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

$$\int\frac{\partial x}{x^4\sqrt[3]{X^2}}=\left(-\frac{1}{3ax^3}+\frac{4b}{9a^2x^2}-\frac{20b^2}{27a^3x}\right)\sqrt[3]{X}-\frac{40b^3}{81a^3}\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

$$\int\frac{\partial x}{x^5\sqrt[3]{X^2}}=\left(-\frac{1}{4ax^4}+\frac{11b}{36a^2x^3}-\frac{11b^2}{27a^3x^2}+\frac{55b^3}{81a^4x}\right)\sqrt[3]{X}+\frac{110b^4}{243a^4}\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

Taf. XIX. $\int \frac{x^m \partial x}{\sqrt[3]{(a+bx)}},\ \int \frac{x^m \partial x}{\sqrt[3]{(a+bx)^2}}$

VZ. $a + bx = X$

$$\int \frac{\partial x}{\sqrt[3]{X}} = \frac{3\sqrt[3]{X^2}}{2b}$$

$$\int \frac{x\partial x}{\sqrt[3]{X}} = \left(\tfrac{1}{5}X - \tfrac{1}{2}a\right)\frac{3\sqrt[3]{X^2}}{b^2}$$

$$\int \frac{x^2\partial x}{\sqrt[3]{X}} = \left(\tfrac{1}{8}X^2 - \tfrac{2}{5}aX + \tfrac{1}{2}a^2\right)\frac{3\sqrt[3]{X^2}}{b^3}$$

$$\int \frac{x^3\partial x}{\sqrt[3]{X}} = \left(\tfrac{1}{11}X^3 - \tfrac{3}{8}aX^2 + \tfrac{3}{5}a^2X - \tfrac{1}{2}a^3\right)\frac{3\sqrt[3]{X^2}}{b^4}$$

$$\int \frac{x^4\partial x}{\sqrt[3]{X}} = \left(\tfrac{1}{14}X^4 - \tfrac{4}{11}aX^3 + \tfrac{3}{4}a^2X^2 - \tfrac{4}{5}a^3X + \tfrac{1}{2}a^4\right)\frac{3\sqrt[3]{X^2}}{b^5}$$

$$\int \frac{x^5\partial x}{\sqrt[3]{X}} = \left(\tfrac{1}{17}X^5 - \tfrac{5}{14}aX^4 + \tfrac{10}{11}a^2X^3 - \tfrac{5}{4}a^3X^2 + a^4X - \tfrac{1}{2}a^5\right)\frac{3\sqrt[3]{X^2}}{b^6}$$

$$\int \frac{\partial x}{\sqrt[3]{X^2}} = \frac{3\sqrt[3]{X}}{b}$$

$$\int \frac{x\partial x}{\sqrt[3]{X^2}} = \left(\tfrac{1}{4}X - a\right)\frac{3\sqrt[3]{X}}{b^2}$$

$$\int \frac{x^2\partial x}{\sqrt[3]{X^2}} = \left(\tfrac{1}{7}X^2 - \tfrac{1}{2}aX + a^2\right)\frac{3\sqrt[3]{X}}{b^3}$$

$$\int \frac{x^3\partial x}{\sqrt[3]{X^2}} = \left(\tfrac{1}{10}X^3 - \tfrac{3}{7}aX^2 + \tfrac{3}{4}a^2X - a^3\right)\frac{3\sqrt[3]{X}}{b^4}$$

$$\int \frac{x^4\partial x}{\sqrt[3]{X^2}} = \left(\tfrac{1}{13}X^4 - \tfrac{2}{5}aX^3 + \tfrac{6}{7}a^2X^2 - a^3X + a^4\right)\frac{3\sqrt[3]{X}}{b^5}$$

$$\int \frac{x^5\partial x}{\sqrt[3]{X^2}} = \left(\tfrac{1}{16}X^5 - \tfrac{5}{13}aX^4 + a^2X^3 - \tfrac{10}{7}a^3X^2 + \tfrac{5}{4}a^4X - a^5\right)\frac{3\sqrt[3]{X}}{b^6}$$

$$\int \frac{\partial x}{x^m \sqrt[3]{(a+bx)}},\ \int \frac{\partial x}{x^m \sqrt[3]{(a+bx)^2}}$$ **Taf. XX.**

VZ. $a+bx = X$

$$\frac{\partial x}{x\sqrt[3]{X}} = \frac{1}{\sqrt[3]{a}}\left[\frac{3}{2}\log\frac{\sqrt[3]{X}-\sqrt[3]{a}}{\sqrt[3]{x}} + \sqrt{3}\,.\ \mathrm{Arc\,Tang}\frac{\sqrt{3}\,.\sqrt[3]{X}}{\sqrt[3]{X}+2\sqrt[3]{a}}\right]$$

$$\frac{\partial x}{x^2\sqrt[3]{X}} = -\frac{\sqrt[3]{X^2}}{ax} - \frac{b}{3a}\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\frac{\partial x}{x^3\sqrt[3]{X}} = \left(-\frac{1}{2ax^2} + \frac{2b}{3a^2x}\right)\sqrt[3]{X^2} + \frac{2b^2}{9a^2}\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\frac{\partial x}{x^4\sqrt[3]{X}} = \left(-\frac{1}{3ax^3} + \frac{7b}{18a^2x^2} - \frac{14b^2}{27a^3x}\right)\sqrt[3]{X^2} - \frac{14b^3}{81a^3}\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\frac{\partial x}{x^5\sqrt[3]{X}} = \left(-\frac{1}{4ax^4} + \frac{5b}{18a^2x^3} - \frac{35b^2}{108a^3x^2} + \frac{35b^3}{81a^4x}\right)\sqrt[3]{X^2} + \frac{35b^4}{243a^4}\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\frac{\partial x}{x\sqrt[3]{X^2}} = \frac{1}{\sqrt[3]{a^2}}\left[\frac{3}{2}\log\frac{\sqrt[3]{X}-\sqrt[3]{a}}{\sqrt[3]{x}} - \sqrt{3}\,.\ \mathrm{Arc\,Tang}\frac{\sqrt{3}\,.\sqrt[3]{X}}{\sqrt[3]{X}+2\sqrt[3]{a}}\right]$$

$$\frac{\partial x}{x^2\sqrt[3]{X^2}} = -\frac{\sqrt[3]{X}}{ax} - \frac{2b}{3a}\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

$$\frac{\partial x}{x^3\sqrt[3]{X^2}} = \left(-\frac{1}{2ax^2} + \frac{5b}{6a^2x}\right)\sqrt[3]{X} + \frac{5b^2}{9a^2}\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

$$\frac{\partial x}{x^4\sqrt[3]{X^2}} = \left(-\frac{1}{3ax^3} + \frac{4b}{9a^2x^2} - \frac{20b^2}{27a^3x}\right)\sqrt[3]{X} - \frac{40b^3}{81a^3}\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

$$\frac{\partial x}{x^5\sqrt[3]{X^2}} = \left(-\frac{1}{4ax^4} + \frac{11b}{36a^2x^3} - \frac{11b^2}{27a^3x^2} + \frac{55b^3}{81a^4x}\right)\sqrt[3]{X} + \frac{110b^4}{243a^4}\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

Taf. XXI. $\int x^m \partial x \sqrt[3]{(a+bx)}; \int x^m \partial x \sqrt[3]{(a+bx)^2}$

VZ. $a + bx = X$

$$\int \partial x \sqrt[3]{X} = \frac{3X\sqrt[3]{X}}{4b}$$

$$\int x \partial x \sqrt[3]{X} = \left(\frac{1}{7}X - \frac{1}{4}a\right)\frac{3X\sqrt[3]{X}}{b^2}$$

$$\int x^2 \partial x \sqrt[3]{X} = \left(\frac{1}{10}X^2 - \frac{2}{7}aX + \frac{1}{4}a^2\right)\frac{3X\sqrt[3]{X}}{b^3}$$

$$\int x^3 \partial x \sqrt[3]{X} = \left(\frac{1}{13}X^3 - \frac{3}{10}aX^2 + \frac{3}{7}a^2X - \frac{1}{4}a^3\right)\frac{3X\sqrt[3]{X}}{b^4}$$

$$\int x^4 \partial x \sqrt[3]{X} = \left(\frac{1}{16}X^4 - \frac{4}{13}aX^3 + \frac{3}{5}a^2X^2 - \frac{4}{7}a^3X + \frac{1}{4}a^4\right)\frac{3X\sqrt[3]{X}}{b^5}$$

$$\int x^5 \partial x \sqrt[3]{X} = \left(\frac{1}{19}X^5 - \frac{5}{16}aX^4 + \frac{10}{13}a^2X^3 - a^3X^2 + \frac{5}{7}a^4X - \frac{1}{5}a^5\right)\frac{3X\sqrt[3]{X}}{b^6}$$

$$\int \partial x \sqrt[3]{X^2} = \frac{3X\sqrt[3]{X^2}}{5b}$$

$$\int x \partial x \sqrt[3]{X^2} = \left(\frac{1}{8}X - \frac{1}{5}a\right)\frac{3X\sqrt[3]{X^2}}{b^2}$$

$$\int x^2 \partial x \sqrt[3]{X^2} = \left(\frac{1}{11}X^2 - \frac{1}{4}aX + \frac{1}{5}a^2\right)\frac{3X\sqrt[3]{X^2}}{b^3}$$

$$\int x^3 \partial x \sqrt[3]{X^2} = \left(\frac{1}{14}X^3 - \frac{3}{11}aX^2 + \frac{3}{8}a^2X - \frac{1}{5}a^3\right)\frac{3X\sqrt[3]{X^2}}{b^4}$$

$$\int x^4 \partial x \sqrt[3]{X^2} = \left(\frac{1}{17}X^4 - \frac{2}{7}aX^3 + \frac{6}{11}a^2X^2 - \frac{1}{2}a^3X + \frac{1}{5}a^4\right)\frac{3X\sqrt[3]{X^2}}{b^5}$$

$$\int x^5 \partial x \sqrt[3]{X^2} = \left(\frac{1}{20}X^5 - \frac{5}{17}aX^4 + \frac{5}{7}a^2X^3 - \frac{10}{11}a^3X^2 + \frac{5}{8}a^4X - \frac{1}{5}a^5\right)\frac{3X\sqrt[3]{X^2}}{b^6}$$

$$\int\frac{\partial x\sqrt[3]{(a+bx)}}{x^m},\quad\int\frac{\partial x\sqrt[3]{(a+bx)^2}}{x^m}$$ *Taf. XXII.*

VZ. $a+bx=X$

$$\int\frac{\partial x\sqrt[3]{X}}{x}=3\sqrt[3]{X}+a\int\frac{\partial x}{x\sqrt[3]{X^2}}$$

$$\int\frac{\partial x\sqrt[3]{X}}{x^2}=-\frac{X\sqrt[3]{X}}{ax}+\frac{b}{3a}\int\frac{\partial x\sqrt[3]{X}}{x}$$

$$\int\frac{\partial x\sqrt[3]{X}}{x^3}=\left(-\frac{1}{2ax^2}+\frac{b}{3a^2x}\right)X\sqrt[3]{X}-\frac{b^2}{9a^2}\int\frac{\partial x\sqrt[3]{X}}{x}$$

$$\int\frac{\partial x\sqrt[3]{X}}{x^4}=\left(-\frac{1}{3ax^3}+\frac{5b}{18a^2x^2}-\frac{5b^2}{27a^3x}\right)X\sqrt[3]{X}+\frac{5b^3}{81a^3}\int\frac{\partial x\sqrt[3]{X}}{x}$$

$$\int\frac{\partial x\sqrt[3]{X}}{x^5}=\left(-\frac{1}{4ax^4}+\frac{2b}{9a^2x^3}-\frac{5b^2}{27a^3x^2}+\frac{10b^3}{81a^4x}\right)X\sqrt[3]{X}-\frac{10b^4}{243a^4}\int\frac{\partial x\sqrt[3]{X}}{x}$$

$$\int\frac{\partial x\sqrt[3]{X^2}}{x}=\frac{3}{2}\sqrt[3]{X^2}+a\int\frac{\partial x}{x\sqrt[3]{X}}$$

$$\int\frac{\partial x\sqrt[3]{X^2}}{x^2}=-\frac{X\sqrt[3]{X^2}}{ax}+\frac{2b}{3a}\int\frac{\partial x\sqrt[3]{X^2}}{x}$$

$$\int\frac{\partial x\sqrt[3]{X^2}}{x^3}=\left(-\frac{1}{2ax^2}+\frac{b}{6a^2x}\right)X\sqrt[3]{X^3}-\frac{b^2}{9a^2}\int\frac{\partial x\sqrt[3]{X^2}}{x}$$

$$\int\frac{\partial x\sqrt[3]{X^2}}{x^4}=\left(-\frac{1}{3ax^3}+\frac{2b}{9a^2x^2}-\frac{2b^2}{27a^3x}\right)X\sqrt[3]{X^2}+\frac{4b^3}{81a^3}\int\frac{\partial x\sqrt[3]{X^2}}{x}$$

$$\int\frac{\partial x\sqrt[3]{X^2}}{x^5}=\left(-\frac{1}{4ax^4}+\frac{7b}{36a^2x^3}-\frac{7b^2}{54a^3x^2}+\frac{7b^3}{162a^4x}\right)X\sqrt[3]{X^2}$$
$$-\frac{7b^4}{243a^4}\int\frac{\partial x\sqrt[3]{X^2}}{x}$$

Taf. XXIII.

$$\int \frac{\partial x}{(a+bx^2)^{\frac{n}{2}}}$$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{1}{2}}} = \int \frac{\partial x}{\sqrt{X}}$$ (Man s. die folgende Seite.)

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = \frac{x}{a\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{5}{2}}} = \left(\frac{1}{3aX} + \frac{2}{3a^2}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{7}{2}}} = \left(\frac{1}{5aX^2} + \frac{4}{15a^2X} + \frac{8}{15a^3}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{9}{2}}} = \left(\frac{1}{7aX^3} + \frac{6}{35a^2X^2} + \frac{8}{35a^3X} + \frac{16}{35a^4}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{11}{2}}} = \left(\frac{1}{9aX^4} + \frac{8}{63a^2X^3} + \frac{16}{105a^3X^2} + \frac{64}{315a^4X} + \frac{128}{315a^5}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{13}{2}}} = \left(\frac{1}{11aX^5} + \frac{10}{99a^2X^4} + \frac{80}{693a^3X^3} + \frac{32}{231a^4X^2} + \frac{128}{693a^5X} + \frac{256}{693a^6}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{15}{2}}} = \left(\frac{1}{13aX^6} + \frac{12}{143a^2X^5} + \frac{40}{429a^3X^4} + \frac{320}{3003a^4X^3} + \frac{128}{1001a^5X^2} + \frac{512}{3003a^6X} + \frac{1024}{3003a^7}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{17}{2}}} = \left(\frac{1}{15aX^7} + \frac{14}{195a^2X^6} + \frac{56}{715a^3X^5} + \frac{112}{1287a^4X^4} + \frac{128}{1287a^5X^3} + \frac{256}{2145a^6X^2} + \frac{1024}{6435a^7X} + \frac{2048}{6435a^8}\right)\frac{x}{\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{19}{2}}} = \left(\frac{1}{17aX^8} + \frac{16}{255a^2X^7} + \frac{224}{3315a^3X^6} + \frac{896}{12155a^4X^5} + \frac{1792}{21879a^5X^4} + \frac{2048}{21879a^6X^3} + \frac{4096}{36465a^7X^2} + \frac{16384}{109395a^8X} + \frac{32768}{109395a^9}\right)\frac{x}{\sqrt{X}}$$

Anmerkung zur vorhergehenden Tafel.

Es ist im Allgemeinen

$$\int \frac{\partial x}{\sqrt{(a+bx^2)}} = \frac{1}{\sqrt{b}} \log [x\sqrt{b} + \sqrt{(a+bx^2)}] + \text{Const.}$$

oder $$\int \frac{\partial x}{\sqrt{(a+bx^2)}} = \frac{1}{\sqrt{-b}} \text{Arc Sin } x\sqrt{-\frac{b}{a}} + \text{Const.}$$

er erste Ausdruck wird reell, wenn b positiv, der zweite wird es, wenn negativ ist. Zugleich können a und b nicht negativ seyn. Hieraus ergibt sich:

I. $$\int \frac{\partial x}{\sqrt{(\pm a+bx^2)}} = \frac{1}{\sqrt{b}} \log [x\sqrt{b} + \sqrt{(\pm a + bx^2)}] + \text{Const.}$$

II. $$\int \frac{\partial x}{\sqrt{(a-bx^2)}} = \frac{1}{\sqrt{b}} \text{Arc Sin } x\sqrt{\frac{b}{a}} = \frac{1}{\sqrt{b}} \text{Arc Cos } \sqrt{\frac{a-bx^2}{a}}$$

$$= \frac{1}{2\sqrt{b}} \text{Arc Cos } \frac{a-2bx^2}{a} = \frac{1}{\sqrt{b}} \text{Arc Tang } \frac{x\sqrt{b}}{\sqrt{(a-bx^2)}}$$

$$= \frac{1}{\sqrt{b}} \text{Arc Cot } \frac{\sqrt{(a-bx^2)}}{x\sqrt{b}} = \frac{1}{\sqrt{b}} \text{Arc Sec } \sqrt{\frac{a}{a-bx^2}}$$

$$= \frac{1}{\sqrt{b}} \text{Arc Cosec } \sqrt{\frac{a}{bx^2}} = \frac{1}{2\sqrt{b}} \text{Arc Sin vers } \frac{2bx^2}{a}.$$

(Die Kreisbogen sind hier sämmtlich für $x = 0$ verschwindend genommen.)

besondere ist

$$\int \frac{\partial x}{\sqrt{(1+x^2)}} = \log [x + \sqrt{(1+x^2)}] + \text{Const.}$$

$$\int \frac{\partial x}{\sqrt{(x^2-1)}} = \log [x + \sqrt{(x^2-1)}] + \text{Const.}$$

$$\int \frac{\partial x}{\sqrt{(1-x^2)}} = \text{Arc Sin } x = \text{Arc Cos } \sqrt{(1-x^2)} = \tfrac{1}{2} \text{Arc Cos } (1-2x^2)$$

$$= \text{Arc Tang } \frac{x}{\sqrt{(1-x^2)}} = \text{Arc Cot } \frac{\sqrt{(1-x^2)}}{x} = \text{Arc Sec } \frac{1}{\sqrt{(1-x^2)}}$$

$$= \text{Arc Cosec } \frac{1}{x} = \tfrac{1}{2} \text{Arc Sin vers } 2x^2.$$

s Integral $\int \frac{\partial x}{\sqrt{(\pm a+bx^2)}}$ kann nur alsdann, wenn das obere Zeichen t, für $x = 0$ verschwinden, und in diesem Falle ist

$$\int \frac{\partial x}{\sqrt{(+a+bx^2)}} = \frac{1}{\sqrt{b}} \log \left(x\sqrt{\frac{b}{a}} + \sqrt{\frac{a+bx^2}{a}} \right)$$

Taf. XXIV. $\int \frac{x^m \partial x}{\sqrt{(a+bx^2)}}$

VZ. $a + bx^2 = X$

$\int \frac{\partial x}{\sqrt{X}} = \int \frac{\partial x}{\sqrt{X}}$ (Man s. die vorhergehende Seite.)

$\int \frac{x\partial x}{\sqrt{X}} = \frac{\sqrt{X}}{b}$

$\int \frac{x^2\partial x}{\sqrt{X}} = \frac{x\sqrt{X}}{2b} - \frac{a}{2b}\int \frac{\partial x}{\sqrt{X}}$

$\int \frac{x^3\partial x}{\sqrt{X}} = \left(\frac{x^2}{3b} - \frac{2a}{3b^2}\right)\sqrt{X}$

$\int \frac{x^4\partial x}{\sqrt{X}} = \left(\frac{x^3}{4b} - \frac{3ax}{8b^2}\right)\sqrt{X} + \frac{3a^2}{8b^2}\int \frac{\partial x}{\sqrt{X}}$

$\int \frac{x^5\partial x}{\sqrt{X}} = \left(\frac{x^4}{5b} - \frac{4ax^2}{15b^2} + \frac{8a^2}{15b^3}\right)\sqrt{X}$

$\int \frac{x^6\partial x}{\sqrt{X}} = \left(\frac{x^5}{6b} - \frac{5ax^3}{24b^2} + \frac{5a^2x}{16b^3}\right)\sqrt{X} - \frac{5a^3}{16b^3}\int \frac{\partial x}{\sqrt{X}}$

$\int \frac{x^7\partial x}{\sqrt{X}} = \left(\frac{x^6}{7b} - \frac{6ax^4}{35b^2} + \frac{8a^2x^2}{35b^3} - \frac{16a^3}{35b^4}\right)\sqrt{X}$

$\int \frac{x^8\partial x}{\sqrt{X}} = \left(\frac{x^7}{8b} - \frac{7ax^5}{48b^2} + \frac{35a^2x^3}{192b^3} - \frac{35a^3x}{128b^4}\right)\sqrt{X} + \frac{35a^4}{128b^4}\int \frac{\partial x}{\sqrt{X}}$

$\int \frac{x^9\partial x}{\sqrt{X}} = \left(\frac{x^8}{9b} - \frac{8ax^6}{63b^2} + \frac{16a^2x^4}{105b^3} - \frac{64a^3x^2}{315b^4} + \frac{128a^4}{315b^5}\right)\sqrt{X}$

$$\int \frac{x^{10}\partial x}{\sqrt{X}} = \left(\frac{x^9}{10b} - \frac{9ax^7}{80b^2} + \frac{21a^2x^5}{160b^3} - \frac{21a^3x}{128b^4} + \frac{63a^4x}{256b^5}\right)\sqrt{X} - \frac{63a^5}{256b^5}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^{11}\partial x}{\sqrt{X}} = \left(\frac{x^{10}}{11b} - \frac{10ax^8}{99b^2} + \frac{80a^2x^6}{693b^3} - \frac{32a^3x^4}{231b^4} + \frac{128a^4x^2}{693b^5} - \frac{256a^5}{693b^6}\right)\sqrt{X}$$

$$\int \frac{x^{12}\partial x}{\sqrt{X}} = \left(\frac{x^{11}}{12b} - \frac{11ax^9}{120b^2} + \frac{33a^2x^7}{320b^3} - \frac{77a^3x^5}{640b^4} + \frac{77a^4x}{512b^5} - \frac{231a^5x}{1024b^6}\right)\sqrt{X} + \frac{231a^6}{1024b^6}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^m \partial x}{\sqrt{(1-x^2)}}$$

Taf. XXV.

VZ. $1 - x^2 = X$

$$\frac{\partial x}{\sqrt{X}} = \text{Arc Sin } x$$ (Die andern Formen Seite 141)

$$\frac{x\partial x}{\sqrt{X}} = -\sqrt{X}$$

$$\frac{x^2\partial x}{\sqrt{X}} = -\tfrac{1}{2}x\sqrt{X} + \tfrac{1}{2}\int\frac{\partial x}{\sqrt{X}}$$

$$\frac{x^3\partial x}{\sqrt{X}} = -\left(\tfrac{1}{3}x^2 + \tfrac{2}{3}\right)\sqrt{X}$$

$$\frac{x^4\partial x}{\sqrt{X}} = -\left(\tfrac{1}{4}x^3 + \tfrac{3}{8}x\right)\sqrt{X} + \tfrac{3}{8}\int\frac{\partial x}{\sqrt{X}}$$

$$\frac{x^5\partial x}{\sqrt{X}} = -\left(\tfrac{1}{5}x^4 + \tfrac{4}{15}x^2 + \tfrac{8}{15}\right)\sqrt{X}$$

$$\frac{x^6\partial x}{\sqrt{X}} = -\left(\tfrac{1}{6}x^5 + \tfrac{5}{24}x^3 + \tfrac{5}{16}x\right)\sqrt{X} + \tfrac{5}{16}\int\frac{\partial x}{\sqrt{X}}$$

$$\frac{x^7\partial x}{\sqrt{X}} = -\left(\tfrac{1}{7}x^6 + \tfrac{6}{35}x^4 + \tfrac{8}{35}x^2 + \tfrac{16}{35}\right)\sqrt{X}$$

$$\frac{x^8\partial x}{\sqrt{X}} = -\left(\tfrac{1}{8}x^7 + \tfrac{7}{48}x^5 + \tfrac{35}{192}x^3 + \tfrac{35}{128}x\right)\sqrt{X} + \tfrac{35}{128}\int\frac{\partial x}{\sqrt{X}}$$

$$\frac{x^9\partial x}{\sqrt{X}} = -\left(\tfrac{1}{9}x^8 + \tfrac{8}{63}x^6 + \tfrac{16}{105}x^4 + \tfrac{64}{315}x^2 + \tfrac{128}{315}\right)\sqrt{X}$$

$$\frac{x^{10}\partial x}{\sqrt{X}} = -\left(\tfrac{1}{10}x^9 + \tfrac{9}{80}x^7 + \tfrac{21}{160}x^5 + \tfrac{21}{128}x^3 + \tfrac{63}{256}x\right)\sqrt{X} + \tfrac{63}{256}\int\frac{\partial x}{\sqrt{X}}$$

$$\frac{x^{11}\partial x}{\sqrt{X}} = -\left(\tfrac{1}{11}x^{10} + \tfrac{10}{99}x^8 + \tfrac{80}{693}x^6 + \tfrac{32}{231}x^4 + \tfrac{128}{693}x^2 + \tfrac{256}{693}\right)\sqrt{X}$$

$$\frac{x^{12}\partial x}{\sqrt{X}} = -\left(\tfrac{1}{12}x^{11} + \tfrac{11}{120}x^9 + \tfrac{33}{320}x^7 + \tfrac{77}{640}x^5 + \tfrac{77}{512}x^3 + \tfrac{231}{1024}x\right)\sqrt{X}$$
$$+ \tfrac{231}{1024}\int\frac{\partial x}{\sqrt{X}}$$

Taf. XXVI.

$$\int \frac{\partial x}{x^m \sqrt{(a+bx^2)}}$$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{x\sqrt{X}} = \int \frac{\partial x}{x\sqrt{X}}$$ (Man s. die folgende Seite.)

$$\int \frac{\partial x}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax}$$

$$\int \frac{\partial x}{x^3\sqrt{X}} = -\frac{\sqrt{X}}{2ax^2} - \frac{b}{2a}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^4\sqrt{X}} = \left(-\frac{1}{3ax^3} + \frac{2b}{3a^2x}\right)\sqrt{X}$$

$$\int \frac{\partial x}{x^5\sqrt{X}} = \left(-\frac{1}{4ax^4} + \frac{3b}{8a^2x^2}\right)\sqrt{X} + \frac{3b^2}{8a^2}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^6\sqrt{X}} = \left(-\frac{1}{5ax^5} + \frac{4b}{15a^2x^3} - \frac{8b^2}{15a^3x}\right)\sqrt{X}$$

$$\int \frac{\partial x}{x^7\sqrt{X}} = \left(-\frac{1}{6ax^6} + \frac{5b}{24a^2x^4} - \frac{5b^2}{16a^3x^2}\right)\sqrt{X} - \frac{5b^3}{16a^3}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^8\sqrt{X}} = \left(-\frac{1}{7ax^7} + \frac{6b}{35a^2x^5} - \frac{8b^2}{35a^3x^3} + \frac{16b^3}{35a^4x}\right)\sqrt{X}$$

$$\int \frac{\partial x}{x^9\sqrt{X}} = \left(-\frac{1}{8ax^8} + \frac{7b}{48a^2x^6} - \frac{35b^2}{192a^3x^4} + \frac{35b^3}{128a^4x^2}\right)\sqrt{X} + \frac{35b^3}{128a^4}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^{10}\sqrt{X}} = \left(-\frac{1}{9ax^9} + \frac{8b}{63a^2x^7} - \frac{16b^2}{105a^3x^5} + \frac{64b^3}{315a^4x^3} - \frac{128b^4}{315a^5x}\right)\sqrt{X}$$

$$\int \frac{\partial x}{x^{11}\sqrt{X}} = \left(-\frac{1}{10ax^{10}} + \frac{9b}{80a^2x^8} - \frac{21b^2}{160a^3x^6} + \frac{21b^3}{128a^4x^4} - \frac{63b^4}{256a^5x^2}\right)\sqrt{X} - \frac{63b^4}{256a^5}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^{12}\sqrt{X}} = \left(-\frac{1}{11ax^{11}} + \frac{10b}{99a^2x^9} - \frac{80b^2}{693a^3x^7} + \frac{32b^3}{231a^4x^5} - \frac{128b^4}{693a^5x^3} + \frac{256b^5}{693a^6x}\right)\sqrt{X}$$

$$\int \frac{\partial x}{x^m(a+bx^2)^{\frac{3}{2}}}$$

Taf. XXVIII.

NZ. $a + bx^2 = X$

$$\int \frac{\partial x}{xX^{\frac{3}{2}}} = \frac{1}{a\sqrt{X}} + \frac{1}{a}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{ax} - \frac{2bx}{a^2}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} - \frac{3b}{2a^2}\right)\frac{1}{\sqrt{X}} - \frac{3b}{2a^2}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{3ax^3} + \frac{4b}{3a^2x} + \frac{8b^2x}{3a^3}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{4ax^4} + \frac{5b}{8a^2x^2} + \frac{15b^2}{8a^3}\right)\frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^5} + \frac{2b}{5a^2x^3} - \frac{8b^2}{5a^3x} - \frac{16b^3x}{5a^4}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^7X^{\frac{3}{2}}} = \left(-\frac{1}{6ax^6} + \frac{7b}{24a^2x^4} - \frac{35b^2}{48a^3x^2} - \frac{35b^3}{16a^4}\right)\frac{1}{\sqrt{X}} - \frac{35b^3}{16a^4}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^8X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^7} + \frac{8b}{35a^2x^5} - \frac{16b^2}{35a^3x^3} + \frac{64b^3}{35a^4x} + \frac{128b^4x}{35a^5}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{8ax^8} + \frac{3b}{16a^2x^6} - \frac{21b^2}{64a^3x^4} + \frac{105b^3}{128a^4x^2} + \frac{315b^4}{128a^5}\right)\frac{1}{\sqrt{X}} + \frac{315b^4}{128a^5}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^{10}X^{\frac{3}{2}}} = \left(-\frac{1}{9ax^9} + \frac{10b}{63a^2x^7} - \frac{16b^2}{63a^3x^5} + \frac{32b^3}{63a^4x^3} - \frac{128b^4}{63a^5x} - \frac{256b^5x}{63a^6}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^{11}X^{\frac{3}{2}}} = \left(\frac{1}{10ax^{10}} + \frac{11b}{80a^2x^8} - \frac{33b^2}{160a^3x^6} + \frac{231b^3}{640a^4x^4} - \frac{231b^4}{256a^5x^2} - \frac{693b^5}{256a^6}\right)\frac{1}{\sqrt{X}} - \frac{693b^5}{256a^6}\int\frac{\partial x}{x\sqrt{X}}$$

Taf. XXVII. $$\int \frac{x^m \partial x}{(a+bx^2)^{\frac{3}{2}}}$$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = \frac{x}{aVX}$$

$$\int \frac{x\partial x}{X^{\frac{3}{2}}} = -\frac{1}{bVX}$$

$$\int \frac{x^2\partial x}{X^{\frac{3}{2}}} = -\frac{x}{bVX} + \frac{1}{b}\int \frac{\partial x}{VX}$$

$$\int \frac{x^3\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^2}{b} + \frac{2a}{b^2}\right)\frac{1}{VX}$$

$$\int \frac{x^4\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^3}{2b} + \frac{3ax}{2b^2}\right)\frac{1}{VX} - \frac{3a}{2b^2}\int \frac{\partial x}{VX}$$

$$\int \frac{x^5\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^4}{3b} - \frac{4ax^2}{3b^2} - \frac{8a^2}{3b^3}\right)\frac{1}{VX}$$

$$\int \frac{x^6\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^5}{4b} - \frac{5ax^3}{8b^2} - \frac{15a^2x}{8b^3}\right)\frac{1}{VX} + \frac{15a^2}{8b^3}\int \frac{\partial x}{VX}$$

$$\int \frac{x^7\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^6}{5b} - \frac{2ax^4}{5b^2} + \frac{8a^2x^2}{5b^3} + \frac{16a^3}{5b^4}\right)\frac{1}{VX}$$

$$\int \frac{x^8\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^7}{6b} - \frac{7ax^5}{24b^2} + \frac{35a^2x^3}{48b^3} + \frac{35a^3x}{16b^4}\right)\frac{1}{VX} - \frac{35a^3}{16b^4}\int \frac{\partial x}{VX}$$

$$\int \frac{x^9\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^8}{7b} - \frac{8ax^6}{35b^2} + \frac{16a^2x^4}{35b^3} - \frac{64a^3x^2}{35b^4} - \frac{128a^4}{35b^5}\right)\frac{1}{VX}$$

$$\int \frac{x^{10}\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^9}{8b} - \frac{3ax^7}{16b^2} + \frac{21a^2x^5}{64b^3} - \frac{105a^3x^3}{128b^4} - \frac{315a^4x}{128b^5}\right)\frac{1}{VX} + \frac{315a^4}{128b^5}\int \frac{\partial x}{VX}$$

$$\int \frac{x^{11}\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^{10}}{9b} - \frac{10ax^8}{63b^2} + \frac{16a^2x^6}{63b^3} - \frac{32a^3x^4}{63b^4} + \frac{128a^4x^2}{63b^5} + \frac{256a^5}{63b^6}\right)\frac{1}{VX}$$

$$\int \frac{\partial x}{x^m (a+bx^2)^{\frac{3}{2}}}$$ **Taf. XXVIII.**

NZ. $a + bx^2 = X$

$$\int \frac{\partial x}{x X^{\frac{3}{2}}} = \frac{1}{a\sqrt{X}} + \frac{1}{a}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2 X^{\frac{3}{2}}} = \left(-\frac{1}{ax} - \frac{2bx}{a^2}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^3 X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} - \frac{3b}{2a^2}\right)\frac{1}{\sqrt{X}} - \frac{3b}{2a^2}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^4 X^{\frac{3}{2}}} = \left(-\frac{1}{3ax^3} + \frac{4b}{3a^2 x} + \frac{8b^2 x}{3a^3}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^5 X^{\frac{3}{2}}} = \left(-\frac{1}{4ax^4} + \frac{5b}{8a^2x^2} + \frac{15b^2}{8a^3}\right)\frac{1}{\sqrt{X}} + \frac{15b^2}{8a^3}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^6 X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^5} + \frac{2b}{5a^2x^3} - \frac{8b^2}{5a^3x} - \frac{16b^3x}{5a^4}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^7 X^{\frac{3}{2}}} = \left(-\frac{1}{6ax^6} + \frac{7b}{24a^2x^4} - \frac{35b^2}{48a^3x^2} - \frac{35b^3}{16a^4}\right)\frac{1}{\sqrt{X}} - \frac{35b^3}{16a^4}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^8 X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^7} + \frac{8b}{35a^2x^5} - \frac{16b^2}{35a^3x^3} + \frac{64b^3}{35a^4x} + \frac{128b^4x}{35a^5}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^9 X^{\frac{3}{2}}} = \left(-\frac{1}{8ax^8} + \frac{3b}{16a^2x^6} - \frac{21b^2}{64a^3x^4} + \frac{105b^3}{128a^4x^2} + \frac{315b^4}{128a^5}\right)\frac{1}{\sqrt{X}}$$
$$+ \frac{315b^4}{128a^5}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^{10} X^{\frac{3}{2}}} = \left(-\frac{1}{9ax^9} + \frac{10b}{63a^2x^7} - \frac{16b^2}{63a^3x^5} + \frac{32b^3}{63a^4x^3} - \frac{128b^4}{63a^5x}\right.$$
$$\left. - \frac{256b^5x}{63a^6}\right)\frac{1}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^{11} X^{\frac{3}{2}}} = \left(\frac{1}{10ax^{10}} + \frac{11b}{80a^2x^8} - \frac{33b^2}{160a^3x^6} + \frac{231b^3}{640a^4x^4} - \frac{231b^4}{256a^5x^2}\right.$$
$$\left. - \frac{693b^5}{256a^6}\right)\frac{1}{\sqrt{X}} - \frac{693b^5}{256a^6}\int \frac{\partial x}{x\sqrt{X}}$$

Taf. XXIX. $\int \frac{x^m \partial x}{(a+bx^2)^{\frac{5}{2}}}$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{5}{2}}} = \left(\frac{2bx^3}{3a^2} + \frac{x}{a}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{5}{2}}} = -\frac{1}{3bX\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{5}{2}}} = \frac{x^3}{3aX\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{5}{2}}} = \left(-\frac{x^2}{b} - \frac{2a}{3b^2}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{5}{2}}} = \left(-\frac{4x^3}{3b} - \frac{ax}{b^2}\right)\frac{1}{X\sqrt{X}} + \frac{1}{b^2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^4}{b} + \frac{4ax^2}{b^2} + \frac{8a^2}{3b^3}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^5}{2b} + \frac{10ax^3}{3b^2} + \frac{5a^2x}{2b^3}\right)\frac{1}{X\sqrt{X}} - \frac{5a}{2b^3}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^6}{3b} - \frac{2ax^4}{b^2} - \frac{8a^2x^2}{b^3} - \frac{16a^3}{3b^4}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x^8\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^7}{4b} - \frac{7ax^5}{8b^2} - \frac{35a^2x^3}{6b^3} - \frac{35a^3x}{8b^4}\right)\frac{1}{X\sqrt{X}} + \frac{35a^2}{8b^4}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^9\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^8}{5b} - \frac{8ax^6}{15b^2} + \frac{16a^2x^4}{5b^3} + \frac{64a^3x^2}{5b^4} + \frac{128a^4}{15b^5}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{x^{10}\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^9}{6b} - \frac{3ax^7}{8b^2} + \frac{21a^2x^5}{16b^3} + \frac{35a^3x^3}{4b^4} + \frac{105a^4x}{16b^5}\right)\frac{1}{X\sqrt{X}} - \frac{105a^3}{16b^5}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^{11}\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^{10}}{7b} - \frac{2ax^8}{7b^2} + \frac{16a^2x^6}{21b^3} - \frac{32a^3x^4}{7b^4} - \frac{128a^4x^2}{7b^5} - \frac{256a^5}{21b^6}\right)\frac{1}{X\sqrt{X}}$$

$$\int \frac{\partial x}{x^m(a+bx^2)^{\frac{5}{2}}}$$ **Taf. XXX.**

VZ. $a+bx^2 = X$

$$\int \frac{\partial x}{xX^{\frac{5}{2}}} = \left(\frac{4}{3a}+\frac{bx^2}{a^2}\right)\frac{1}{X\sqrt{X}}+\frac{1}{a^2}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{5}{2}}} = -\frac{1}{axX\sqrt{X}}-\frac{4b}{a}\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{5}{2}}} = -\frac{1}{2ax^2X\sqrt{X}}-\frac{5b}{2a}\int\frac{\partial x}{xX^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{5}{2}}} = \left(-\frac{1}{3ax^3}+\frac{2b}{a^2x}\right)\frac{1}{X\sqrt{X}}+\frac{8b^2}{a^2}\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{5}{2}}} = \left(-\frac{1}{4ax^4}+\frac{7b}{8a^2x^2}\right)\frac{1}{X\sqrt{X}}+\frac{35b^2}{8a^2}\int\frac{\partial x}{xX^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{5}{2}}} = \left(-\frac{1}{5ax^5}+\frac{8b}{15a^2x^3}-\frac{16b^2}{5a^3x}\right)\frac{1}{X\sqrt{X}}-\frac{64b^3}{5a^3}\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{5}{2}}} = \left(-\frac{1}{6ax^6}+\frac{3b}{8a^2x^4}-\frac{21b^2}{16a^3x^2}\right)\frac{1}{X\sqrt{X}}-\frac{105b^3}{16a^3}\int\frac{\partial x}{xX^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{5}{2}}} = \left(-\frac{1}{7ax^7}+\frac{2b}{7a^2x^5}-\frac{16b^2}{21a^3x^3}+\frac{32b^3}{7a^4x}\right)\frac{1}{X\sqrt{X}}+\frac{128b^4}{7a^4}\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{5}{2}}} = \left(-\frac{1}{8ax^8}+\frac{11b}{48a^2x^6}-\frac{33b^2}{64a^3x^4}+\frac{231b^3}{128a^4x^2}\right)\frac{1}{X\sqrt{X}}$$
$$+\frac{1155b^4}{128a^4}\int\frac{\partial x}{xX^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^{10}X^{\frac{5}{2}}} = \left(-\frac{1}{9ax^9}+\frac{4b}{21a^2x^7}-\frac{8b^2}{21a^3x^5}+\frac{64b^3}{63a^4x^3}-\frac{128b^4}{21a^5x}\right)\frac{1}{X\sqrt{X}}$$
$$-\frac{512b^5}{21a^5}\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^{11}X^{\frac{5}{2}}} = \left(-\frac{1}{10ax^{10}}+\frac{13b}{80a^2x^8}-\frac{143b^2}{480a^3x^6}+\frac{429b^3}{640a^4x^4}-\frac{3003b^4}{1280a^5x^2}\right)\frac{1}{X\sqrt{X}}$$
$$-\frac{3003b^5}{256a^5}\int\frac{\partial x}{xX^{\frac{5}{2}}}$$

Taf. XXXI. $\displaystyle\int \frac{x^m \partial x}{(a+bx^2)^{\frac{7}{2}}}$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{7}{2}}} = \left(\frac{8b^2x^5}{15a^3} + \frac{4bx^3}{3a^2} + \frac{x}{a}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{7}{2}}} = -\frac{1}{5bX^2\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{7}{2}}} = \left(\frac{2bx^5}{15a^2} + \frac{x^3}{3a}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{7}{2}}} = \left(-\frac{x^2}{3b} - \frac{2a}{15b^2}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{7}{2}}} = \frac{x^5}{5aX^2\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{7}{2}}} = \left(-\frac{x^4}{b} - \frac{4ax^2}{3b^2} - \frac{8a^2}{15b^3}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{7}{2}}} = \left(-\frac{23x^5}{15b} - \frac{7ax^3}{3b^2} - \frac{a^2x}{b^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{1}{b^3}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^6}{b} + \frac{6ax^4}{b^2} + \frac{8a^2x^2}{b^3} + \frac{16a^3}{5b^4}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^8\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^7}{2b} + \frac{161ax^5}{30b^2} + \frac{49a^2x^3}{6b^3} + \frac{7a^3x}{2b^4}\right)\frac{1}{X^2\sqrt{X}} - \frac{7a}{2b^4}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^9\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^8}{3b} - \frac{8ax^6}{3b^2} - \frac{16a^2x^4}{b^3} - \frac{64a^3x^2}{3b^4} - \frac{128a^4}{15b^5}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{x^{10}\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^9}{4b} - \frac{9ax^7}{8b^2} - \frac{483a^2x^5}{40b^3} - \frac{147a^3x^3}{8b^4} - \frac{63a^4x}{8b^5}\right)\frac{1}{X^2\sqrt{X}} + \frac{63a^2}{8b^5}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^{11}\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^{10}}{5b} - \frac{2ax^8}{3b^2} + \frac{16a^2x^6}{3b^3} + \frac{32a^3x^4}{b^4} + \frac{128a^4x}{3b^5} + \frac{256a^5}{15b^6}\right)\frac{1}{X^2\sqrt{X}}$$

$$\int \frac{\partial x}{x^m(a+bx^2)^{\frac{7}{2}}}$$

Taf. XXXII.

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{xX^{\frac{7}{2}}} = \left(\frac{23}{15a} + \frac{7bx^2}{3a^2} + \frac{b^2x^4}{a^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{1}{a^3}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{7}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{6b}{a}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{7}{2}}} = -\frac{1}{2ax^2X^2\sqrt{X}} - \frac{7b}{2a}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{7}{2}}} = \left(-\frac{1}{3ax^3} + \frac{8b}{3a^2x}\right)\frac{1}{X^2\sqrt{X}} + \frac{16b^2}{a^2}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{7}{2}}} = \left(-\frac{1}{4ax^4} + \frac{9b}{8a^2x^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{63b^2}{8a^2}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{7}{2}}} = \left(-\frac{1}{5ax^5} + \frac{2b}{3a^2x^3} - \frac{16b^2}{3a^3x}\right)\frac{1}{X^2\sqrt{X}} - \frac{32b^3}{a^3}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{7}{2}}} = \left(-\frac{1}{6ax^6} + \frac{11b}{24a^2x^4} - \frac{33b^2}{16a^3x^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{231b^3}{16a^3}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{7}{2}}} = \left(-\frac{1}{7ax^7} + \frac{12b}{35a^2x^5} - \frac{8b^2}{7a^3x^3} + \frac{64b^3}{7a^4x}\right)\frac{1}{X^2\sqrt{X}} + \frac{384b^4}{7a^4}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{7}{2}}} = \left(-\frac{1}{8ax^8} + \frac{13b}{48a^2x^6} - \frac{143b^2}{192a^3x^4} + \frac{429b^3}{128a^4x^2}\right)\frac{1}{X^2\sqrt{X}} + \frac{3003b^4}{128a^4}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^{10}X^{\frac{7}{2}}} = \left(-\frac{1}{9ax^9} + \frac{2b}{9a^2x^7} - \frac{8b^2}{15a^3x^5} + \frac{16b^3}{9a^4x^3} - \frac{128b^4}{9a^5x}\right)\frac{1}{X^2\sqrt{X}} - \frac{256b^5}{3a^5}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^{11}X^{\frac{7}{2}}} = \left(-\frac{1}{10ax^{10}} + \frac{3b}{16a^2x^8} - \frac{13b^2}{32a^3x^6} + \frac{143b^3}{128a^4x^4} - \frac{1287b^4}{256a^5x^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{9009b^5}{256a^5}\int\frac{\partial x}{xX^{\frac{7}{2}}}$$

Taf. XXXIII. $\int \frac{x^m \partial x}{(a+bx^2)^{\frac{9}{2}}}, \int \frac{\partial x}{x^m (a+bx^2)^{\frac{9}{2}}}$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{9}{2}}} = \left(\frac{16b^3x^7}{35a^4} + \frac{8b^2x^5}{5a^3} + \frac{2bx^3}{a^2} + \frac{x}{a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{9}{2}}} = -\frac{1}{7bX^3\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{9}{2}}} = \left(\frac{8b^2x^7}{105a^3} + \frac{4bx^5}{15a^2} + \frac{x^3}{3a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{9}{2}}} = \left(-\frac{x^2}{5b} - \frac{2a}{35b^2}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{9}{2}}} = \left(\frac{2bx^7}{35a^2} + \frac{x^5}{5a}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{9}{2}}} = \left(-\frac{x^4}{3b} - \frac{4ax^2}{15b^2} - \frac{8a^2}{105b^3}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{9}{2}}} = \frac{x^7}{7aX^3\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X^{\frac{9}{2}}} = \left(-\frac{x^6}{b} - \frac{2ax^4}{b^2} - \frac{8a^2x^2}{5b^3} - \frac{16a^3}{35b^4}\right)\frac{1}{X^3\sqrt{X}}$$

$$\int \frac{\partial x}{xX^{\frac{9}{2}}} = \left(\frac{176}{105a} + \frac{58bx^2}{15a^2} + \frac{10b^2x^4}{3a^3} + \frac{b^3x^6}{a^4}\right)\frac{1}{X^3\sqrt{X}} + \frac{1}{a^4}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{9}{2}}} = -\frac{1}{axX^3\sqrt{X}} - \frac{8b}{a}\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{9}{2}}} = -\frac{1}{2ax^2X^3\sqrt{X}} - \frac{9b}{2a}\int \frac{\partial x}{xX^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{9}{2}}} = \left(-\frac{1}{3ax^3} + \frac{10b}{3a^2x}\right)\frac{1}{X^3\sqrt{X}} + \frac{80b^2}{3a^2}\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{9}{2}}} = \left(-\frac{1}{4ax^4} + \frac{11b}{8a^2x^2}\right)\frac{1}{X^3\sqrt{X}} + \frac{99b^2}{8a^2}\int \frac{\partial x}{xX^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{9}{2}}} = \left(-\frac{1}{5ax^5} + \frac{4b}{5a^2x^3} - \frac{8b^2}{a^3x}\right)\frac{1}{X^3\sqrt{X}} - \frac{64b^3}{a^3}\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{9}{2}}} = \left(-\frac{1}{6ax^6} + \frac{13b}{24a^2x^4} - \frac{143b^2}{48a^3x^2}\right)\frac{1}{X^3\sqrt{X}} - \frac{429b^3}{16a^3}\int \frac{\partial x}{xX^{\frac{9}{2}}}$$

$$\int x^m \partial x \sqrt{(a+bx^2)}$$ Taf. XXXIV.

VZ. $a + bx^2 = X$

$$\int \partial x \sqrt{X} = \frac{x\sqrt{X}}{2} + \frac{a}{2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x \sqrt{X} = \frac{X\sqrt{X}}{3b}$$

$$\int x^2\partial x \sqrt{X} = \frac{xX\sqrt{X}}{4b} - \frac{a}{4b}\int \partial x \sqrt{X}$$

$$\int x^3\partial x \sqrt{X} = \left(\frac{x^2}{5b} - \frac{2a}{15b^2}\right)X\sqrt{X}$$

$$\int x^4\partial x \sqrt{X} = \left(\frac{x^3}{6b} - \frac{ax}{8b^2}\right)X\sqrt{X} + \frac{a^2}{8b^2}\int \partial x \sqrt{X}$$

$$\int x^5\partial x \sqrt{X} = \left(\frac{x^4}{7b} - \frac{4ax^2}{35b^2} + \frac{8a^2}{105b^3}\right)X\sqrt{X}$$

$$\int x^6\partial x \sqrt{X} = \left(\frac{x^5}{8b} - \frac{5ax^3}{48b^2} + \frac{5a^2x}{64b^3}\right)X\sqrt{X} - \frac{5a^3}{64b^3}\int \partial x \sqrt{X}$$

$$\int x^7\partial x \sqrt{X} = \left(\frac{x^6}{9b} - \frac{2ax^4}{21b^2} + \frac{8a^2x^2}{105b^3} - \frac{16a^3}{315b^4}\right)X\sqrt{X}$$

$$\int x^8\partial x \sqrt{X} = \left(\frac{x^7}{10b} - \frac{7ax^5}{80b^2} + \frac{7a^2x^3}{96b^3} - \frac{7a^3x}{128b^5}\right)X\sqrt{X} + \frac{7a^4}{128b^4}\int \partial x \sqrt{X}$$

$$\int x^9\partial x \sqrt{X} = \left(\frac{x^8}{11b} - \frac{8ax^6}{99b^2} + \frac{16a^2x^4}{231b^3} - \frac{64a^3x^2}{1155b^4} + \frac{128a^4}{3465b^5}\right)X\sqrt{X}$$

$$\int x^{10}\partial x \sqrt{X} = \left(\frac{x^9}{12b} - \frac{3ax^7}{40b^2} + \frac{21a^2x^5}{320b^3} - \frac{7a^3x^3}{128b^4} + \frac{21a^4x}{512b^5}\right)X\sqrt{X}$$
$$- \frac{21a^5}{512b^5}\int \partial x \sqrt{X}$$

$$\int x^{11}\partial x \sqrt{X} = \left(\frac{x^{10}}{13b} - \frac{10ax^8}{143b^2} + \frac{80a^2x^6}{1287b^3} - \frac{160a^3x^4}{3003b^4} + \frac{128a^4x^2}{3003b^5}\right.$$
$$\left. - \frac{256a^5}{9009b^6}\right)X\sqrt{X}$$

Taf. XXXV. $\int \frac{\partial x \sqrt{(a+bx^2)}}{x^m}$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x \sqrt{X}}{x} = \sqrt{X} + a \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + b \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^3} = -\frac{\sqrt{X}}{2x^2} + \frac{b}{2} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^4} = -\frac{X\sqrt{X}}{3ax^3}$$

$$\int \frac{\partial x \sqrt{X}}{x^5} = -\frac{X\sqrt{X}}{4ax^4} + \frac{b\sqrt{X}}{8ax^2} - \frac{b^2}{8a} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^6} = \left(-\frac{1}{5ax^5} + \frac{2b}{15a^2x^3}\right) X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{8a^2x^4}\right) X\sqrt{X} - \frac{b^2\sqrt{X}}{16a^2x^2} + \frac{b^3}{16a^2} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^8} = \left(-\frac{1}{7ax^7} + \frac{4b}{35a^2x^5} - \frac{8b^2}{105a^3x^3}\right) X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{5b}{48a^2x^6} - \frac{5b^2}{64a^3x^4}\right) X\sqrt{X} + \frac{5b^3\sqrt{X}}{128a^3x^2} - \frac{5b^4}{128a^3} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{2b}{21a^2x^7} - \frac{8b^2}{105a^3x^5} + \frac{16b^3}{315a^4x^3}\right) X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^{11}} = \left(-\frac{1}{10ax^{10}} + \frac{7b}{80a^2x^8} - \frac{7b^2}{96a^3x^6} + \frac{7b^3}{128a^4x^4}\right) X\sqrt{X} - \frac{7b^4\sqrt{X}}{256a^4x^2} + \frac{7b^5}{256a^4} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^{12}} = \left(-\frac{1}{11ax^{11}} + \frac{8b}{99a^2x^9} - \frac{16b^2}{231a^3x^7} + \frac{64b^3}{1155a^4x^5} - \frac{128b^4}{3465a^5x^3}\right) X\sqrt{X}$$

$\int x^m \partial x (a+bx^2)^{\frac{3}{2}}$ Taf. XXXVI.

VZ. $a + bx^2 = X$

$$\int \partial x X^{\frac{3}{2}} = \left(\frac{X}{4} + \frac{3a}{8}\right) x\sqrt{X} + \frac{3a^2}{8}\int \frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{3}{2}} = \frac{X^2\sqrt{X}}{5b}$$

$$\int x^2 \partial x X^{\frac{3}{2}} = \frac{xX^2\sqrt{X}}{6b} - \frac{a}{6b}\int \partial x X^{\frac{3}{2}}$$

$$\int x^3 \partial x X^{\frac{3}{2}} = \left(\frac{x^2}{7b} - \frac{2a}{35b^2}\right) X^2\sqrt{X}$$

$$\int x^4 \partial x X^{\frac{3}{2}} = \left(\frac{x^3}{8b} - \frac{ax}{16b^2}\right) X^2\sqrt{X} + \frac{a^2}{16b^2}\int \partial x X^{\frac{3}{2}}$$

$$\int x^5 \partial x X^{\frac{3}{2}} = \left(\frac{x^4}{9b} - \frac{4ax^2}{63b^2} + \frac{8a^2}{315b^3}\right) X^2\sqrt{X}$$

$$\int x^6 \partial x X^{\frac{3}{2}} = \left(\frac{x^5}{10b} - \frac{ax^3}{16b^2} + \frac{a^2x}{32b^3}\right) X^2\sqrt{X} - \frac{a^3}{32b^3}\int \partial x X^{\frac{3}{2}}$$

$$\int x^7 \partial x X^{\frac{3}{2}} = \left(\frac{x^6}{11b} - \frac{2ax^4}{33b^2} + \frac{8a^2x^2}{231b^3} - \frac{16a^3}{1155b^4}\right) X^2\sqrt{X}$$

$$\int x^8 \partial x X^{\frac{3}{2}} = \left(\frac{x^7}{12b} - \frac{7ax^5}{120b^2} + \frac{7a^2x^3}{192b^3} - \frac{7a^3x}{384b^4}\right) X^2\sqrt{X} + \frac{7a^4}{384b^4}\int \partial x X^{\frac{3}{2}}$$

$$\int x^9 \partial x X^{\frac{3}{2}} = \left(\frac{x^8}{13b} - \frac{8ax^6}{143b^2} + \frac{16a^2x^4}{429b^3} - \frac{64a^3x^2}{3003b^4} + \frac{128a^4}{15015b^5}\right) X^2\sqrt{X}$$

$$\int x^{10} \partial x X^{\frac{3}{2}} = \left(\frac{x^9}{14b} - \frac{3ax^7}{56b^2} + \frac{3a^2x^5}{80b^3} - \frac{3a^3x^3}{128b^4} + \frac{3a^4x}{256b^5}\right) X^2\sqrt{X} - \frac{3a^5}{256b^5}\int \partial x X^{\frac{3}{2}}$$

$$\int x^{11} \partial x X^{\frac{3}{2}} = \left(\frac{x^{10}}{15b} - \frac{2ax^8}{39b^2} + \frac{16a^2x^6}{429b^3} - \frac{32a^3x^4}{1287b^4} + \frac{128a^4x^2}{9009b^5} - \frac{256a^5}{45045b^6}\right) X^2\sqrt{X}$$

Taf. **XXXVII.** $\int \frac{\partial x(a+bx^2)^{\frac{3}{2}}}{x^m}$

VZ. $a+bx^2 = X$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x} = \left(\frac{X}{3} + a\right)\sqrt{X} + a^2 \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^2} = -\frac{X^2\sqrt{X}}{ax} + \frac{4b}{a}\int \partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^3} = -\frac{X^2\sqrt{X}}{2ax^2} + \frac{3b}{2a}\int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^4} = \left(-\frac{1}{3ax^3} - \frac{2b}{3a^2x}\right)X^2\sqrt{X} + \frac{8b^2}{3a^2}\int \partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^5} = \left(-\frac{1}{4ax^4} - \frac{b}{8a^2x^2}\right)X^2\sqrt{X} + \frac{3b^2}{8a^2}\int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^6} = -\frac{X^2\sqrt{X}}{5ax^5}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{24a^2x^4} + \frac{b^2}{48a^3x^2}\right)X^2\sqrt{X} - \frac{b^3}{16a^3}\int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^8} = \left(-\frac{1}{7ax^7} + \frac{2b}{35a^2x^5}\right)X^2\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^9} = \left(-\frac{1}{8ax^8} + \frac{b}{16a^2x^6} - \frac{b^2}{64a^3x^4} - \frac{b^3}{128a^4x^2}\right)X^2\sqrt{X} + \frac{3b^4}{128a^4}\int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{4b}{63a^2x^7} - \frac{8b^2}{315a^3x^5}\right)X^2\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^{11}} = \left(-\frac{1}{10ax^{10}} + \frac{b}{16a^2x^8} - \frac{b^2}{32a^3x^6} + \frac{b^3}{128a^4x^4} + \frac{b^4}{256a^5x^2}\right)X^2\sqrt{X} - \frac{3b^5}{256a^5}\int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$\int x^m \partial x (a+bx^2)^{\frac{7}{2}}$$ *Taf. XL.*

VZ. $a + bx^2 = X$

$$\int \partial x X^{\frac{7}{2}} = \left(\frac{X^3}{8} + \frac{7aX^2}{48} + \frac{35a^2X}{192} + \frac{35a^3}{128}\right)x\sqrt{X} + \frac{35a^4}{128}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{7}{2}} = \frac{X^4\sqrt{X}}{9b}$$

$$\int x^2\partial x X^{\frac{7}{2}} = \frac{xX^4\sqrt{X}}{10b} - \frac{a}{10b}\int \partial x X^{\frac{7}{2}}$$

$$\int x^3\partial x X^{\frac{7}{2}} = \left(\frac{x^2}{11b} - \frac{2a}{99b^2}\right)X^4\sqrt{X}$$

$$\int x^4\partial x X^{\frac{7}{2}} = \left(\frac{x^3}{12b} - \frac{ax}{40b^2}\right)X^4\sqrt{X} + \frac{a^2}{40b^2}\int \partial x X^{\frac{7}{2}}$$

$$\int x^5\partial x X^{\frac{7}{2}} = \left(\frac{x^4}{13b} - \frac{4ax^2}{143b^2} + \frac{8a^2}{1287b^3}\right)X^4\sqrt{X}$$

$$\int x^6\partial x X^{\frac{7}{2}} = \left(\frac{x^5}{14b} - \frac{5ax^3}{168b^2} + \frac{a^2x}{112b^3}\right)X^4\sqrt{X} - \frac{a^3}{112b^3}\int \partial x X^{\frac{7}{2}}$$

$$\int x^7\partial x X^{\frac{7}{2}} = \left(\frac{x^6}{15b} - \frac{2ax^4}{65b^2} + \frac{8a^2x^2}{715b^3} - \frac{16a^3}{6435b^4}\right)X^4\sqrt{X}$$

$$\int x^8\partial x X^{\frac{7}{2}} = \left(\frac{x^7}{16b} - \frac{ax^5}{32b^2} + \frac{5a^2x^3}{384b^3} - \frac{a^3x}{256b^4}\right)X^4\sqrt{X} + \frac{7a^4}{256b^4}\int \partial x X^{\frac{7}{2}}$$

$$\int x^9\partial x X^{\frac{7}{2}} = \left(\frac{x^8}{17b} - \frac{8ax^6}{255b^2} + \frac{16a^2x^4}{1105b^3} - \frac{64a^3x^2}{12155b^4} + \frac{128b^5}{109395b^5}\right)X^4\sqrt{X}$$

$$\int x^{10}\partial x X^{\frac{7}{2}} = \left(\frac{x^9}{18b} - \frac{ax^7}{32b^2} + \frac{a^2x^5}{64b^3} - \frac{5a^3x^3}{768b^4} + \frac{a^4x}{512b^5}\right)X^4\sqrt{X} - \frac{7a^5}{512b^5}\int \partial x X^{\frac{7}{2}}$$

$$\int x^{11}\partial x X^{\frac{7}{2}} = \left(\frac{x^{10}}{19b} - \frac{10ax^8}{323b^2} + \frac{16a^2x^6}{969b^3} - \frac{32a^3x^4}{4199b^4} + \frac{128a^4x^2}{46189b^5} - \frac{256a^5}{415701b^6}\right)X^4\sqrt{X}$$

Taf. XXXIX.

$$\int\frac{\partial x(a+bx^2)^{\frac{5}{2}}}{x^m}$$

VZ. $a + bx^2 = X$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x}=\left(\frac{X^2}{5}+\frac{aX}{3}+a^2\right)\sqrt{X}+a^3\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^2}=-\frac{X^3\sqrt{X}}{ax}+\frac{6b}{a}\int\partial x X^{\frac{5}{2}}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^3}=-\frac{X^3\sqrt{X}}{2ax^2}+\frac{5b}{2a}\int\frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^4}=\left(-\frac{1}{3ax^3}-\frac{4b}{3a^2x}\right)X^3\sqrt{X}+\frac{8b^2}{a^2}\int\partial x X^{\frac{5}{2}}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^5}=\left(-\frac{1}{4ax^4}-\frac{3b}{8a^2x^2}\right)X^3\sqrt{X}+\frac{15b^2}{8a^2}\int\frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^6}=\left(-\frac{1}{5ax^5}-\frac{2b}{15a^2x^3}-\frac{8b^2}{15a^3x}\right)X^3\sqrt{X}+\frac{16b^3}{5a^3}\int\partial x X^{\frac{5}{2}}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^7}=\left(-\frac{1}{6ax^6}-\frac{b}{24a^2x^4}-\frac{b^2}{16a^3x^2}\right)X^3\sqrt{X}+\frac{5b^3}{16a^3}\int\frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^8}=-\frac{X^3\sqrt{X}}{7ax^7}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^9}=\left(-\frac{1}{8ax^8}+\frac{b}{48a^2x^6}+\frac{b^2}{192a^3x^4}+\frac{b^3}{128a^4x^2}\right)X^3\sqrt{X}$$
$$-\frac{5b^4}{128a^4}\int\frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^{10}}=\left(-\frac{1}{9ax^9}+\frac{2b}{63a^2x^7}\right)X^3\sqrt{X}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^{11}}=\left(-\frac{1}{10ax^{10}}+\frac{3b}{80a^2x^8}-\frac{b^2}{160a^3x^6}-\frac{b^3}{640a^4x^4}-\frac{3b^4}{1280a^5x^2}\right)X^3\sqrt{X}$$
$$+\frac{3b^5}{256a^5}\int\frac{\partial x X^{\frac{5}{2}}}{x}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^{12}}=\left(-\frac{1}{11ax^{11}}+\frac{4b}{99a^2x^9}-\frac{8b^2}{693a^3x^7}\right)X^3\sqrt{X}$$

$$\int x^m \partial x (a+bx^2)^{\frac{7}{2}}$$ *Taf. XL.*

VZ. $a + bx^2 = X$

$$\int \partial x X^{\frac{7}{2}} = \left(\frac{X^3}{8} + \frac{7aX^2}{48} + \frac{35a^2X}{192} + \frac{35a^3}{128}\right) x\sqrt{X} + \frac{35a^4}{128}\int \frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{7}{2}} = \frac{X^4\sqrt{X}}{9b}$$

$$\int x^2 \partial x X^{\frac{7}{2}} = \frac{xX^4\sqrt{X}}{10b} - \frac{a}{10b}\int \partial x X^{\frac{7}{2}}$$

$$\int x^3 \partial x X^{\frac{7}{2}} = \left(\frac{x^2}{11b} - \frac{2a}{99b^2}\right) X^4\sqrt{X}$$

$$\int x^4 \partial x X^{\frac{7}{2}} = \left(\frac{x^3}{12b} - \frac{ax}{40b^2}\right) X^4\sqrt{X} + \frac{a^2}{40b^2}\int \partial x X^{\frac{7}{2}}$$

$$\int x^5 \partial x X^{\frac{7}{2}} = \left(\frac{x^4}{13b} - \frac{4ax^2}{143b^2} + \frac{8a^2}{1287b^3}\right) X^4\sqrt{X}$$

$$\int x^6 \partial x X^{\frac{7}{2}} = \left(\frac{x^5}{14b} - \frac{5ax^3}{168b^2} + \frac{a^2x}{112b^3}\right) X^4\sqrt{X} - \frac{a^3}{112b^3}\int \partial x X^{\frac{7}{2}}$$

$$\int x^7 \partial x X^{\frac{7}{2}} = \left(\frac{x^6}{15b} - \frac{2ax^4}{65b^2} + \frac{8a^2x^2}{715b^3} - \frac{16a^3}{6435b^4}\right) X^4\sqrt{X}$$

$$\int x^8 \partial x X^{\frac{7}{2}} = \left(\frac{x^7}{16b} - \frac{ax^5}{32b^2} + \frac{5a^2x^3}{384b^3} - \frac{a^3x}{256b^4}\right) X^4\sqrt{X} + \frac{7a^4}{256b^4}\int \partial x X^{\frac{7}{2}}$$

$$\int x^9 \partial x X^{\frac{7}{2}} = \left(\frac{x^8}{17b} - \frac{8ax^6}{255b^2} + \frac{16a^2x^4}{1105b^3} - \frac{64a^3x^2}{12155b^4} + \frac{128b^5}{109395b^5}\right) X^4\sqrt{X}$$

$$\int x^{10} \partial x X^{\frac{7}{2}} = \left(\frac{x^9}{18b} - \frac{ax^7}{32b^2} + \frac{a^2x^5}{64b^3} - \frac{5a^3x^3}{768b^4} + \frac{a^4x}{512b^5}\right) X^4\sqrt{X} - \frac{7a^5}{512b^5}\int \partial x X^{\frac{7}{2}}$$

$$\int x^{11} \partial x X^{\frac{7}{2}} = \left(\frac{x^{10}}{19b} - \frac{10ax^8}{323b^2} + \frac{16a^2x^6}{969b^3} - \frac{32a^3x^4}{4199b^4} + \frac{128a^4x^2}{46189b^5} - \frac{256a^5}{415701b^6}\right) X^4\sqrt{X}$$

Taf. XLIII. $$\int \frac{\partial x}{(ax+bx^2)^{\frac{n}{2}}}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{1}{2}}} = \int \frac{\partial x}{\sqrt{X}}$$ (Man s. die folgende Seite.)

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = -\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{5}{2}}} = \left(-\frac{1}{3X} + \frac{8b}{3a^2}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{7}{2}}} = \left(-\frac{1}{5X^2} + \frac{4^2b}{15a^2X} - \frac{2.4^3b^2}{15a^4}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{9}{2}}} = \left(-\frac{1}{7X^3} + \frac{6.4b}{35a^2X^2} - \frac{2.4^3b^2}{35a^4X} + \frac{4^5b^3}{35a^6}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{11}{2}}} = \left(-\frac{1}{9X^4} + \frac{2.4^2b}{63a^2X^3} - \frac{4^4b^2}{105a^4X^2} + \frac{4^6b^3}{315a^6X} - \frac{2.4^7b^4}{315a^8}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{13}{2}}} = \left(-\frac{1}{11X^5} + \frac{10.4b}{99a^2X^4} - \frac{5.4^4b^2}{693a^4X^3} + \frac{2.4^5b^3}{231a^6X^2} - \frac{2.4^7b^4}{693a^8X} + \frac{4^9b^5}{693a^{10}}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{15}{2}}} = \left(-\frac{1}{13X^6} + \frac{3.4^2b}{143a^2X^5} - \frac{10.4^3b^2}{429a^4X^4} + \frac{5.4^6b^3}{3003a^6X^3} - \frac{2.4^7b^4}{1001a^8X^2} + \frac{2.4^9b^5}{3003a^{10}X} - \frac{4^{11}b^6}{3003a^{12}}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{17}{2}}} = \left(-\frac{1}{15X^7} + \frac{14b}{195a^2X^6} - \frac{14.4^3b^2}{715a^4X^5} + \frac{7.4^5b^3}{1287a^6X^4} - \frac{2.4^7b^4}{1287a^8X^3} + \frac{4^9b^5}{2145a^{10}X^2} - \frac{4^{11}b^6}{6435a^{12}X} + \frac{2.4^{12}b^7}{6435a^{14}}\right)\frac{2(2bx+a)}{a^2\sqrt{X}}$$

$$\int \frac{\partial x}{X^{\frac{19}{2}}} = -\frac{2(2bx+a)}{17a^2X^{\frac{17}{2}}} - \frac{4^3b}{17a^2}\int \frac{\partial x}{X^{\frac{17}{2}}}$$

$$\int x^m \partial x (a+bx^2)^{\frac{9}{2}};\quad \int \frac{\partial x (a+bx^2)^{\frac{9}{2}}}{x^m} \qquad \textit{Taf. XLII.}$$

VZ. $a + bx^2 = X$

$$\int \partial x X^{\frac{9}{2}} = \left(\frac{X^4}{10}+\frac{9aX^3}{80}+\frac{21a^2X^2}{160}+\frac{21a^3X}{128}+\frac{63a^4}{256}\right)x\sqrt{X}+\frac{63a^5}{256}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{9}{2}} = \frac{X^5\sqrt{X}}{11b}$$

$$\int x^2\partial x X^{\frac{9}{2}} = \frac{xX^5\sqrt{X}}{12b}-\frac{a}{12b}\int\partial x X^{\frac{9}{2}}$$

$$\int x^3\partial x X^{\frac{9}{2}} = \left(\frac{x^2}{13b}-\frac{2a}{143b^2}\right)X^5\sqrt{X}$$

$$\int x^4\partial x X^{\frac{9}{2}} = \left(\frac{x^3}{14b}-\frac{ax}{56b^2}\right)X^5\sqrt{X}+\frac{a^2}{56b^2}\int\partial x X^{\frac{9}{2}}$$

$$\int x^5\partial x X^{\frac{9}{2}} = \left(\frac{x^4}{15b}-\frac{4ax^2}{195b^2}+\frac{8a^2}{2145b^3}\right)X^5\sqrt{X}$$

$$\int x^6\partial x X^{\frac{9}{2}} = \left(\frac{x^5}{16b}-\frac{5ax^3}{224b^2}+\frac{5a^2x}{896b^3}\right)X^5\sqrt{X}-\frac{5a^3}{896b^3}\int\partial x X^{\frac{9}{2}}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x} = \left(\frac{X^4}{9}+\frac{aX^3}{7}+\frac{a^2X^2}{5}+\frac{a^3X}{3}+a^4\right)\sqrt{X}+a^5\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^2} = -\frac{X^5\sqrt{X}}{ax}+\frac{10b}{a}\int\partial x X^{\frac{9}{2}}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^3} = -\frac{X^5\sqrt{X}}{2ax^2}+\frac{9b}{2a}\int\frac{\partial x X^{\frac{9}{2}}}{x}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^4} = \left(-\frac{1}{3ax^3}-\frac{8b}{3a^2x}\right)X^5\sqrt{X}+\frac{80b^2}{3a^2}\int\partial x X^{\frac{9}{2}}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^5} = \left(-\frac{1}{4ax^4}-\frac{7b}{8a^2x^2}\right)X^5\sqrt{X}+\frac{63b^2}{8a^2}\int\frac{\partial x X^{\frac{9}{2}}}{x}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^6} = \left(-\frac{1}{5ax^5}-\frac{2b}{5a^2x^3}-\frac{16b^2}{5a^3x}\right)X^5\sqrt{X}+\frac{32b^3}{a^3}\int\partial x X^{\frac{9}{2}}$$

Taf. XLIII.

$$\int \frac{\partial x}{(ax+bx^2)^{\frac{n}{2}}}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{1}{2}}} = \int \frac{\partial x}{VX}$$ (Man s. die folgende Seite.)

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = -\frac{2(2bx+a)}{a^2VX}$$

$$\int \frac{\partial x}{X^{\frac{5}{2}}} = \left(-\frac{1}{3X} + \frac{8b}{3a^2}\right)\frac{2(2bx+a)}{a^2VX}$$

$$\int \frac{\partial x}{X^{\frac{7}{2}}} = \left(-\frac{1}{5X^2} + \frac{4^2b}{15a^2X} - \frac{2.4^3b^2}{15a^4}\right)\frac{2(2bx+a)}{a^2VX}$$

$$\int \frac{\partial x}{X^{\frac{9}{2}}} = \left(-\frac{1}{7X^3} + \frac{6.4b}{35a^2X^2} - \frac{2.4^3b^2}{35a^4X} + \frac{4^5b^3}{35a^6}\right)\frac{2(2bx+a)}{a^2VX}$$

$$\int \frac{\partial x}{X^{\frac{11}{2}}} = \left(-\frac{1}{9X^4} + \frac{2.4^2b}{63a^2X^3} - \frac{4^4b^2}{105a^4X^2} + \frac{4^6b^3}{315a^6X} - \frac{2.4^7b^4}{315a^8}\right)\frac{2(2bx+a)}{a^2VX}$$

$$\int \frac{\partial x}{X^{\frac{13}{2}}} = \left(-\frac{1}{11X^5} + \frac{10.4b}{99a^2X^4} - \frac{5.4^4b^2}{693a^4X^3} + \frac{2.4^5b^3}{231a^6X^2} - \frac{2.4^7b^4}{693a^8X} + \frac{4^9b^5}{693a^{10}}\right)\frac{2(2bx+a)}{a^2VX}$$

$$\int \frac{\partial x}{X^{\frac{15}{2}}} = \left(-\frac{1}{13X^6} + \frac{3.4^2b}{143a^2X^5} - \frac{10.4^3b^2}{429a^4X^4} + \frac{5.4^6b^3}{3003a^6X^3} - \frac{2.4^7b^4}{1001a^8X^2} + \frac{2.4^9b^5}{3003a^{10}X} - \frac{4^{11}b^6}{3003a^{12}}\right)\frac{2(2bx+a)}{a^2VX}$$

$$\int \frac{\partial x}{X^{\frac{17}{2}}} = \left(-\frac{1}{15X^7} + \frac{14b}{195a^2X^6} - \frac{14.4^3b^2}{715a^4X^5} + \frac{7.4^5b^3}{1287a^6X^4} - \frac{2.4^7b^4}{1287a^8X^3} + \frac{4^9b^5}{2145a^{10}X^2} - \frac{4^{11}b^6}{6435a^{12}X} + \frac{2.4^{12}b^7}{6435a^{14}}\right)\frac{2(2bx+a)}{a^2VX}$$

$$\int \frac{\partial x}{X^{\frac{19}{2}}} = -\frac{2(2bx+a)}{17a^2X^{\frac{17}{2}}} - \frac{4^3b}{17a^2}\int \frac{\partial x}{X^{\frac{17}{2}}}$$

Anmerkung zur vorhergehenden Tafel.

Es ist im Allgemeinen

$$\int \frac{\partial x}{\sqrt{(ax+bx^2)}} = \frac{1}{\sqrt{b}} \log \frac{\sqrt{(ax+bx^2)} + x\sqrt{b}}{\sqrt{(ax+bx^2)} - x\sqrt{b}}$$

oder $$\int \frac{\partial x}{\sqrt{(ax+bx^2)}} = \frac{2}{\sqrt{-b}} \text{ Arc Tang} \frac{x\sqrt{-b}}{\sqrt{(ax+bx^2)}},$$

woraus sich ergiebt, daſs in jedem Falle der erste Ausdruck reell wird, wenn b positiv, der zweite, wenn b negativ ist. Hieraus erhält man

$$\text{I.} \quad \int \frac{\partial x}{\sqrt{(ax+bx^2)}} = \pm \frac{1}{\sqrt{b}} \log \frac{\sqrt{(ax+bx^2)} \pm x\sqrt{b}}{\sqrt{(ax+bx^2)} \mp x\sqrt{b}}$$

$$= \pm \frac{1}{\sqrt{b}} \log \frac{\sqrt{(a+bx)} \pm \sqrt{bx}}{\sqrt{(a+bx)} \mp \sqrt{bx}}$$

$$= \pm \frac{1}{\sqrt{b}} \log \frac{2bx + a \pm 2\sqrt{b} \cdot \sqrt{(ax+bx^2)}}{a}$$

$$= \pm \frac{2}{\sqrt{b}} \log \frac{\sqrt{(a+bx)} \pm \sqrt{bx}}{\sqrt{a}}.$$

Die oberen Zeichen gehören hier zusammen, und eben so die unteren.

$$\text{II.} \quad \int \frac{\partial x}{\sqrt{(ax-bx^2)}} = \frac{2}{\sqrt{b}} \text{Arc Tang} \frac{x\sqrt{b}}{\sqrt{(ax-bx^2)}} = \frac{2}{\sqrt{b}} \text{ArcTang} \sqrt{\frac{bx}{a-bx}}$$

$$= \frac{2}{\sqrt{b}} \text{Arc Cot} \sqrt{\frac{a-bx}{bx}} = \frac{2}{\sqrt{b}} \text{ Arc Sec} \sqrt{\frac{a}{a-bx}}$$

$$= \frac{2}{\sqrt{b}} \text{Arc Cosec} \sqrt{\frac{a}{bx}} = \frac{2}{\sqrt{b}} \text{ Arc Sin} \sqrt{\frac{bx}{a}}$$

$$= \frac{2}{\sqrt{b}} \text{ Arc Cos} \sqrt{\frac{a-bx}{a}} = \frac{1}{\sqrt{b}} \text{ Arc Cos} \frac{a-2bx}{a}$$

$$= \frac{1}{\sqrt{b}} \text{Arc Sin vers} \frac{2bx}{a}.$$

Sämmtliche Integrale auf dieser Seite verschwinden für $x = 0$.

Insbesondere ist

$$\int \frac{\partial x}{\sqrt{(x^2+x)}} = \pm \log [2x + 1 \pm 2\sqrt{(x^2+x)}]$$

$$\int \frac{\partial x}{\sqrt{(x^2-x)}} = \pm \log [1 - 2x \mp 2\sqrt{(x^2-x)}].$$

Taf. XLIV. $\int \frac{x^m \partial x}{\sqrt{(ax+bx^2)}}$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{\sqrt{X}} = \int \frac{\partial x}{\sqrt{X}}$$ (Man s. die vorhergehende Seite.)

$$\int \frac{x\partial x}{\sqrt{X}} = \frac{\sqrt{X}}{b} - \frac{a}{2b}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^2\partial x}{\sqrt{X}} = \left(\frac{x}{2b} - \frac{3a}{4b^2}\right)\sqrt{X} + \frac{3a^2}{8b^2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^3\partial x}{\sqrt{X}} = \left(\frac{x^2}{3b} - \frac{5ax}{12b^2} + \frac{5a^2}{8b^3}\right)\sqrt{X} - \frac{5a^3}{16b^3}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^4\partial x}{\sqrt{X}} = \left(\frac{x^3}{4b} - \frac{7ax^2}{24b^2} + \frac{35a^2x}{96b^3} - \frac{35a^3}{64b^4}\right)\sqrt{X} + \frac{35a^4}{128b^4}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^5\partial x}{\sqrt{X}} = \left(\frac{x^4}{5b} - \frac{9ax^3}{40b^2} + \frac{21a^2x^2}{80b^3} - \frac{21a^3x}{64b^4} + \frac{63a^4}{128b^5}\right)\sqrt{X} - \frac{63a^5}{256b^5}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^6\partial x}{\sqrt{X}} = \left(\frac{x^5}{6b} - \frac{11ax^4}{60b^2} + \frac{33a^2x^3}{160b^3} - \frac{77a^3x^2}{320b^4} + \frac{77a^4x}{256b^5} - \frac{231a^5}{512b^6}\right)\sqrt{X}$$
$$+ \frac{231a^6}{1024b^6}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^7\partial x}{\sqrt{X}} = \left(\frac{x^6}{7b} - \frac{13ax^5}{84b^2} + \frac{143a^2x^4}{840b^3} - \frac{429a^3x^3}{2240b^4} + \frac{143a^4x^2}{640b^5}\right.$$
$$\left. - \frac{143a^5x}{512b^6} + \frac{429a^6}{1024b^7}\right)\sqrt{X} - \frac{429a^7}{2048b^7}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^8\partial x}{\sqrt{X}} = \left(\frac{x^7}{8b} - \frac{15ax^6}{112b^2} + \frac{65a^2x^5}{448b^3} - \frac{143a^3x^4}{896b^4} + \frac{1287a^4x^3}{7168b^5}\right.$$
$$\left. - \frac{429a^5x^2}{2048b^6} + \frac{2145a^6x}{8192b^7} - \frac{6435a^7}{16384b^8}\right)\sqrt{X} + \frac{6435a^8}{32768b^8}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^9\partial x}{\sqrt{X}} = \frac{x^8\sqrt{X}}{9b} - \frac{17a}{18b}\int \frac{x^8\partial x}{\sqrt{X}}$$

$$\int \frac{x^{10}\partial x}{\sqrt{X}} = \left(\frac{x^9}{10b} - \frac{19ax^8}{180b^2}\right)\sqrt{X} + \frac{323a^2}{360b^2}\int \frac{x^8\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x}{x^m(ax+bx^2)^{\frac{3}{2}}}$$ *Taf. XLVII.*

VZ. $ax+bx^2 = X$

$$\int \frac{\partial x}{xX^{\frac{3}{2}}} = -\frac{2}{3ax\sqrt{X}} - \frac{4b}{3a}\int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^2} + \frac{2b}{5a^2x}\right)\frac{2}{\sqrt{X}} + \frac{8b^2}{5a^2}\int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^3} + \frac{8b}{35a^2x^2} - \frac{16b^2}{35a^3x}\right)\frac{2}{\sqrt{X}} - \frac{64b^3}{35a^3}\int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{9ax^4} + \frac{10b}{63a^2x^3} - \frac{16b^2}{63a^3x^2} + \frac{32b^3}{63a^4x}\right)\frac{2}{\sqrt{X}} + \frac{128b^4}{63a^4}\int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{11ax^5} + \frac{4b}{33a^2x^4} - \frac{40b^2}{231a^3x^3} + \frac{64b^3}{231a^4x^2} - \frac{128b^4}{231a^5x}\right)\frac{2}{\sqrt{X}}$$
$$- \frac{512b^5}{231a^5}\int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{13ax^6} + \frac{14b}{143a^2x^5} - \frac{56b^2}{429a^3x^4} + \frac{80b^3}{429a^4x^3} - \frac{128b^4}{429a^5x^2}\right.$$
$$\left. + \frac{256b^5}{429a^6x}\right)\frac{2}{\sqrt{X}} + \frac{1024b^6}{429a^6}\int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{3}{2}}} = \left(-\frac{1}{15ax^7} + \frac{16b}{195a^2x^6} - \frac{224b^2}{2145a^3x^5} + \frac{896b^3}{6435a^4x^4}\right.$$
$$\left. - \frac{256b^4}{1287a^5x^3} + \frac{2048b^5}{6435a^6x^2} - \frac{4096b^6}{6435a^7x}\right)\frac{2}{\sqrt{X}} - \frac{16384b^7}{6435a^7}\int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{3}{2}}} = -\frac{2}{17ax^8\sqrt{X}} - \frac{18b}{17a}\int \frac{\partial x}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{19ax^9} + \frac{20b}{323a^2x^8}\right)\frac{2}{\sqrt{X}} + \frac{360b^2}{323a^2}\int \frac{\partial x}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^{10}X^{\frac{3}{2}}} = \left(-\frac{1}{21a^2x^{10}} + \frac{22b}{399a^2x^9} - \frac{440b^2}{6783a^3x^8}\right)\frac{2}{\sqrt{X}} - \frac{2640b^3}{2261a^3}\int \frac{\partial x}{x^7X^{\frac{3}{2}}}$$

Taf. XLVI.

$$\int \frac{x^m \partial x}{(ax+bx^2)^{\frac{3}{2}}}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = -\frac{2(2bx+a)}{a^2 \sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{3}{2}}} = \frac{2x}{a\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{3}{2}}} = -\frac{2x}{b\sqrt{X}} + \frac{1}{b}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^2}{b} + \frac{3ax}{b^2}\right)\frac{1}{\sqrt{X}} - \frac{3a}{2b^2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^3}{2b} - \frac{5ax^2}{4b^2} - \frac{15a^2x}{4b^3}\right)\frac{1}{\sqrt{X}} + \frac{15a^2}{8b^3}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^4}{3b} - \frac{7ax^3}{12b^2} + \frac{35a^2x^2}{24b^3} + \frac{35a^3x}{8b^4}\right)\frac{1}{\sqrt{X}} - \frac{35a^3}{16b^4}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^5}{4b} - \frac{3ax^4}{8b^2} + \frac{21a^2x^3}{32b^3} - \frac{105a^3x^2}{64b^4} - \frac{315a^4x}{64b^5}\right)\frac{1}{\sqrt{X}} + \frac{315a^4}{128b^5}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^6}{5b} - \frac{11ax^5}{40b^2} + \frac{33a^2x^4}{80b^3} - \frac{231a^3x^2}{320b^4} + \frac{231a^4x^2}{128b^5} + \frac{693a^5x}{128b^6}\right)\frac{1}{\sqrt{X}} - \frac{693a^5}{256b^6}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^8\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^7}{6b} - \frac{13ax^6}{60b^2} + \frac{143a^2x^5}{480b^3} - \frac{143a^3x^4}{320b^4} + \frac{1001a^4x^2}{1280b^5} - \frac{1001a^5x^2}{512b^6} - \frac{3003a^6x}{512b^7}\right)\frac{1}{\sqrt{X}} + \frac{3003a^6}{1024b^7}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^9\partial x}{X^{\frac{3}{2}}} = \frac{x^8}{7b\sqrt{X}} - \frac{15a}{14b}\int \frac{x^8\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{x^{10}\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^9}{8b} - \frac{17a}{112b^2}\right)\frac{1}{\sqrt{X}} - \frac{255a^2}{224b^2}\int \frac{x^8\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^m(ax+bx^2)^{\frac{3}{2}}}$$

Taf. XLVII.

VZ. $ax+bx^2=X$

$$\int \frac{\partial x}{xX^{\frac{3}{2}}} = -\frac{2}{3ax\sqrt{X}} - \frac{4b}{3a}\int\frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^2}+\frac{2b}{5a^2x}\right)\frac{2}{\sqrt{X}} + \frac{8b^2}{5a^2}\int\frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{7ax^3}+\frac{8b}{35a^2x^2}-\frac{16b^2}{35a^3x}\right)\frac{2}{\sqrt{X}} - \frac{64b^3}{35a^3}\int\frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{3}{2}}} = \left(-\frac{1}{9ax^4}+\frac{10b}{63a^2x^3}-\frac{16b^2}{63a^3x^2}+\frac{32b^3}{63a^4x}\right)\frac{2}{\sqrt{X}} + \frac{128b^4}{63a^4}\int\frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{3}{2}}} = \left(-\frac{1}{11ax^5}+\frac{4b}{33a^2x^4}-\frac{40b^2}{231a^3x^3}+\frac{64b^3}{231a^4x^2}-\frac{128b^4}{231a^5x}\right)\frac{2}{\sqrt{X}}$$
$$-\frac{512b^5}{231a^5}\int\frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{13ax^6}+\frac{14b}{143a^2x^5}-\frac{56b^2}{429a^3x^4}+\frac{80b^3}{429a^4x^3}-\frac{128b^4}{429a^5x^2}\right.$$
$$\left.+\frac{256b^5}{429a^6x}\right)\frac{2}{\sqrt{X}} + \frac{1024b^6}{429a^6}\int\frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{3}{2}}} = \left(-\frac{1}{15ax^7}+\frac{16b}{195a^2x^6}-\frac{224b^2}{2145a^3x^5}+\frac{896b^3}{6435a^4x^4}\right.$$
$$\left.-\frac{256b^4}{1287a^5x^3}+\frac{2048b^5}{6435a^6x^2}-\frac{4096b^6}{6435a^7x}\right)\frac{2}{\sqrt{X}} - \frac{16384b^7}{6435a^7}\int\frac{\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{3}{2}}} = -\frac{2}{17ax^8\sqrt{X}} - \frac{18b}{17a}\int\frac{\partial x}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{19ax^9}+\frac{20b}{323a^2x^8}\right)\frac{2}{\sqrt{X}} + \frac{360b^2}{323a^2}\int\frac{\partial x}{x^7X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^{10}X^{\frac{3}{2}}} = \left(-\frac{1}{21a^2x^{10}}+\frac{22b}{399a^2x^9}-\frac{440b^2}{6783a^3x^8}\right)\frac{2}{\sqrt{X}} - \frac{2640b^3}{2261a^3}\int\frac{\partial x}{x^7X^{\frac{3}{2}}}$$

Taf. XLVIII. $$\int \frac{x^m \partial x}{(ax+bx^2)^{\frac{5}{2}}}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{5}{2}}} = \left(-\frac{2}{3X} + \frac{16b}{3a^2}\right)\frac{2bx+a}{a^2\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{5}{2}}} = \frac{2x^1}{3aX\sqrt{X}} - \frac{8(2bx+a)}{3a^3\sqrt{X}} = \left(\frac{1}{a+bx} - \frac{4(2bx+a)}{a^2}\right)\frac{2}{3a\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{5}{2}}} = \left(\frac{2x^2}{3aX} + \frac{4x}{3a^2}\right)\frac{1}{\sqrt{X}} = \left(\frac{x}{a+bx} + \frac{2x}{a}\right)\frac{2}{3a\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{5}{2}}} = \frac{2x^3}{3aX\sqrt{X}} = \frac{2x^2}{3a(a+bx)\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{5}{2}}} = \left(-\frac{8x^3}{3b} - \frac{2ax^2}{b^2}\right)\frac{1}{X\sqrt{X}} + \frac{1}{b^2}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^4}{b} + \frac{20ax^3}{3b^2} + \frac{5a^2x^2}{b^3}\right)\frac{1}{X\sqrt{X}} - \frac{5a}{2b^3}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^5}{2b} - \frac{7ax^4}{4b^2} - \frac{35a^2x^3}{3b^3} - \frac{35a^3x^2}{4b^4}\right)\frac{1}{X\sqrt{X}} + \frac{35a^2}{8b^4}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^6}{3b} - \frac{3ax^5}{4b^2} + \frac{21a^2x^4}{8b^3} + \frac{35a^3x^3}{2b^4} + \frac{105a^4x^2}{8b^5}\right)\frac{1}{X\sqrt{X}} - \frac{105a^3}{16b^5}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^8\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^7}{4b} - \frac{11ax^6}{24b^2} + \frac{33a^2x^5}{32b^3} - \frac{231a^3x^4}{64b^4} - \frac{385a^4x^3}{16b^5} - \frac{1155a^5x^2}{64b^6}\right)\frac{1}{X\sqrt{X}} + \frac{1155a^4}{128b^6}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^9\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^8}{5b} - \frac{13ax^7}{40b^2} + \frac{143a^2x^6}{240b^3} - \frac{429a^3x^5}{320b^4} + \frac{3003a^4x^4}{640b^5} + \frac{1001a^5x^3}{32b^6} + \frac{3003a^6x^2}{128b^7}\right)\frac{1}{X\sqrt{X}} - \frac{3003a^5}{256b^7}\int\frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^{10}\partial x}{X^{\frac{5}{2}}} = \frac{x^9}{6bX\sqrt{X}} - \frac{5a}{4b}\int\frac{x^9\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^m(ax+bx^2)^{\frac{5}{2}}}$$ *Taf. XLIX.*

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{xX^{\frac{5}{2}}} = -\frac{2}{5axX\sqrt{X}} - \frac{8b}{5a}\int \frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^2X^{\frac{5}{2}}} = \left(-\frac{1}{7ax^2} + \frac{2b}{7a^2x}\right)\frac{2}{X\sqrt{X}} + \frac{16b^2}{7a^2}\int \frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{5}{2}}} = \left(-\frac{1}{9ax^3} + \frac{4b}{21a^2x^2} - \frac{8b^2}{21a^3x}\right)\frac{2}{X\sqrt{X}} - \frac{64b^3}{21a^3}\int \frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{5}{2}}} = \left(-\frac{1}{11ax^4} + \frac{14b}{99a^2x^3} - \frac{8b^2}{33a^3x^2} + \frac{16b^3}{33a^4x}\right)\frac{2}{X\sqrt{X}} - \frac{128b^4}{33a^4}\int \frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{5}{2}}} = \left(-\frac{1}{13ax^5} + \frac{16b}{143a^2x^4} - \frac{224b^2}{1287a^3x^3} + \frac{128b^3}{429a^4x^2} - \frac{256b^4}{429a^5x}\right)\frac{2}{X\sqrt{X}}$$
$$- \frac{2048b^5}{429a^5}\int \frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{5}{2}}} = \left(-\frac{1}{15ax^6} + \frac{6b}{65a^2x^5} - \frac{96b^2}{715a^3x^4} + \frac{448b^3}{2145a^4x^3} - \frac{256b^4}{715a^5x^2}\right.$$
$$\left. + \frac{512b^5}{715a^6x}\right)\frac{2}{X\sqrt{X}} + \frac{4096b^6}{715a^6}\int \frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{5}{2}}} = \left(-\frac{1}{17ax^7} + \frac{4b}{51a^2x^6} - \frac{24b^2}{221a^3x^5} + \frac{384b^3}{2431a^4x^4} - \frac{1792b^4}{7293a^5x^3}\right.$$
$$\left. + \frac{1024b^5}{2431a^6x^2} - \frac{2048b^6}{2431a^7x}\right)\frac{2}{X\sqrt{X}} - \frac{16384b^7}{2431a^7}\int \frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{5}{2}}} = -\frac{2}{19ax^8X\sqrt{X}} - \frac{22b}{19a}\int \frac{\partial x}{x^7X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{5}{2}}} = \left(-\frac{1}{21ax^9} + \frac{8b}{133a^2x^8}\right)\frac{2}{X\sqrt{X}} + \frac{176b^2}{133a^2}\int \frac{\partial x}{x^7X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^{10}X^{\frac{5}{2}}} = \left(-\frac{1}{23ax^{10}} + \frac{26b}{483a^2x^9} - \frac{208b^2}{3059a^3x^8}\right)\frac{2}{X\sqrt{X}} - \frac{4576b^3}{3059a^3}\int \frac{\partial x}{x^7X^{\frac{5}{2}}}$$

Taf. L. $$\int \frac{x^m \partial x}{(ax+bx^2)^{\frac{7}{2}}}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{X^{\frac{7}{2}}} = \left(-\frac{1}{X^2} + \frac{16b}{3a^2X} - \frac{128b^2}{3a^4}\right)\frac{2(2bx+a)}{5a^2\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{7}{2}}} = \frac{2x}{5aX^2\sqrt{X}} - \left(\frac{1}{X} - \frac{8b}{a^2}\right)\frac{16(2bx+a)}{15a^3\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^2}{X^2} + \frac{2x}{aX}\right)\frac{2}{5a\sqrt{X}} - \frac{16(2bx+a)}{5a^4\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^3}{X^2} + \frac{4x^2}{3aX} + \frac{8x}{3a^2}\right)\frac{2}{5a\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^4}{X^2} + \frac{2x^3}{3aX}\right)\frac{2}{5a\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X^{\frac{7}{2}}} = \frac{2x^5}{5aX^2\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X^{\frac{7}{2}}} = \frac{2x^6}{5aX^2\sqrt{X}} - \frac{2}{5a}\int \frac{x^5\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^7\partial x}{X^{\frac{7}{2}}} = -\frac{2x^6}{5bX^2\sqrt{X}} + \frac{7}{5b}\int \frac{x^5\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^8\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^7}{2b} + \frac{9ax^6}{10b^2}\right)\frac{1}{X^2\sqrt{X}} - \frac{63a}{20b^2}\int \frac{x^5\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^9\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^8}{3b} - \frac{11ax^7}{12b^2} - \frac{33a^2x^6}{20b^3}\right)\frac{1}{X^2\sqrt{X}} + \frac{231a^2}{40b^3}\int \frac{x^5\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^{10}\partial x}{X^{\frac{7}{2}}} = \left(\frac{x^9}{4b} - \frac{13ax^8}{24b^2} + \frac{143a^2x^7}{96b^3} + \frac{429a^3x^6}{160b^4}\right)\frac{1}{X^2\sqrt{X}} - \frac{3003a^3}{320b^4}\int \frac{x^5\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^m (ax+bx^2)^{\frac{7}{2}}}$$

Taf. LI.

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{x X^{\frac{7}{2}}} = -\frac{2}{7axX^2\sqrt{X}} - \frac{12b}{7a}\int \frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^2 X^{\frac{7}{2}}} = \left(-\frac{1}{9ax^2} + \frac{2b}{9a^2x}\right)\frac{2}{X^2\sqrt{X}} + \frac{8b^2}{3a^2}\int \frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^3 X^{\frac{7}{2}}} = \left(-\frac{1}{11ax^3} + \frac{16b}{99a^2x^2} - \frac{32b^2}{99a^3x}\right)\frac{2}{X^2\sqrt{X}} - \frac{128b^3}{33a^3}\int \frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^4 X^{\frac{7}{2}}} = \left(-\frac{1}{13ax^4} + \frac{18b}{143a^2x^3} - \frac{32b^2}{143a^3x^2} + \frac{64b^3}{143a^4x}\right)\frac{2}{X^2\sqrt{X}} + \frac{768b^4}{143a^4}\int \frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^5 X^{\frac{7}{2}}} = \left(-\frac{1}{15ax^5} + \frac{4b}{39a^2x^4} - \frac{24b^2}{143a^3x^3} + \frac{128b^3}{429a^4x^2} - \frac{256b^4}{429a^5x}\right)\frac{2}{X^2\sqrt{X}} - \frac{1024b^5}{143a^5}\int \frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^6 X^{\frac{7}{2}}} = \left(-\frac{1}{17ax^6} + \frac{22b}{255a^2x^5} - \frac{88b^2}{663a^3x^4} + \frac{48b^3}{221a^4x^3} - \frac{256b^4}{663a^5x^2} + \frac{512b^5}{663a^6x}\right)\frac{2}{X^2\sqrt{X}} + \frac{2048b^6}{221a^6}\int \frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^7 X^{\frac{7}{2}}} = \left(-\frac{1}{19ax^7} + \frac{24b}{323a^2x^6} - \frac{176b^2}{1615a^3x^5} + \frac{704b^3}{4199a^4x^4} - \frac{1152b^4}{4199a^5x^3} + \frac{2048b^5}{4199a^6x^2} - \frac{4096b^6}{4199a^7x}\right)\frac{2}{X^2\sqrt{X}} - \frac{49152b^7}{4199a^7}\int \frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^8 X^{\frac{7}{2}}} = -\frac{2}{21ax^8X^2\sqrt{X}} - \frac{26b}{21a}\int \frac{\partial x}{x^7 X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^9 X^{\frac{7}{2}}} = \left(-\frac{1}{23ax^9} + \frac{4b}{69a^2x^8}\right)\frac{2}{X^2\sqrt{X}} + \frac{104b^2}{69a^2}\int \frac{\partial x}{x^7 X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^{10} X^{\frac{7}{2}}} = \left(-\frac{1}{25ax^{10}} + \frac{6b}{115a^2x^9} - \frac{8b^2}{115a^3x^8}\right)\frac{2}{X^2\sqrt{X}} - \frac{208b^3}{115a^3}\int \frac{\partial x}{x^7 X^{\frac{7}{2}}}$$

Taf. LII. $$\int\frac{x^m\partial x}{(ax+bx^2)^{\frac{9}{2}}},\ \int\frac{\partial x}{x^m(ax+bx^2)^{\frac{9}{2}}}$$

VZ. $ax+bx^2=X$

$$\int\frac{\partial x}{X^{\frac{9}{2}}}=\left(-\frac{1}{X^3}+\frac{24b}{5a^2X^2}-\frac{128b^2}{5a^4X}+\frac{1024b^3}{5a^6}\right)\frac{2(2bx+a)}{7a^2\sqrt{X}}$$

$$\int\frac{x\partial x}{X^{\frac{9}{2}}}=\frac{2x}{7aX^3\sqrt{X}}-\left(\frac{1}{X^2}-\frac{16b}{3a^2X}+\frac{128b^2}{3a^4}\right)\frac{24(2bx+a)}{35a^3\sqrt{X}}$$

$$\int\frac{x^2\partial x}{X^{\frac{9}{2}}}=\left(\frac{x^2}{X^3}+\frac{2x}{aX^2}\right)\frac{2}{7a\sqrt{X}}-\left(\frac{1}{X}-\frac{8b}{a^2}\right)\frac{32(2bx+a)}{21a^4\sqrt{X}}$$

$$\int\frac{x^3\partial x}{X^{\frac{9}{2}}}=\left(\frac{x^3}{X^3}+\frac{8x^2}{5aX^2}+\frac{16x}{5a^2X}\right)\frac{2}{7a\sqrt{X}}-\frac{128(2bx+a)}{35a^5\sqrt{X}}$$

$$\int\frac{x^4\partial x}{X^{\frac{9}{2}}}=\left(\frac{x^4}{X^3}+\frac{6x^3}{5aX^2}+\frac{8x^2}{5a^2X}+\frac{16x}{5a^3}\right)\frac{2}{7a\sqrt{X}}$$

$$\int\frac{x^5\partial x}{X^{\frac{9}{2}}}=\left(\frac{x^5}{X^3}+\frac{4x^4}{5aX^2}+\frac{8x^3}{15a^2X}\right)\frac{2}{7a\sqrt{X}}$$

$$\int\frac{x^6\partial x}{X^{\frac{9}{2}}}=\left(\frac{x^6}{X^3}+\frac{2x^5}{5aX^2}\right)\frac{2}{7a\sqrt{X}}=\left(\frac{1}{(a+bx)^3}+\frac{2}{5a(a+bx)^2}\right)\frac{2x^3}{7a\sqrt{X}}$$

$$\int\frac{x^7\partial x}{X^{\frac{9}{2}}}=\frac{2x^7}{7aX^3\sqrt{X}}=\frac{2x^4}{7a(a+bx)^3\sqrt{X}}.$$

$$\int\frac{\partial x}{xX^{\frac{9}{2}}}=-\frac{2}{9axX^3\sqrt{X}}-\frac{16b}{9a}\int\frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int\frac{\partial x}{x^2X^{\frac{9}{2}}}=\left(-\frac{1}{11ax^2}+\frac{2b}{11a^2x}\right)\frac{2}{X^3\sqrt{X}}+\frac{32b^2}{11a^2}\int\frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int\frac{\partial x}{x^3X^{\frac{9}{2}}}=\left(-\frac{1}{13ax^3}+\frac{20b}{143a^2x^2}-\frac{40b^2}{143a^3x}\right)\frac{2}{X^3\sqrt{X}}-\frac{640b^3}{143a^3}\int\frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int\frac{\partial x}{x^4X^{\frac{9}{2}}}=\left(-\frac{1}{15ax^4}+\frac{22b}{195a^2x^3}-\frac{8b^2}{39a^3x^2}+\frac{16b^3}{39a^4x}\right)\frac{2}{X^3\sqrt{X}}+\frac{256b^4}{39a^4}\int\frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int\frac{\partial x}{x^5X^{\frac{9}{2}}}=\left(-\frac{1}{17ax^5}+\frac{8b}{85a^2x^4}-\frac{176b^2}{1105a^3x^3}+\frac{64b^3}{221a^4x^2}-\frac{128b^4}{221a^5x}\right)\frac{2}{X^3\sqrt{X}}$$
$$-\frac{2048b^5}{221a^5}\int\frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int x^m \partial x \sqrt{(ax+bx^2)}$$ **Taf. LIII.**

VZ. $ax+bx^2 = X$

$$\int \partial x \sqrt{X} = \left(\frac{x}{2}+\frac{a}{4b}\right)\sqrt{X} - \frac{a^2}{8b}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\,\partial x \sqrt{X} = \frac{X\sqrt{X}}{3b} - \frac{a}{2b}\int \partial x \sqrt{X}$$

$$\int x^2 \partial x \sqrt{X} = \left(\frac{x}{4b}-\frac{5a}{24b^2}\right)X\sqrt{X} + \frac{5a^2}{16b^2}\int \partial x \sqrt{X}$$

$$\int x^3 \partial x \sqrt{X} = \left(\frac{x^2}{5b}-\frac{7ax}{40b^2}+\frac{7a^2}{48b^3}\right)X\sqrt{X} - \frac{7a^3}{32b^3}\int \partial x \sqrt{X}$$

$$\int x^4 \partial x \sqrt{X} = \left(\frac{x^3}{6b}-\frac{3ax^2}{20b^2}+\frac{21a^2x}{160b^3}-\frac{7a^3}{64b^4}\right)X\sqrt{X} + \frac{21a^4}{128b^4}\int \partial x \sqrt{X}$$

$$\int x^5 \partial x \sqrt{X} = \left(\frac{x^4}{7b}-\frac{11ax^3}{84b^2}+\frac{33a^2x^2}{280b^3}-\frac{33a^3x}{320b^4}+\frac{11a^4}{128b^5}\right)X\sqrt{X} - \frac{33a^5}{256b^5}\int \partial x \sqrt{X}$$

$$\int x^6 \partial x \sqrt{X} = \left(\frac{x^5}{8b}-\frac{13ax^4}{112b^2}+\frac{143a^2x^3}{1344b^3}-\frac{429a^3x^2}{4480b^4}+\frac{429a^4x}{5120b^5}-\frac{143a^5}{2048b^6}\right)X\sqrt{X} + \frac{429a^6}{4096b^6}\int \partial x \sqrt{X}$$

$$\int x^7 \partial x \sqrt{X} = \left(\frac{x^6}{9b}-\frac{5ax^5}{48b^2}+\frac{65a^2x^4}{672b^3}-\frac{715a^3x^3}{8064b^4}+\frac{715a^4x^2}{8960b^4}-\frac{143a^5x}{2048b^6}+\frac{715a^6}{12288b^6}\right)X\sqrt{X} - \frac{715a^7}{8192b^7}\int \partial x \sqrt{X}$$

$$\int x^8 \partial x \sqrt{X} = \frac{x^7 X\sqrt{X}}{10b} - \frac{17a}{20b}\int x^7 \partial x \sqrt{X}$$

$$\int x^9 \partial x \sqrt{X} = \left(\frac{x^8}{11b}-\frac{19ax^7}{220b^2}\right)X\sqrt{X} + \frac{323a^2}{440b^2}\int x^7 \partial x \sqrt{X}$$

$$\int x^{10} \partial x \sqrt{X} = \left(\frac{x^9}{12b}-\frac{7ax^8}{88b^2}+\frac{133a^2x^7}{1760b^3}\right)X\sqrt{X} - \frac{2261a^3}{3520b^3}\int x^7 \partial x \sqrt{X}$$

Taf. LIV. $$\int \frac{\partial x \sqrt{(ax+bx^2)}}{x^m}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x \sqrt{X}}{x} = \sqrt{X} + \frac{a}{2} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^2} = -\frac{2\sqrt{X}}{x} + b \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^3} = -\frac{2X\sqrt{X}}{3ax^3} = -\frac{2(a+bx)\sqrt{X}}{3ax^2}$$

$$\int \frac{\partial x \sqrt{X}}{x^4} = \left(-\frac{1}{5ax^4} + \frac{2b}{15a^2x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^5} = \left(-\frac{1}{7ax^5} + \frac{4b}{35a^2x^4} - \frac{8b^2}{105a^3x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^6} = \left(-\frac{1}{9ax^6} + \frac{2b}{21a^2x^5} - \frac{8b^2}{105a^3x^4} + \frac{16b^3}{315a^4x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^7} = \left(-\frac{1}{11ax^7} + \frac{8b}{99a^2x^6} - \frac{16b^2}{231a^3x^5} + \frac{64b^3}{1155a^4x^4} - \frac{128b^4}{3465a^5x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^8} = \left(-\frac{1}{13ax^8} + \frac{10b}{143a^2x^7} - \frac{80b^2}{1287a^3x^6} + \frac{160b^3}{3003a^4x^5} - \frac{128b^4}{3003a^5x^4} + \frac{256b^5}{9009a^6x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^9} = \left(-\frac{1}{15ax^9} + \frac{4b}{65a^2x^8} - \frac{8b^2}{143a^3x^7} + \frac{64b^3}{1287a^4x^6} - \frac{128b^4}{3003a^5x^5} + \frac{512b^5}{15015a^6x^4} - \frac{1024b^6}{45045a^7x^3}\right) 2X\sqrt{X}$$

$$\int \frac{\partial x \sqrt{X}}{x^{10}} = -\frac{2X\sqrt{X}}{17ax^{10}} - \frac{14b}{17a} \int \frac{\partial x \sqrt{X}}{x^9}$$

$$\int \frac{\partial x \sqrt{X}}{x^{11}} = \left(-\frac{1}{19ax^{11}} + \frac{16b}{323a^2x^{10}}\right) 2X\sqrt{X} + \frac{224b^2}{323a^2} \int \frac{\partial x \sqrt{X}}{x^9}$$

$$\int x^m \partial x (a + bx^2)^{\frac{3}{2}}$$ Taf. LV.

VZ. $ax + bx^2 = X$

$$\int \partial x X^{\frac{3}{2}} = \left(\frac{X}{b} - \frac{3a^2}{8b^2}\right)\frac{2bx+a}{8}\sqrt{X} + \frac{3a^4}{128b^2}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{3}{2}} = \frac{X^2\sqrt{X}}{5b} - \frac{a}{2b}\int\partial x X^{\frac{3}{2}}$$

$$\int x^2\partial x X^{\frac{3}{2}} = \left(\frac{x}{6b} - \frac{7a}{60b^2}\right)X^2\sqrt{X} + \frac{7a^2}{24b^2}\int\partial x X^{\frac{3}{2}}$$

$$\int x^3\partial x X^{\frac{3}{2}} = \left(\frac{x^2}{7b} - \frac{3ax}{28b^2} + \frac{3a^2}{40b^3}\right)X^2\sqrt{X} - \frac{3a^3}{16b^3}\int\partial x X^{\frac{3}{2}}$$

$$\int x^4\partial x X^{\frac{3}{2}} = \left(\frac{x^3}{8b} - \frac{11ax^2}{112b^2} + \frac{33a^2x}{448b^3} - \frac{33a^3}{640b^4}\right)X^2\sqrt{X} + \frac{33a^4}{256b^4}\int\partial x X^{\frac{3}{2}}$$

$$\int x^5\partial x X^{\frac{3}{2}} = \left(\frac{x^4}{9b} - \frac{13ax^3}{144b^2} + \frac{143a^2x^2}{2016b^3} - \frac{143a^3x}{2688b^4} + \frac{143a^4}{3840b^5}\right)X^2\sqrt{X}$$
$$- \frac{143a^5}{1536b^5}\int\partial x X^{\frac{3}{2}}$$

$$\int x^6\partial x X^{\frac{3}{2}} = \left(\frac{x^5}{10b} - \frac{ax^4}{12b^2} + \frac{13a^2x^3}{192b^8} - \frac{143a^3x^2}{2688b^4} + \frac{143a^4x}{3584b^5}\right.$$
$$\left. - \frac{143a^5}{5120b^6}\right)X^2\sqrt{X} + \frac{143a^6}{2048b^6}\int\partial x X^{\frac{3}{2}}$$

$$\int x^7\partial x X^{\frac{3}{2}} = \left(\frac{x^6}{11b} - \frac{17ax^5}{220b^2} + \frac{17a^2x^4}{264b^3} - \frac{221a^3x^3}{4224b^4} + \frac{221a^4x^2}{5376b^5}\right.$$
$$\left. - \frac{221a^5x}{7168b^6} + \frac{221a^6}{10240b^7}\right)X^2\sqrt{X} - \frac{221a^7}{4096b^7}\int\partial x X^{\frac{3}{2}}$$

$$\int x^8\partial x X^{\frac{3}{2}} = \frac{x^7X^2\sqrt{X}}{12b} - \frac{19a}{24b}\int x^7\partial x X^{\frac{3}{2}}$$

$$\int x^9\partial x X^{\frac{3}{2}} = \left(\frac{x^8}{13b} - \frac{7ax^7}{104b^2}\right)X^2\sqrt{X} + \frac{133a^2}{208b^2}\int x^7\partial x X^{\frac{3}{2}}$$

$$\int x^{10}\partial x X^{\frac{3}{2}} = \left(\frac{x^9}{14b} - \frac{23ax^8}{364b^2} + \frac{23a^2x^7}{416b^3}\right)X^2\sqrt{X} - \frac{437a^3}{832b^3}\int x^7\partial x X^{\frac{3}{2}}$$

Taf. LVI.

$$\int \frac{\partial x (ax+bx^2)^{\frac{3}{2}}}{x^m}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x} = \frac{X\sqrt{X}}{3} + \frac{a}{2}\int \partial x \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^2} = \frac{X\sqrt{X}}{2x} + \frac{3a}{4}\sqrt{X} + \frac{3a^2}{8}\int \frac{\partial x}{\sqrt{X}}$$

$$= \left(\frac{5a}{4} + \frac{bx}{2}\right)\sqrt{X} + \frac{3a^2}{8}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^3} = \frac{X\sqrt{X}}{x^2} - \frac{3a\sqrt{X}}{x} + \frac{3ab}{2}\int \frac{\partial x}{\sqrt{X}}$$

$$= \left(b - \frac{2a}{x}\right)\sqrt{X} + \frac{3ab}{2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^4} = -\frac{2X^2\sqrt{X}}{3ax^4} + \frac{2b}{3a}\int \frac{\partial x X^{\frac{3}{2}}}{x^3}$$

$$= -\left(\frac{2a}{3x^2} + \frac{8b}{3x}\right)\sqrt{X} + b^2\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^5} = -\frac{2X^2\sqrt{X}}{5ax^5} = -\frac{2(a+bx)^2\sqrt{X}}{5ax^3}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^6} = \left(-\frac{1}{7ax^6} + \frac{2b}{35a^2x^5}\right)2X^2\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^7} = \left(-\frac{1}{9ax^7} + \frac{4b}{63a^2x^6} - \frac{8b^2}{315a^3x^5}\right)2X^2\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^8} = \left(-\frac{1}{11ax^8} + \frac{2b}{33a^2x^7} - \frac{8b^2}{231a^3x^6} + \frac{16b^3}{1155a^4x^5}\right)2X^2\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^9} = \left(-\frac{1}{13ax^9} + \frac{8b}{143a^2x^8} - \frac{16b^2}{429a^3x^7} + \frac{64b^3}{3003a^4x^6} - \frac{128b^4}{15015a^5x^5}\right)2X^2\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^{10}} = \left(-\frac{1}{15ax^{10}} + \frac{2b}{39a^2x^9} - \frac{16b^2}{429a^3x^8} + \frac{32b^3}{1287a^4x^7} - \frac{128b^4}{9009a^5x^6} + \frac{256b^5}{45045a^6x^5}\right)2X^2\sqrt{X}$$

$$\int x^m \partial x (ax+bx^2)^{\frac{5}{2}}$$ **Taf. LVII.**

Vz. $ax + bx^2 = X$

$$\int \partial x X^{\frac{5}{2}} = \left(\frac{X^2}{b} - \frac{5a^2X}{16b^2} + \frac{15a^4}{128b^3}\right)\frac{2bx+a}{12}\sqrt{X} - \frac{5a^6}{1024b^3}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{5}{2}} = \frac{X^3\sqrt{X}}{7b} - \frac{a}{2b}\int\partial x X^{\frac{5}{2}}$$

$$\int x^2\partial x X^{\frac{5}{2}} = \left(\frac{x}{8b} - \frac{9a}{112b^2}\right)X^3\sqrt{X} + \frac{9a^2}{32b^2}\int\partial x X^{\frac{5}{2}}$$

$$\int x^3\partial x X^{\frac{5}{2}} = \left(\frac{x^2}{9b} - \frac{11ax}{144b^2} + \frac{11a^2}{224b^3}\right)X^3\sqrt{X} - \frac{11a^3}{64b^3}\int\partial x X^{\frac{5}{2}}$$

$$\int x^4\partial x X^{\frac{5}{2}} = \left(\frac{x^3}{10b} - \frac{13ax^2}{180b^2} + \frac{143a^2x}{2880b^3} - \frac{143a^3}{4480b^4}\right)X^3\sqrt{X} + \frac{143a^4}{1280b^4}\int\partial x X^{\frac{5}{2}}$$

$$\int x^5\partial x X^{\frac{5}{2}} = \left(\frac{x^4}{11b} - \frac{3ax^3}{44b^2} + \frac{39a^2x^2}{792b^3} - \frac{39a^3x}{1152b^4} + \frac{39a^4}{1792b^5}\right)X^3\sqrt{X} - \frac{39a^5}{512b^5}\int\partial x X^{\frac{5}{2}}$$

$$\int x^6\partial x X^{\frac{5}{2}} = \left(\frac{x^5}{12b} - \frac{17ax^4}{264b^2} + \frac{17a^2x^3}{352b^3} - \frac{221a^3x^2}{6336b^4} + \frac{221a^4x}{9216b^5} - \frac{221a^5}{14336b^6}\right)X^3\sqrt{X} + \frac{221a^6}{4096b^6}\int\partial x X^{\frac{5}{2}}$$

$$\int x^7\partial x X^{\frac{5}{2}} = \frac{x^6X^3\sqrt{X}}{13b} - \frac{19a}{26b}\int x^6\partial x X^{\frac{5}{2}}$$

$$\int x^8\partial x X^{\frac{5}{2}} = \left(\frac{x^7}{14b} - \frac{3ax^6}{52b^2}\right)X^3\sqrt{X} + \frac{57a^2}{104b^2}\int x^6\partial x X^{\frac{5}{2}}$$

$$\int x^9\partial x X^{\frac{5}{2}} = \left(\frac{x^8}{15b} - \frac{23ax^7}{420b^2} + \frac{23a^2x^6}{520b^3}\right)X^3\sqrt{X} - \frac{437a^3}{1040b^3}\int x^6\partial x X^{\frac{5}{2}}$$

$$\int x^{10}\partial x X^{\frac{5}{2}} = \left(\frac{x^9}{16b} - \frac{5ax^8}{96b^2} + \frac{115a^2x^7}{2688b^3} - \frac{115a^3x^6}{3328b^4}\right)X^3\sqrt{X} + \frac{2185a^4}{6656b^4}\int x^6\partial x X^{\frac{5}{2}}$$

Taf. LVIII.

$$\int \frac{\partial x (ax+bx^2)^{\frac{5}{2}}}{x^m}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x} = \frac{X^2 \sqrt{X}}{5} + \frac{a}{2} \int \partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^2} = \left(\frac{X^2}{4x} + \frac{5aX}{24}\right)\sqrt{X} + \frac{5a^2}{16} \int \partial x \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^3} = \left(\frac{X^2}{3x^2} + \frac{5aX}{12x} + \frac{5a^2}{8}\right)\sqrt{X} + \frac{5a^3}{16} \int \frac{\partial x}{\sqrt{X}}$$
$$= \left(\frac{11a^2}{8} + \frac{13abx}{12} + \frac{b^2x^2}{3}\right)\sqrt{X} + \frac{5a^3}{16} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^4} = \left(\frac{X^2}{2x^3} + \frac{5aX}{4x^2} - \frac{15a^2}{4x}\right)\sqrt{X} + \frac{15a^2b}{8} \int \frac{\partial x}{\sqrt{X}}$$
$$= \left(-\frac{2a^2}{x} + \frac{9ab}{4} + \frac{b^2x}{2}\right)\sqrt{X} + \frac{15a^2b}{8} \int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^5} = -\frac{2X^3\sqrt{X}}{3ax^5} + \frac{4b}{3a} \int \frac{\partial x X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^6} = \left(-\frac{1}{5ax^6} - \frac{2b}{15a^2x^5}\right)2X^3\sqrt{X} + \frac{8b^2}{15a^2} \int \frac{\partial x X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^7} = -\frac{2X^3\sqrt{X}}{7ax^7} = -\frac{2(a+bx)^3\sqrt{X}}{7ax^4}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^8} = \left(-\frac{1}{9ax^8} + \frac{2b}{63a^2x^7}\right)2X^3\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^9} = \left(-\frac{1}{11ax^9} + \frac{4b}{99a^2x^8} - \frac{8b^2}{693a^3x^7}\right)2X^3\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^{10}} = \left(-\frac{1}{13ax^{10}} + \frac{6b}{143a^2x^9} - \frac{8b^2}{429a^3x^8} + \frac{16b^3}{3003a^4x^7}\right)2X^3\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^{11}} = \left(-\frac{1}{15ax^{11}} + \frac{8b}{195a^2x^{10}} - \frac{16b^2}{715a^3x^9} + \frac{64b^3}{6435a^4x^8}\right.$$
$$\left. - \frac{128b^4}{45045a^5x^7}\right)2X^3\sqrt{X}$$

$$\int x^m \partial x (ax+bx^2)^{\frac{7}{2}}$$ *Taf. LIX.*

VZ. $ax + bx^2 = X$

$$\int \partial x X^{\frac{7}{2}} = \left(\frac{X^3}{b} - \frac{7a^2X^2}{24b^2} + \frac{35a^4X}{384b^3} - \frac{35a^6}{1024b^4}\right)\frac{2bx+a}{16}\sqrt{X} + \frac{35a^8}{32768b^4}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{7}{2}} = \frac{X^4\sqrt{X}}{9b} - \frac{a}{2b}\int \partial x X^{\frac{7}{2}}$$

$$\int x^2\partial x X^{\frac{7}{2}} = \left(\frac{x}{10b} - \frac{11a}{180b^2}\right)X^4\sqrt{X} + \frac{11a^2}{40b^2}\int\partial x X^{\frac{7}{2}}$$

$$\int x^3\partial x X^{\frac{7}{2}} = \left(\frac{x^2}{11b} - \frac{13ax}{220b^2} + \frac{13a^2}{360b^3}\right)X^4\sqrt{X} - \frac{13a^3}{80b^3}\int\partial x X^{\frac{7}{2}}$$

$$\int x^4\partial x X^{\frac{7}{2}} = \left(\frac{x^3}{12b} - \frac{5ax^2}{88b^2} + \frac{13a^2x}{352b^3} - \frac{13a^3}{576b^4}\right)X^4\sqrt{X} + \frac{13a^4}{128b^4}\int\partial x X^{\frac{7}{2}}$$

$$\int x^5\partial x X^{\frac{7}{2}} = \left(\frac{x^4}{13b} - \frac{17ax^3}{312b^2} + \frac{85a^2x^2}{2288b^3} - \frac{17a^3x}{704b^4} + \frac{17a^4}{1152b^5}\right)X^4\sqrt{X} - \frac{17a^5}{256b^5}\int\partial x X^{\frac{7}{2}}$$

$$\int x^6\partial x X^{\frac{7}{2}} = \left(\frac{x^5}{14b} - \frac{19ax^4}{364b^2} + \frac{323a^2x^3}{8736b^3} - \frac{1615a^3x^2}{64064b^4} + \frac{323a^4x}{19712b^5} - \frac{323a^5}{32256b^6}\right)X^4\sqrt{X} + \frac{323a^6}{7168b^6}\int\partial x X^{\frac{7}{2}}$$

$$\int x^7\partial x X^{\frac{7}{2}} = \frac{x^6X^4\sqrt{X}}{15b} - \frac{7a}{10b}\int x^6\partial x X^{\frac{7}{2}}$$

$$\int x^8\partial x X^{\frac{7}{2}} = \left(\frac{x^7}{16b} - \frac{23ax^6}{480b^2}\right)X^4\sqrt{X} + \frac{161a^2}{320b^2}\int x^6\partial x X^{\frac{7}{2}}$$

$$\int x^9\partial x X^{\frac{7}{2}} = \left(\frac{x^8}{17b} - \frac{25ax^7}{544b^2} + \frac{115a^2x^6}{3264b^3}\right)X^4\sqrt{X} - \frac{805a^3}{2176b^3}\int x^6\partial x X^{\frac{7}{2}}$$

$$\int x^{10}\partial x X^{\frac{7}{2}} = \left(\frac{x^9}{18b} - \frac{3ax^8}{68b^2} + \frac{75a^2x^7}{2176b^3} - \frac{345a^3x^6}{13056b^4}\right)X^4\sqrt{X} + \frac{2415a^4}{8704b^4}\int x^6\partial x X^{\frac{7}{2}}$$

Taf. LX.

$$\int \frac{\partial x (ax+bx^2)^{\frac{7}{2}}}{x^m}$$

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x} = \frac{X^3\sqrt{X}}{7} + \frac{a}{2}\int \partial x X^{\frac{5}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^2} = \left(\frac{X^3}{6x} + \frac{7aX^2}{60}\right)\sqrt{X} + \frac{7a^2}{24}\int \partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^3} = \left(\frac{X^3}{5x^2} + \frac{7aX^2}{40x} + \frac{7a^2X}{48}\right)\sqrt{X} + \frac{7a^3}{32}\int \partial x \sqrt{X}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^4} = \left(\frac{X^3}{4x^3} + \frac{7aX^2}{24x^2} + \frac{35a^2X}{96x} + \frac{35a^3}{64}\right)\sqrt{X} + \frac{35a^4}{128}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^5} = \left(\frac{X^3}{3x^4} + \frac{7aX^2}{12x^3} + \frac{35a^2X}{24x^2} - \frac{35a^3}{8x}\right)\sqrt{X} + \frac{35a^3b}{16}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^6} = -\frac{2X^3\sqrt{X}}{3x^5} + \frac{7b}{3}\int \frac{\partial x X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^7} = \left(-\frac{1}{5x^6} - \frac{7b}{15ax^5}\right)2X^3\sqrt{X} + \frac{28b^2}{15a}\int \frac{\partial x X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^8} = \left(-\frac{1}{7x^7} - \frac{b}{5ax^6} - \frac{2b^2}{15a^2x^5}\right)2X^3\sqrt{X} + \frac{8b^3}{15a^2}\int \frac{\partial x X^{\frac{5}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^9} = -\frac{2X^4\sqrt{X}}{9ax^9} = -\frac{2(a+bx)^4\sqrt{X}}{9ax^5}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^{10}} = \left(-\frac{1}{11ax^{10}} + \frac{2b}{99a^2x^9}\right)2X^4\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^{11}} = \left(-\frac{1}{13ax^{11}} + \frac{4b}{143a^2x^{10}} - \frac{8b^2}{1287a^3x^9}\right)2X^4\sqrt{X}$$

$$\int x^m \partial x (ax+bx^2)^{\frac{9}{2}}, \quad \int \frac{\partial x (ax+bx^2)^{\frac{9}{2}}}{x^m}$$ Taf. LXI.

VZ. $ax + bx^2 = X$

$$\int \partial x X^{\frac{9}{2}} = \left(\frac{X^4}{b} - \frac{9a^2X^3}{32b^2} + \frac{21a^4X^2}{256b^3} - \frac{105a^6X}{4096b^4} + \frac{315a^8}{32768b^5}\right)\frac{2bx+a}{20}\sqrt{X} - \frac{63a^{10}}{262144b^5}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{9}{2}} = \frac{X^5\sqrt{X}}{11b} - \frac{a}{2b}\int \partial x X^{\frac{9}{2}}$$

$$\int x^2\partial x X^{\frac{9}{2}} = \left(\frac{x}{12b} - \frac{13a}{264b^2}\right)X^5\sqrt{X} + \frac{13a^2}{48b^2}\int \partial x X^{\frac{9}{2}}$$

$$\int x^3\partial x X^{\frac{9}{2}} = \left(\frac{x^2}{13b} - \frac{5ax}{104b^2} + \frac{5a^2}{176b^3}\right)X^5\sqrt{X} - \frac{5a^3}{32b^3}\int \partial x X^{\frac{9}{2}}$$

$$\int x^4\partial x X^{\frac{9}{2}} = \frac{x^3X^5\sqrt{X}}{14b} - \frac{17a}{28b}\int x^3\partial x X^{\frac{9}{2}}$$

$$\int x^5\partial x X^{\frac{9}{2}} = \left(\frac{x^4}{15b} - \frac{19ax^3}{420b^2}\right)X^5\sqrt{X} + \frac{323a^2}{840b^2}\int x^3\partial x X^{\frac{9}{2}}$$

$$\int x^6\partial x X^{\frac{9}{2}} = \left(\frac{x^5}{16b} - \frac{7ax^4}{160b^2} + \frac{19a^2x^3}{640b^3}\right)X^5\sqrt{X} - \frac{323a^3}{1280b^3}\int x^3\partial x X^{\frac{9}{2}}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x} = \frac{X^4\sqrt{X}}{9} + \frac{a}{2}\int \partial x X^{\frac{7}{2}}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^2} = \left(\frac{X^4}{8x} + \frac{9aX^3}{112}\right)\sqrt{X} + \frac{9a^2}{32}\int \partial x X^{\frac{5}{2}}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^3} = \left(\frac{X^4}{7x^2} + \frac{3aX^3}{28x} + \frac{3a^2X^2}{40}\right)\sqrt{X} + \frac{3a^3}{16}\int \partial x X^{\frac{3}{2}}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^4} = \left(\frac{X^4}{6x^3} + \frac{3aX^3}{20x^2} + \frac{21a^2X^2}{160x} + \frac{7a^3X}{64}\right)\sqrt{X} + \frac{21a^4}{128}\int \partial x\sqrt{X}$$

$$\int\frac{\partial x X^{\frac{9}{2}}}{x^5} = \left(\frac{X^4}{5x^4} + \frac{9aX^3}{40x^3} + \frac{21a^2X^2}{80x^2} + \frac{21a^3X}{64x} + \frac{63a^4}{128}\right)\sqrt{X} + \frac{63a^5}{256}\int\frac{\partial x}{\sqrt{X}}$$

Taf. LXII.

$$\int\frac{\partial x}{(a+bx+cx^2)^{\frac{n}{2}}}$$

VZ. $a+bx+cx^2=X,\ 4ac-b^2=k$

$$\int\frac{\partial x}{X^{\frac{1}{2}}}=\int\frac{\partial x}{\sqrt{X}}$$ (Man s. die folgende Seite.)

$$\int\frac{\partial x}{X^{\frac{3}{2}}}=\frac{2(2cx+b)}{k\sqrt{X}}$$

$$\int\frac{\partial x}{X^{\frac{5}{2}}}=\left(\frac{1}{3kX}+\frac{8c}{3k^2}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{\partial x}{X^{\frac{7}{2}}}=\left(\frac{1}{5kX^2}+\frac{4^2c}{15k^2X}+\frac{2.4^3c^2}{15k^3}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{\partial x}{X^{\frac{9}{2}}}=\left(\frac{1}{7kX^3}+\frac{6.4c}{35k^2X^2}+\frac{2.4^3c^2}{35k^3X}+\frac{4^5c^3}{35k^4}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{\partial x}{X^{\frac{11}{2}}}=\left(\frac{1}{9kX^4}+\frac{2.4^2c}{63k^2X^3}+\frac{4^4c^2}{105k^3X^2}+\frac{4^6c^3}{315k^4X}+\frac{2.4^7c^4}{315k^5}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{\partial x}{X^{\frac{13}{2}}}=\left(\frac{1}{11kX^5}+\frac{10.4c}{99k^2X^4}+\frac{5.4^4c^2}{693k^3X^3}+\frac{2.4^5c^3}{231k^4X^2}+\frac{2.4^7c^4}{693k^5X}+\frac{4^9c^5}{693k^6}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{\partial x}{X^{\frac{15}{2}}}=\left(\frac{1}{13kX^6}+\frac{3.4^2c}{143k^2X^5}+\frac{10.4^3c^2}{429k^3X^4}+\frac{5.4^6c^3}{3003k^4X^3}+\frac{2.4^7c^4}{1001k^5X^2}+\frac{2.4^9c^5}{3003k^6X}+\frac{4^{11}c^6}{3003k^7}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{\partial x}{X^{\frac{17}{2}}}=\left(\frac{1}{15kX^7}+\frac{14.4c}{195k^2X^6}+\frac{14.4^3c^2}{715k^3X^5}+\frac{7.4^5c^3}{1287k^4X^4}+\frac{2.4^7c^4}{1287k^5X^3}+\frac{4^9c^5}{2145k^6X^2}+\frac{4^{11}c^6}{6435k^7X}+\frac{2.4^{12}c^7}{6435k^8}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{\partial x}{X^{\frac{19}{2}}}=\frac{2(2cx+b)}{17kX^8\sqrt{X}}+\frac{64c}{17k}\int\frac{\partial x}{X^{\frac{17}{2}}}$$

Anmerkung zur vorhergehenden Tafel.

Es ist im Allgemeinen

$$\int \frac{\partial x}{\sqrt{(a+bx+cx^2)}} = \frac{1}{\sqrt{c}} \log \left[2cx+b+2\sqrt{c}.\sqrt{(a+bx+cx^2)}\right] + \text{Const.}$$

oder auch

$$\int \frac{\partial x}{\sqrt{(a+bx+cx^2)}} = \frac{-1}{\sqrt{-c}} \text{ Arc Sin } \frac{2cx+b}{\sqrt{(b^2-4ac)}} + \text{Const.}$$

Die erste Form wird reell, wenn c positiv, die zweite, wenn c negativ ist. Hieraus ergiebt sich:

$$\text{I.} \quad \int \frac{\partial x}{\sqrt{X}} = \int \frac{\partial x}{\sqrt{(a+bx+cx^2)}} = \pm \frac{1}{\sqrt{c}} \log (2cx+b \pm 2\sqrt{c}.\sqrt{X})$$

und wenn das Integral für $x = o$ verschwinden soll,

$$\int \frac{\partial x}{\sqrt{X}} = \pm \frac{1}{\sqrt{c}} \log \frac{2cx+b \pm 2\sqrt{c}.\sqrt{X}}{b \pm 2\sqrt{ac}}.$$

Die oberen Zeichen müssen hier zugleich genommen werden, und eben so die unteren.

$$\text{II.} \quad \int \frac{\partial x}{\sqrt{X}} = \int \frac{\partial x}{\sqrt{(a+bx-cx^2)}} = \frac{1}{\sqrt{c}} \text{ Arc Sin } \frac{2cx-b}{\sqrt{(b^2+4ac)}}$$

$$= \frac{1}{\sqrt{c}} \text{ Arc Cos } \frac{2\sqrt{cX}}{\sqrt{(b^2+4ac)}} = \frac{1}{\sqrt{c}} \text{ Arc Tang } \frac{2cx-b}{2\sqrt{cX}}$$

$$= \frac{1}{\sqrt{c}} \text{ Arc Cot } \frac{2\sqrt{cX}}{2cx-b} = \frac{1}{\sqrt{c}} \text{ Arc Sec } \frac{\sqrt{(b^2+4ac)}}{2\sqrt{cX}}$$

$$= \frac{1}{\sqrt{c}} \text{ Arc Cosec } \frac{\sqrt{(b^2+4ac)}}{2cx-b} = \frac{1}{2\sqrt{c}} \text{ Arc Sin vers } \frac{2(2cx-b)^2}{b^2+4ac},$$

und diese Kreisbogen verschwinden sämmtlich für $x = \frac{b}{2c}$. Sollen sie für $x = o$ verschwinden, so ist

$$\int \frac{\partial x}{\sqrt{X}} = \int \frac{\partial x}{\sqrt{(a+bx-cx^2)}} = \frac{1}{\sqrt{c}} \text{ Arc Sin } \frac{2(2cx-b)\sqrt{ac}+2b\sqrt{cX}}{b^2+4ac}$$

$$= \frac{1}{\sqrt{c}} \text{ Arc Cos } \frac{4c\sqrt{aX}-b(2cx-b)}{b^2+4ac} = \text{etc.}$$

In der Ausübung dürfte es besser seyn den Bogen mit der Constante nicht zusammen zu ziehen.

Taf. LXIII. $\int \partial x (a+bx+cx^2)^{\frac{n}{2}}$

VZ. $a+bx+cx^2 = X, \quad 4ac - b^2 = k$

$$\int \partial x X^{\frac{1}{2}} = \frac{(2cx+b)\sqrt{X}}{4c} + \frac{k}{8c}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{3}{2}} = \left(\frac{X}{8c} + \frac{3k}{64c^2}\right)(2cx+b)\sqrt{X} + \frac{3k^2}{128c^2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{5}{2}} = \left(\frac{X^2}{12c} + \frac{5kX}{192c^2} + \frac{5k^2}{512c^3}\right)(2cx+b)\sqrt{X} + \frac{5k^3}{1024c^3}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{7}{2}} = \left(\frac{X^3}{16c} + \frac{7kX^2}{6.4^3c^2} + \frac{35k^2X}{6.4^5c^3} + \frac{35k^3}{4^7c^4}\right)(2cx+b)\sqrt{X} + \frac{35k^4}{2.4^7c^4}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{9}{2}} = \left(\frac{X^4}{20c} + \frac{9kX^3}{10.4^3c^2} + \frac{21k^2X^2}{5.4^5c^3} + \frac{21k^3X}{4^7c^4} + \frac{63k^4}{2.4^8c^5}\right) \times (2cx+b)\sqrt{X} + \frac{63k^5}{4^9c^5}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{11}{2}} = \left(\frac{X^5}{24c} + \frac{11kX^4}{15.4^3c^2} + \frac{33k^2X^3}{10.4^5c^3} + \frac{77k^3X^2}{5.4^7c^4} + \frac{77k^4X}{4^9c^5} + \frac{231k^5}{2.4^{10}c^6}\right)(2cx+b)\sqrt{X} + \frac{231k^6}{4^{11}c^6}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{13}{2}} = \left(\frac{X^6}{28c} + \frac{13kX^5}{21.4^3c^2} + \frac{143k^2X^4}{210.4^4c^3} + \frac{429k^3X^3}{35.4^7c^4} + \frac{143k^4X^2}{10.4^8c^5} + \frac{143k^5X}{2.4^{10}c^6} + \frac{429k^6}{4^{12}c^7}\right)(2cx+b)\sqrt{X} + \frac{429k^7}{2.4^{12}c^7}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \partial x X^{\frac{15}{2}} = \left(\frac{X^7}{32c} + \frac{15kX^6}{7.4^4c^2} + \frac{65k^2X^6}{7.4^6c^3} + \frac{143k^3X^4}{14.4^7c^4} + \frac{1287k^4X^3}{7.4^{10}c^5} + \frac{429k^5X^2}{2.4^{11}c^6} + \frac{2145k^6X}{2.4^{13}c^7} + \frac{6435k^7}{4^{15}c^8}\right)(2cx+b)\sqrt{X} + \frac{6435k^8}{2.4^{15}c^8}\int \frac{\partial x}{\sqrt{X}}$$

$$\int\frac{x^m\partial x}{\sqrt{(a+bx+cx^2)}}$$ Taf. LXIV.

VZ. $a+bx+cx^2=X$

$$\int\frac{\partial x}{\sqrt X}=\int\frac{\partial x}{\sqrt X}$$ (Seite 183.)

$$\int\frac{x\partial x}{\sqrt X}=\frac{\sqrt X}{c}-\frac{b}{2c}\int\frac{\partial x}{\sqrt X}$$

$$\int\frac{x^2\partial x}{\sqrt X}=\left(\frac{x}{2c}-\frac{3b}{4c^2}\right)\sqrt X+\left(\frac{3b^2}{8c^2}-\frac{a}{2c}\right)\int\frac{\partial x}{\sqrt X}$$

$$\int\frac{x^3\partial x}{\sqrt X}=\left(\frac{x^2}{3c}-\frac{5bx}{12c^2}+\frac{5b^2}{8c^3}-\frac{2a}{3c^2}\right)\sqrt X-\left(\frac{5b^3}{16c^3}-\frac{3ab}{4c^2}\right)\int\frac{\partial x}{\sqrt X}$$

$$\int\frac{x^4\partial x}{\sqrt X}=\left[\frac{x^3}{4c}-\frac{7bx^2}{24c^2}+\left(\frac{35b^2}{96c^3}-\frac{3a}{8c^2}\right)x-\frac{35b^3}{64c^4}+\frac{55ab}{48c^3}\right]\sqrt X$$
$$+\left(\frac{35b^4}{128c^4}-\frac{15ab^2}{16c^3}+\frac{3a^2}{8c^2}\right)\int\frac{\partial x}{\sqrt X}$$

$$\int\frac{x^5\partial x}{\sqrt X}=\frac{x^4\sqrt X}{5c}-\frac{4a}{5c}\int\frac{x^3\partial x}{\sqrt X}-\frac{9b}{10c}\int\frac{x^4\partial x}{\sqrt X}$$

$$\int\frac{x^6\partial x}{\sqrt X}=\left(\frac{x^5}{6c}-\frac{11bx^4}{60c^2}\right)\sqrt X+\frac{11ab}{15c^2}\int\frac{x^3\partial x}{\sqrt X}+\left(\frac{33b^2}{40c^2}-\frac{5a}{6c}\right)\int\frac{x^4\partial x}{\sqrt X}$$

$$\int\frac{x^7\partial x}{\sqrt X}=\left[\frac{x^6}{7c}-\frac{13bx^5}{84c^2}+\left(\frac{143b^2}{840c^3}-\frac{6a}{35c^2}\right)x^4\right]\sqrt X$$
$$-\left(\frac{143ab^2}{210c^3}-\frac{24a^2}{35c^2}\right)\int\frac{x^3\partial x}{\sqrt X}-\left(\frac{429b^3}{56c^3}-\frac{649ab}{420c^2}\right)\int\frac{x^4\partial x}{\sqrt X}$$

$$\int\frac{x^8\partial x}{\sqrt X}=\left[\frac{x^7}{8c}-\frac{15bx^6}{112c^2}+\left(\frac{65b^2}{448c^3}-\frac{7a}{48c^2}\right)x^5-\right.$$
$$\left.\left(\frac{143b^3}{896c^4}-\frac{1079ab}{3360c^3}\right)x^4\right]\sqrt X+\left(\frac{143ab^3}{224c^4}-\frac{1079a^2b}{840c^3}\right)\int\frac{x^3\partial x}{\sqrt X}$$
$$+\left(\frac{6435b^4}{896c^4}-\frac{2431ab^2}{1120c^3}+\frac{35a^2}{48c^2}\right)\int\frac{x^4\partial x}{\sqrt X}$$

$$\int\frac{x^9\partial x}{\sqrt X}=\frac{x^8\sqrt X}{9c}-\frac{8a}{9c}\int\frac{x^7\partial x}{\sqrt X}-\frac{17b}{18c}\int\frac{x^8\partial x}{\sqrt X}$$

Taf. LXV. $$\int \frac{\partial x}{x^m \sqrt{(a+bx+cx^2)}}$$

VZ. $a + bx + cx^2 = X$

$$\int \frac{\partial x}{x\sqrt{X}} = \int \frac{\partial x}{x\sqrt{X}}$$ (Man s. die folgende Seite.)

$$\int \frac{\partial x}{x^2\sqrt{X}} = -\frac{\sqrt{X}}{ax} - \frac{b}{2a}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^3\sqrt{X}} = \left(-\frac{1}{2ax^2} + \frac{3b}{4a^2x}\right)\sqrt{X} + \left(\frac{3b^2}{8a^2} - \frac{c}{2a}\right)\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^4\sqrt{X}} = \left[-\frac{1}{3ax^3} + \frac{5b}{12a^2x^2} - \left(\frac{5b^2}{8a^3} - \frac{2c}{3a^2}\right)\frac{1}{x}\right]\sqrt{X} - \left(\frac{5b^3}{16a^3} - \frac{3bc}{4a^2}\right)\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^5\sqrt{X}} = \left[-\frac{1}{4ax^4} + \frac{7b}{24a^2x^3} - \left(\frac{35b^2}{96a^3} - \frac{3c}{8a^2}\right)\frac{1}{x^2} + \left(\frac{35b^3}{64a^4} - \frac{55bc}{48a^3}\right)\frac{1}{x}\right]\sqrt{X} + \left(\frac{35b^4}{128a^4} - \frac{15b^2c}{16a^3} + \frac{3c^2}{8a^2}\right)\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^6\sqrt{X}} = -\frac{\sqrt{X}}{5ax^5} - \frac{9b}{10a}\int \frac{\partial x}{x^5\sqrt{X}} - \frac{4c}{5a}\int \frac{\partial x}{x^4\sqrt{X}}$$

$$\int \frac{\partial x}{x^7\sqrt{X}} = \left(-\frac{1}{6ax^6} + \frac{11b}{60a^2x^5}\right)\sqrt{X} + \left(\frac{33b^2}{40a^2} - \frac{5c}{6a}\right)\int \frac{\partial x}{x^5\sqrt{X}} + \frac{11bc}{15a^2}\int \frac{\partial x}{x^4\sqrt{X}}$$

$$\int \frac{\partial x}{x^8\sqrt{X}} = \left[-\frac{1}{7ax^7} + \frac{13b}{84a^2x^6} - \left(\frac{143b^2}{840a^3} - \frac{6c}{35a^2}\right)\frac{1}{x^5}\right]\sqrt{X} - \left(\frac{429b^3}{560a^3} - \frac{649bc}{420a^2}\right)\int \frac{\partial x}{x^5\sqrt{X}} - \left(\frac{143b^2c}{210a^3} - \frac{24c^2}{35a^2}\right)\int \frac{\partial x}{x^4\sqrt{X}}$$

$$\int \frac{\partial x}{x^9\sqrt{X}} = \left[-\frac{1}{8ax^8} + \frac{15b}{112a^2x^7} - \left(\frac{65b^2}{448a^3} - \frac{7c}{48a^2}\right)\frac{1}{x^6} + \left(\frac{143b^3}{896a^4} - \frac{1079bc}{3360a^3}\right)\frac{1}{x^5}\right]\sqrt{X} + \left(\frac{1287b^4}{1792a^4} - \frac{2431b^2c}{1120a^3} + \frac{35c^2}{48a^2}\right)\int \frac{\partial x}{x^5\sqrt{X}} + \left(\frac{143b^3c}{224a^4} - \frac{1079bc^2}{840a^3}\right)\int \frac{\partial x}{x^4\sqrt{X}}$$

$$\int \frac{\partial x}{x^{10}\sqrt{X}} = -\frac{\sqrt{X}}{9ax^9} - \frac{17b}{18a}\int \frac{\partial x}{x^9\sqrt{X}} - \frac{8c}{9a}\int \frac{\partial x}{x^8\sqrt{X}}$$

Anmerkung zur vorhergehenden Tafel.

Es ist im Allgemeinen

$$\int \frac{\partial x}{x\sqrt{X}} = \frac{1}{\sqrt{a}} \log \frac{2a + bx - 2\sqrt{a}\,.\,\sqrt{X}}{x} + \text{Const.}$$

oder

$$\int \frac{\partial x}{x\sqrt{X}} = \frac{1}{\sqrt{-a}} \operatorname{Arc\,Tang} \frac{2a + bx}{2\sqrt{-a}\,.\,\sqrt{X}} + \text{Const.}$$

Die erste Form wird reell, wenn a positiv, die zweite, wenn a negativ ist. Hieraus ergiebt sich:

$$\text{I.} \quad \int \frac{\partial x}{x\sqrt{X}} = \int \frac{\partial x}{x\sqrt{(a + bx + cx^2)}}$$
$$= \pm \frac{1}{\sqrt{a}} \log \frac{2a + bx \mp 2\sqrt{aX}}{x} + \text{Const.}$$
$$= \pm \frac{1}{\sqrt{a}} \log \frac{2a + bx \mp 2\sqrt{aX}}{kx}.$$

Das k in dem letzteren Ausdrucke bezeichnet eine willkührliche Constante. Die oberen Zeichen gehören hier zusammen, und eben so die unteren. Für $x = o$ kann das Integral nicht verschwinden.

$$\text{II.} \quad \int \frac{\partial x}{x\sqrt{X}} = \int \frac{\partial x}{x\sqrt{(-a + bx + cx^2)}} = \frac{1}{\sqrt{a}} \operatorname{Arc\,Tang} \frac{bx - 2a}{2\sqrt{aX}}$$
$$= \frac{1}{\sqrt{a}} \operatorname{Arc\,Cot} \frac{2\sqrt{aX}}{bx - 2a} = \frac{1}{\sqrt{a}} \operatorname{Arc\,Sec} \frac{x\sqrt{(b^2 + 4ac)}}{2\sqrt{aX}}$$
$$= \frac{1}{\sqrt{a}} \operatorname{Arc\,Cosec} \frac{x\sqrt{(b^2 + 4ac)}}{bx - 2a} = \frac{1}{\sqrt{a}} \operatorname{Arc\,Sin} \frac{bx - 2a}{x\sqrt{(b^2 + 4ac)}}$$
$$= \frac{1}{\sqrt{a}} \operatorname{Arc\,Cos} \frac{2\sqrt{aX}}{x\sqrt{(b^2 + 4ac)}} = \frac{1}{2\sqrt{a}} \operatorname{Arc\,Sin\,vers} \frac{2(bx - 2a)^2}{(b^2 + 4ac)x^2}.$$

Diese Kreisbogen verschwinden sämmtlich für $x = \frac{2a}{b}$. Für $x = o$ können sie nicht verschwinden.

Taf. LXVI.

$$\int \frac{x^m \partial x}{(a+bx+cx^2)^{\frac{3}{2}}}$$

VZ. $a+bx+cx^2 = X$, $4ac-b^2=k$

$$\int \frac{\partial x}{X^{\frac{3}{2}}} = \frac{2(2cx+b)}{k\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{3}{2}}} = -\frac{2(2a+bx)}{k\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X^{\frac{3}{2}}} = -\frac{(4ac-2b^2)x-2ab}{ck\sqrt{X}} + \frac{1}{c}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X^{\frac{3}{2}}} = \frac{x^2}{c\sqrt{X}} - \frac{2a}{c}\int \frac{x\partial x}{X^{\frac{3}{2}}} - \frac{3b}{2c}\int \frac{x^2\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{x^4\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^3}{2c} - \frac{5bx^2}{4c^2}\right)\frac{1}{\sqrt{X}} + \frac{5ab}{2c^2}\int \frac{x\partial x}{X^{\frac{3}{2}}} + \left(\frac{15b^2}{8c^2} - \frac{3a}{2c}\right)\int \frac{x^2\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{x^5\partial x}{X^{\frac{3}{2}}} = \left[\frac{x^4}{3c} - \frac{7bx^3}{12c^2} + \left(\frac{35b^2}{24c^3} - \frac{4a}{3c^2}\right)x^2\right]\frac{1}{\sqrt{X}} - \left(\frac{35ab^2}{12c^3} - \frac{8a^2}{3c^2}\right)\int \frac{x\partial x}{X^{\frac{3}{2}}}$$
$$- \left(\frac{35b^3}{16c^3} - \frac{15ab}{4c^2}\right)\int \frac{x^2\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{x^6\partial x}{X^{\frac{3}{2}}} = \left[\frac{x^5}{4c} - \frac{3bx^4}{8c^2} + \left(\frac{21b^2}{32c^3} - \frac{5a}{8c^2}\right)x^3 - \left(\frac{105b^3}{64c^4} - \frac{49ab}{16c^3}\right)x^2\right]\frac{1}{\sqrt{X}}$$
$$+ \left(\frac{105ab^3}{32c^4} - \frac{49a^2b}{8c^3}\right)\int \frac{x\partial x}{X^{\frac{3}{2}}} + \left(\frac{315b^4}{128c^4} - \frac{105ab^2}{16c^3} + \frac{15a^2}{8c^2}\right)\int \frac{x^2\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{x^7\partial x}{X^{\frac{3}{2}}} = \frac{x^6}{5c\sqrt{X}} - \frac{6a}{5c}\int \frac{x^5\partial x}{X^{\frac{3}{2}}} - \frac{11b}{10c}\int \frac{x^6\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{x^8\partial x}{X^{\frac{3}{2}}} = \left(\frac{x^7}{6c} - \frac{13bx^6}{60c^2}\right)\frac{1}{\sqrt{X}} + \frac{13ab}{10c^2}\int \frac{x^5\partial x}{X^{\frac{3}{2}}} + \left(\frac{143b^2}{120c^2} - \frac{7a}{6c}\right)\int \frac{x^6\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{x^9\partial x}{X^{\frac{3}{2}}} = \left[\frac{x^8}{7c} - \frac{5bx^7}{28c^2} + \left(\frac{13b^2}{56c^3} - \frac{8a}{35c^2}\right)x^6\right]\frac{1}{\sqrt{X}} - \left(\frac{39ab^2}{28c^3} - \frac{48a^2}{35c^2}\right)\int \frac{x^5\partial x}{X^{\frac{3}{2}}}$$
$$- \left(\frac{143b^3}{112c^3} - \frac{351ab}{140c^2}\right)\int \frac{x^6\partial x}{X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^m(a+bx+cx^2)^{\frac{3}{2}}}$$

Taf. LXVII.

VZ. $a + bx + cx^2 = X$

$$\int \frac{\partial x}{xX^{\frac{3}{2}}} = \frac{1}{aVX} - \frac{b}{2a}\int\frac{\partial x}{X^{\frac{3}{2}}} + \frac{1}{a}\int\frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^2X^{\frac{3}{2}}} = \left(-\frac{1}{ax} - \frac{3b}{2a^2}\right)\frac{1}{VX} + \left(\frac{3b^2}{4a^2} - \frac{2c}{a}\right)\int\frac{\partial x}{X^{\frac{3}{2}}} - \frac{3b}{2a^2}\int\frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^3X^{\frac{3}{2}}} = \left(-\frac{1}{2ax^2} + \frac{5b}{4a^2x} + \frac{15b^2}{8a^3} - \frac{3c}{2a^2}\right)\frac{1}{VX} - \left(\frac{15b^3}{16a^3} - \frac{13bc}{4a^2}\right)\int\frac{\partial x}{X^{\frac{3}{2}}}$$
$$+ \left(\frac{15b^2}{8a^3} - \frac{3c}{2a^2}\right)\int\frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^4X^{\frac{3}{2}}} = \left[-\frac{1}{3ax^3} + \frac{7b}{12a^2x^2} - \left(\frac{35b^2}{24a^3} - \frac{4c}{3a^2}\right)\frac{1}{x} - \left(\frac{35b^3}{16a^4} - \frac{15bc}{4a^3}\right)\right]\frac{1}{VX}$$
$$+ \left(\frac{35b^4}{32a^4} - \frac{115b^2c}{24a^3} + \frac{8c^2}{3a^2}\right)\int\frac{\partial x}{X^{\frac{3}{2}}} - \left(\frac{35b^3}{16a^4} - \frac{15bc}{4a^3}\right)\int\frac{\partial x}{xVX}$$

$$\int \frac{\partial x}{x^5X^{\frac{3}{2}}} = -\frac{1}{4ax^4VX} - \frac{9b}{8a}\int\frac{\partial x}{x^4X^{\frac{3}{2}}} - \frac{5c}{4a}\int\frac{\partial x}{x^3X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{3}{2}}} = \left(-\frac{1}{5ax^5} + \frac{11b}{40a^2x^4}\right)\frac{1}{VX} + \left(\frac{99b^2}{80a^2} - \frac{6c}{5a}\right)\int\frac{\partial x}{x^4X^{\frac{3}{2}}}$$
$$+ \frac{11bc}{8a^2}\int\frac{\partial x}{x^3X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{3}{2}}} = \left[-\frac{1}{6ax^6} + \frac{13b}{60a^2x^5} - \left(\frac{143b^2}{480a^3} - \frac{7c}{24a^2}\right)\frac{1}{x^4}\right]\frac{1}{VX}$$
$$-\left(\frac{429b^3}{320a^3} - \frac{209bc}{80a^2}\right)\int\frac{\partial x}{x^4X^{\frac{3}{2}}} - \left(\frac{143b^2c}{96a^3} - \frac{35c^2}{24a^2}\right)\int\frac{\partial x}{x^4X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{3}{2}}} = -\frac{1}{7ax^7VX} - \frac{15b}{14a}\int\frac{\partial x}{x^7X^{\frac{3}{2}}} - \frac{8c}{7a}\int\frac{\partial x}{x^6X^{\frac{3}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{3}{2}}} = \left(-\frac{1}{8ax^8} + \frac{17b}{112a^2x^7}\right)\frac{1}{VX} + \left(\frac{255b^2}{224a^2} - \frac{9c}{8a}\right)\int\frac{\partial x}{x^7X^{\frac{3}{2}}}$$
$$+ \frac{17bc}{14a^2}\int\frac{\partial x}{x^6X^{\frac{3}{2}}}$$

Taf. LXVIII.
$$\int \frac{x^m \partial x}{(a+bx+cx^2)^{\frac{5}{2}}}$$

VZ. $a+bx+cx^2 = X,\ 4ac - b^2 = k$

$$\int \frac{\partial x}{X^{\frac{5}{2}}} = \left(\frac{1}{3kX} + \frac{8c}{3k^2}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{5}{2}}} = -\frac{1}{3cX\sqrt{X}} - \frac{b}{2c}\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^2\partial x}{X^{\frac{5}{2}}} = \left(-\frac{x}{2c} + \frac{b}{12c^2}\right)\frac{1}{X\sqrt{X}} + \left(\frac{b^2}{8c^2} + \frac{a}{2c}\right)\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^3\partial x}{X^{\frac{5}{2}}} = \left(-\frac{x^2}{c} - \frac{bx}{4c^2} + \frac{b^2}{24c^3} - \frac{2a}{3c^2}\right)\frac{1}{X\sqrt{X}} + \left(\frac{b^3}{16c^3} - \frac{3ab}{4c^2}\right)\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^4\partial x}{X^{\frac{5}{2}}} = \frac{1}{c}\int\frac{x^2\partial x}{X^{\frac{3}{2}}} - \frac{a}{c}\int\frac{x^2\partial x}{X^{\frac{5}{2}}} - \frac{b}{c}\int\frac{x^3\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^5\partial x}{X^{\frac{5}{2}}} = \frac{x^4}{cX\sqrt{X}} - \frac{4a}{c}\int\frac{x^3\partial x}{X^{\frac{5}{2}}} - \frac{5b}{2c}\int\frac{x^4\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^6\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^5}{2c} - \frac{7bx^4}{4c^2}\right)\frac{1}{X\sqrt{X}} + \frac{7ab}{c^2}\int\frac{x^3\partial x}{X^{\frac{5}{2}}} + \left(\frac{35b^2}{8c^2} - \frac{5a}{2c}\right)\int\frac{x^4\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^7\partial x}{X^{\frac{5}{2}}} = \left[\frac{x^6}{3c} - \frac{3bx^5}{4c^2} + \left(\frac{21b^2}{8c^3} - \frac{2a}{c^2}\right)x^4\right]\frac{1}{X\sqrt{X}} - \left(\frac{21ab^2}{2c^3} - \frac{8a^2}{c^2}\right)\int\frac{x^3\partial x}{X^{\frac{5}{2}}}$$
$$- \left(\frac{105b^3}{16c^3} - \frac{35ab}{4c^2}\right)\int\frac{x^4\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^8\partial x}{X^{\frac{5}{2}}} = \frac{x^7}{4cX\sqrt{X}} - \frac{7a}{4c}\int\frac{x^6\partial x}{X^{\frac{5}{2}}} - \frac{11b}{8c}\int\frac{x^7\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{x^9\partial x}{X^{\frac{5}{2}}} = \left(\frac{x^8}{5c} - \frac{13bx^7}{40c^2}\right)\frac{1}{X\sqrt{X}} + \frac{91ab}{40c^2}\int\frac{x^6\partial x}{X^{\frac{5}{2}}} + \left(\frac{143b^2}{80c^2} - \frac{8a}{5c}\right)\int\frac{x^7\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^m(a+bx+cx^2)^{\frac{5}{2}}}$$

Taf. LXIX.

VZ. $a+bx+cx^2 = X$

$$\int \frac{\partial x}{xX^{\frac{5}{2}}} = \left(\frac{1}{3aX}+\frac{1}{a^2}\right)\frac{1}{\sqrt{X}} - \frac{b}{2a}\int\frac{\partial x}{X^{\frac{5}{2}}} - \frac{b}{2a^2}\int\frac{\partial x}{X^{\frac{3}{2}}} + \frac{1}{a^2}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{5}{2}}} = -\frac{1}{axX\sqrt{X}} - \frac{5b}{2a}\int\frac{\partial x}{xX^{\frac{5}{2}}} - \frac{4c}{a}\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{5}{2}}} = \left(-\frac{1}{2ax^2}+\frac{7b}{4a^2x}\right)\frac{1}{X\sqrt{X}} + \left(\frac{35b^2}{8a^2}-\frac{5c}{2a}\right)\int\frac{\partial x}{xX^{\frac{5}{2}}} + \frac{7bc}{a^2}\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{5}{2}}} = \left[-\frac{1}{3ax^3}+\frac{3b}{4a^2x^2}-\left(\frac{21b^2}{8a^3}-\frac{2c}{a^2}\right)\frac{1}{x}\right]\frac{1}{X\sqrt{X}} - \left(\frac{105b^3}{16a^3}-\frac{35bc}{4a^2}\right)\int\frac{\partial x}{xX^{\frac{5}{2}}} - \left(\frac{21b^2c}{2a^3}-\frac{8c^2}{a^2}\right)\int\frac{\partial x}{X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{5}{2}}} = -\frac{1}{4ax^4X\sqrt{X}} - \frac{11b}{8a}\int\frac{\partial x}{x^4X^{\frac{5}{2}}} - \frac{7c}{4a}\int\frac{\partial x}{x^3X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{5}{2}}} = \left(-\frac{1}{5ax^5}+\frac{13b}{40a^2x^4}\right)\frac{1}{X\sqrt{X}} + \left(\frac{143b^2}{80a^2}-\frac{8c}{5a}\right)\int\frac{\partial x}{x^4X^{\frac{5}{2}}} + \frac{91bc}{40a^2}\int\frac{\partial x}{x^3X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{5}{2}}} = \left[-\frac{1}{6ax^6}+\frac{b}{4a^2x^5}-\left(\frac{13b^2}{32a^3}-\frac{3c}{8a^2}\right)\frac{1}{x^4}\right]\frac{1}{X\sqrt{X}} - \left(\frac{143b^3}{64a^3}-\frac{65bc}{16a^2}\right)\int\frac{\partial x}{x^4X^{\frac{5}{2}}} - \left(\frac{91b^2c}{64a^3}-\frac{21c^2}{8a^2}\right)\int\frac{\partial x}{x^3X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{5}{2}}} = -\frac{1}{7ax^7X\sqrt{X}} - \frac{17b}{14a}\int\frac{\partial x}{x^7X^{\frac{5}{2}}} - \frac{10c}{7a}\int\frac{\partial x}{x^6X^{\frac{5}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{5}{2}}} = \left(-\frac{1}{8ax^8}+\frac{19b}{112a^2x^7}\right)\frac{1}{X\sqrt{X}} + \left(\frac{323b^2}{224a^2}-\frac{11c}{8a}\right)\int\frac{\partial x}{x^7X^{\frac{5}{2}}} + \frac{95bc}{56a^2}\int\frac{\partial x}{x^6X^{\frac{5}{2}}}$$

Taf. **LXX.** $$\int \frac{x^m \partial x}{(a+bx+cx^2)^{\frac{7}{2}}}$$

VZ. $a+bx+cx^2=X,\ 4ac-b^2=k$

$$\int\frac{\partial x}{X^{\frac{7}{2}}}=\left(\frac{1}{5kX^2}+\frac{16c}{15k^2X}+\frac{128c^2}{15k^3}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int\frac{x\partial x}{X^{\frac{7}{2}}}=-\frac{1}{5cX^2\sqrt{X}}-\frac{b}{2c}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int\frac{x^2\partial x}{X^{\frac{7}{2}}}=\left(-\frac{x}{4c}+\frac{3b}{40c^2}\right)\frac{1}{X^2\sqrt{X}}+\left(\frac{3b^2}{16c^2}+\frac{a}{4c}\right)\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int\frac{x^3\partial x}{X^{\frac{7}{2}}}=\left(-\frac{x^2}{3c}+\frac{bx}{24c^2}-\frac{b^2}{80c^3}-\frac{2a}{15c^2}\right)\frac{1}{X^2\sqrt{X}}-\left(\frac{b^3}{32c^3}+\frac{3ab}{8c^2}\right)\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int\frac{x^4\partial x}{X^{\frac{7}{2}}}=\left[-\frac{x^3}{2c}-\frac{bx^2}{12c^2}+\left(\frac{b^2}{96c^3}-\frac{3a}{8c^2}\right)x-\frac{b^3}{320c^4}+\frac{19ab}{240c^3}\right]\frac{1}{X^2\sqrt{X}}$$
$$-\left(\frac{b^4}{128c^4}-\frac{3ab^2}{16c^3}-\frac{3a^2}{8c^2}\right)\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int\frac{x^5\partial x}{X^{\frac{7}{2}}}=\left[-\frac{x^4}{c}-\frac{3bx^3}{4c^2}-\left(\frac{b^2}{8c^3}+\frac{4a}{3c^2}\right)x^2+\left(\frac{b^3}{64c^4}-\frac{19ab}{48c^3}\right)x\right.$$
$$\left.-\frac{3b^4}{640c^5}+\frac{11ab^2}{160c^4}-\frac{8a^2}{15c^3}\right]\frac{1}{X^2\sqrt{X}}-\left(\frac{3b^5}{256c^5}-\frac{5ab^3}{32c^4}+\frac{15a^2b}{16c^3}\right)\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int\frac{x^6\partial x}{X^{\frac{7}{2}}}=\frac{1}{c}\int\frac{x^4\partial x}{X^{\frac{5}{2}}}-\frac{a}{c}\int\frac{x^4\partial x}{X^{\frac{7}{2}}}-\frac{b}{c}\int\frac{x^5\partial x}{X^{\frac{7}{2}}}$$

$$\int\frac{x^7\partial x}{X^{\frac{7}{2}}}=\frac{x^6}{cX^2\sqrt{X}}-\frac{6a}{c}\int\frac{x^5\partial x}{X^{\frac{7}{2}}}-\frac{7b}{2c}\int\frac{x^6\partial x}{X^{\frac{7}{2}}}$$

$$\int\frac{x^8\partial x}{X^{\frac{7}{2}}}=\left(\frac{x^7}{2c}-\frac{9bx^6}{4c^2}\right)\frac{1}{X^2\sqrt{X}}+\frac{27ab}{2c^2}\int\frac{x^5\partial x}{X^{\frac{7}{2}}}+\left(\frac{63b^2}{8c^2}-\frac{7a}{2c}\right)\int\frac{x^6\partial x}{X^{\frac{7}{2}}}$$

$$\int\frac{x^9\partial x}{X^{\frac{7}{2}}}=\left[\frac{x^8}{3c}-\frac{11bx^7}{12c^2}+\left(\frac{33b^2}{8c^3}-\frac{8a}{3c^2}\right)x^6\right]\frac{1}{X^2\sqrt{X}}$$
$$-\left(\frac{231b^3}{16c^3}-\frac{63ab}{4c^2}\right)\int\frac{x^6\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^m(a+bx+cx^2)^{\frac{7}{2}}}$$ **Taf. LXXI.**

VZ. $a+bx+cx^2 = X$

$$\int \frac{\partial x}{xX^{\frac{7}{2}}} = \left(\frac{1}{5aX^2} + \frac{1}{3a^2X} + \frac{1}{a^3}\right)\frac{1}{\sqrt{X}} - \frac{b}{2a}\int\frac{\partial x}{X^{\frac{7}{2}}} - \frac{b}{2a^2}\int\frac{\partial x}{X^{\frac{5}{2}}} - \frac{b}{2a^3}\int\frac{\partial x}{X^{\frac{3}{2}}} + \frac{1}{a^3}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{7}{2}}} = -\frac{1}{axX^2\sqrt{X}} - \frac{7b}{2a}\int\frac{\partial x}{xX^{\frac{7}{2}}} - \frac{6c}{a}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{7}{2}}} = \left(-\frac{1}{2ax^2} + \frac{9b}{4a^2x}\right)\frac{1}{X^2\sqrt{X}} + \left(\frac{63b^2}{8a^2} - \frac{7c}{2a}\right)\int\frac{\partial x}{xX^{\frac{7}{2}}} + \frac{27bc}{2a^2}\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{7}{2}}} = \left[-\frac{1}{3ax^3} + \frac{11b}{12a^2x^2} - \left(\frac{33b^2}{8a^3} - \frac{8c}{3a^2}\right)\frac{1}{x}\right]\frac{1}{X^2\sqrt{X}} - \left(\frac{231b^3}{16a^3} - \frac{63bc}{4a^2}\right)\int\frac{\partial x}{xX^{\frac{7}{2}}} - \left(\frac{99b^2c}{4a^3} - \frac{16c^2}{a^2}\right)\int\frac{\partial x}{X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^5X^{\frac{7}{2}}} = -\frac{1}{4ax^4X^2\sqrt{X}} - \frac{13b}{8a}\int\frac{\partial x}{x^4X^{\frac{7}{2}}} - \frac{9c}{4a}\int\frac{\partial x}{x^3X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^6X^{\frac{7}{2}}} = \left(-\frac{1}{5ax^5} + \frac{3b}{8a^2x^4}\right)\frac{1}{X^2\sqrt{X}} + \left(\frac{39b^2}{16a^2} - \frac{2c}{a}\right)\int\frac{\partial x}{x^4X^{\frac{7}{2}}} + \frac{27bc}{8a^2}\int\frac{\partial x}{x^3X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^7X^{\frac{7}{2}}} = \left[-\frac{1}{6ax^6} + \frac{17b}{60a^2x^5} - \left(\frac{17b^2}{32a^3} - \frac{11c}{24a^2}\right)\frac{1}{x^4}\right]\frac{1}{X^2\sqrt{X}} - \left(\frac{221b^3}{64a^3} - \frac{93bc}{16a^2}\right)\int\frac{\partial x}{x^4X^{\frac{7}{2}}} - \left(\frac{153b^2c}{32a^3} - \frac{33c^2}{8a^2}\right)\int\frac{\partial x}{x^3X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^8X^{\frac{7}{2}}} = -\frac{1}{7ax^7X^2\sqrt{X}} - \frac{19b}{14a}\int\frac{\partial x}{x^7X^{\frac{7}{2}}} - \frac{12c}{7a}\int\frac{\partial x}{x^6X^{\frac{7}{2}}}$$

$$\int \frac{\partial x}{x^9X^{\frac{7}{2}}} = \left(-\frac{1}{8ax^8} + \frac{3b}{16a^2x^7}\right)\frac{1}{X^2\sqrt{X}} + \left(\frac{57b^2}{32a^2} - \frac{13c}{8a}\right)\int\frac{\partial x}{x^7X^{\frac{7}{2}}} + \frac{9bc}{4a^2}\int\frac{\partial x}{x^6X^{\frac{7}{2}}}$$

Taf. LXXII. $\int \frac{x^m \partial x}{(a+bx+cx^2)^{\frac{9}{2}}}, \int \frac{\partial x}{x^m (a+bx+cx^2)^{\frac{9}{2}}}$

VZ. $a+bx+cx^2 = X, \; 4ac - b^2 = k$

$$\int \frac{\partial x}{X^{\frac{9}{2}}} = \left(\frac{1}{7kX^3} + \frac{24c}{35k^2X^2} + \frac{128c^2}{35k^3X} + \frac{1024c^3}{35k^4}\right)\frac{2(2cx+b)}{\sqrt{X}}$$

$$\int \frac{x\partial x}{X^{\frac{9}{2}}} = -\frac{1}{7cX^3\sqrt{X}} - \frac{b}{2c}\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{x^2\partial x}{X^{\frac{9}{2}}} = \left(-\frac{x}{6c} + \frac{5b}{84c^2}\right)\frac{1}{X^3\sqrt{X}} + \left(\frac{5b^2}{24c^2} + \frac{a}{6c}\right)\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{x^3\partial x}{X^{\frac{9}{2}}} = \left(-\frac{x^2}{5c} + \frac{bx}{20c^2} - \frac{b^2}{56c^3} - \frac{2a}{35c^2}\right)\frac{1}{X^3\sqrt{X}} - \left(\frac{b^3}{16c^3} + \frac{ab}{4c^2}\right)\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{x^4\partial x}{X^{\frac{9}{2}}} = \left[-\frac{x^3}{4c} + \frac{bx^2}{40c^2} - \left(\frac{b^2}{160c^3} + \frac{a}{8c^2}\right)x + \frac{b^3}{448c^4} + \frac{29ab}{560c^3}\right]\frac{1}{X^3\sqrt{X}}$$
$$+ \left(\frac{b^4}{128c^4} + \frac{3ab^2}{16c^3} + \frac{a^2}{8c^2}\right)\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{x^5\partial x}{X^{\frac{9}{2}}} = -\frac{x^4}{3cX^3\sqrt{X}} + \frac{4a}{3c}\int \frac{x^3\partial x}{X^{\frac{9}{2}}} + \frac{b}{6c}\int \frac{x^4\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{xX^{\frac{9}{2}}} = \left(\frac{1}{7aX^3} + \frac{1}{5a^2X^2} + \frac{1}{3a^3X} + \frac{1}{a^4}\right)\frac{1}{\sqrt{X}} - \frac{b}{2a}\int \frac{\partial x}{X^{\frac{9}{2}}}$$
$$- \frac{b}{2a^2}\int \frac{\partial x}{X^{\frac{7}{2}}} - \frac{b}{2a^3}\int \frac{\partial x}{X^{\frac{5}{2}}} - \frac{b}{2a^4}\int \frac{\partial x}{X^{\frac{3}{2}}} + \frac{1}{a^4}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{x^2X^{\frac{9}{2}}} = -\frac{1}{axX^3\sqrt{X}} - \frac{9b}{2a}\int \frac{\partial x}{xX^{\frac{9}{2}}} - \frac{8c}{a}\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^3X^{\frac{9}{2}}} = \left(-\frac{1}{2ax^2} + \frac{11b}{4a^2x}\right)\frac{1}{X^3\sqrt{X}} + \left(\frac{99b^2}{8a^2} - \frac{9c}{2a}\right)\int \frac{\partial x}{xX^{\frac{9}{2}}}$$
$$+ \frac{22bc}{a^2}\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int \frac{\partial x}{x^4X^{\frac{9}{2}}} = \left[-\frac{1}{3ax^3} + \frac{13b}{12a^2x^2} - \left(\frac{143b^2}{24a^3} - \frac{10c}{3a^2}\right)\frac{1}{x}\right]\frac{1}{X^3\sqrt{X}}$$
$$-\left(\frac{429b^3}{16a^3} - \frac{99bc}{4a^2}\right)\int \frac{\partial x}{xX^{\frac{9}{2}}} - \left(\frac{143b^2c}{3a^3} - \frac{80c^2}{3a^2}\right)\int \frac{\partial x}{X^{\frac{9}{2}}}$$

$$\int x^m \partial x (a + bx + cx^2)^{\frac{3}{2}} \qquad \textit{Taf. LXXV.}$$

VZ. $a + bx + cx^2 = X, \; 4ac - b^2 = k$

$$\int \partial x X^{\frac{3}{2}} = \left(\frac{X}{8c} + \frac{3k}{64c^2}\right)(2cx + b)\sqrt{X} + \frac{3k^2}{128c^2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int x \partial x X^{\frac{3}{2}} = \frac{X^2 \sqrt{X}}{5c} - \frac{b}{2c}\int \partial x X^{\frac{3}{2}}$$

$$\int x^2 \partial x X^{\frac{3}{2}} = \left(\frac{x}{6c} - \frac{7b}{60c^2}\right) X^2 \sqrt{X} + \left(\frac{7b^2}{24c^2} - \frac{a}{6c}\right)\int \partial x X^{\frac{3}{2}}$$

$$\int x^3 \partial x X^{\frac{3}{2}} = \left(\frac{x^2}{7c} - \frac{3bx}{28c^2} + \frac{3b^2}{40c^3} - \frac{2a}{35c^2}\right) X^2 \sqrt{X} - \left(\frac{3b^3}{16c^3} - \frac{ab}{4c^2}\right)\int \partial x X^{\frac{3}{2}}$$

$$\int x^4 \partial x X^{\frac{3}{2}} = \left[\frac{x^3}{8c} - \frac{11bx^2}{112c^2} + \left(\frac{33b^2}{448c^3} - \frac{a}{16c^2}\right)x - \frac{33b^3}{640c^4} + \frac{93ab}{1120c^3}\right] X^2 \sqrt{X} + \left(\frac{33b^4}{256c^4} - \frac{9ab^2}{32c^3} + \frac{a^2}{16c^2}\right)\int \partial x X^{\frac{3}{2}}$$

$$\int x^5 \partial x X^{\frac{3}{2}} = \frac{x^4 X^2 \sqrt{X}}{9c} - \frac{4a}{9c}\int x^3 \partial x X^{\frac{3}{2}} - \frac{13b}{18c}\int x^4 \partial x X^{\frac{3}{2}}$$

$$\int x^6 \partial x X^{\frac{3}{2}} = \left(\frac{x^5}{10c} - \frac{bx^4}{12c^2}\right) X^2 \sqrt{X} + \frac{ab}{3c^2}\int x^3 \partial x X^{\frac{3}{2}} + \left(\frac{13b^2}{24c^2} - \frac{a}{2c}\right)\int x^4 \partial x X^{\frac{3}{2}}$$

$$\int x^7 \partial x X^{\frac{3}{2}} = \left[\frac{x^6}{11c} - \frac{17bx^5}{220c^2} + \left(\frac{17b^2}{264c^3} - \frac{2a}{33c^2}\right)x^4\right] X^2 \sqrt{X} - \left(\frac{17ab^2}{66c^3} - \frac{8a^2}{33c^2}\right)\int x^3 \partial x X^{\frac{3}{2}} - \left(\frac{221b^3}{264c^3} - \frac{103ab}{132c^2}\right)\int x^4 \partial x X^{\frac{3}{2}}$$

$$\int x^8 \partial x X^{\frac{3}{2}} = \frac{x^7 X^2 \sqrt{X}}{12c} - \frac{7a}{12c}\int x^6 \partial x X^{\frac{3}{2}} - \frac{19b}{24c}\int x^7 \partial x X^{\frac{3}{2}}$$

$$\int x^9 \partial x X^{\frac{3}{2}} = \left(\frac{x^8}{13c} - \frac{7bx^7}{104c^2}\right) X^2 \sqrt{X} + \frac{49ab}{104c^2}\int x^6 \partial x X^{\frac{3}{2}} + \left(\frac{133b^2}{208c^2} - \frac{8a}{13c}\right)\int x^7 \partial x X^{\frac{3}{2}}$$

Taf. LXXIV. $$\int \frac{\partial x \sqrt{(a+bx+cx^2)}}{x^m}$$

VZ. $a + bx + cx^2 = X$

$$\int \frac{\partial x \sqrt{X}}{x} = \sqrt{X} + a\int \frac{\partial x}{x\sqrt{X}} + \frac{b}{2}\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^2} = -\frac{\sqrt{X}}{x} + \frac{b}{2}\int \frac{\partial x}{x\sqrt{X}} + c\int \frac{\partial x}{\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^3} = -\left(\frac{1}{2x^2} + \frac{b}{4ax}\right)\sqrt{X} - \left(\frac{b^2}{8a} - \frac{c}{2}\right)\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^4} = -\frac{X\sqrt{X}}{3ax^3} + \left(\frac{b}{4ax^2} + \frac{b^2}{8a^2x}\right)\sqrt{X} + \left(\frac{b^3}{16a^2} - \frac{bc}{4a}\right)\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^5} = \left(-\frac{1}{4ax^4} + \frac{5b}{24a^2x^3}\right)X\sqrt{X} - \left[\left(\frac{5b^2}{32a^2} - \frac{c}{8a}\right)\frac{1}{x^2} + \left(\frac{5b^3}{64a^3} - \frac{bc}{16a^2}\right)\frac{1}{x}\right]\sqrt{X} - \left(\frac{5b^4}{128a^3} - \frac{3b^2c}{16a^2} + \frac{c^2}{8a}\right)\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^6} = -\frac{X\sqrt{X}}{5ax^5} - \frac{7b}{10a}\int \frac{\partial x \sqrt{X}}{x^5} - \frac{2c}{5a}\int \frac{\partial x \sqrt{X}}{x^4}$$

$$\int \frac{\partial x \sqrt{X}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{3b}{20a^2x^5}\right)X\sqrt{X} + \left(\frac{21b^2}{40a^2} - \frac{c}{2a}\right)\int \frac{\partial x \sqrt{X}}{x^5} + \frac{3bc}{10a^2}\int \frac{\partial x \sqrt{X}}{x^4}$$

$$\int \frac{\partial x \sqrt{X}}{x^8} = \left[-\frac{1}{7ax^7} + \frac{11b}{84a^2x^6} - \left(\frac{33b^2}{280a^3} - \frac{4c}{35a^2}\right)\frac{1}{x^5}\right]X\sqrt{X} - \left(\frac{33b^3}{80a^3} - \frac{111bc}{140a^2}\right)\int \frac{\partial x \sqrt{X}}{x^5} - \left(\frac{33b^2c}{140a^3} - \frac{8c^2}{35a^2}\right)\int \frac{\partial x \sqrt{X}}{x^4}$$

$$\int \frac{\partial x \sqrt{X}}{x^9} = -\frac{X\sqrt{X}}{8ax^8} - \frac{13b}{16a}\int \frac{\partial x \sqrt{X}}{x^8} - \frac{5c}{8a}\int \frac{\partial x}{x^7\sqrt{X}}$$

$$\int \frac{\partial x \sqrt{X}}{x^{10}} = \left(-\frac{1}{9ax^9} + \frac{5b}{48a^2x^8}\right)X\sqrt{X} + \left(\frac{65b^2}{96a^2} - \frac{2c}{3a}\right)\int \frac{\partial x \sqrt{X}}{x^8} + \frac{25bc}{48a^2}\int \frac{\partial x}{x^7\sqrt{X}}$$

$\int x^m \partial x (a+bx+cx^2)^{\frac{5}{2}}$ Taf. LXXVII.

VZ. $a+bx+cx^2 = X,\ 4ac-b^2 = k$

$$\int \partial x X^{\frac{5}{2}} = \left(\frac{X^2}{12c}+\frac{5kX}{192c^2}+\frac{5k^2}{512c^3}\right)(2cx+b)\sqrt{X}+\frac{5k^3}{1024c^3}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{5}{2}} = \frac{X^3\sqrt{X}}{7c}-\frac{b}{2c}\int\partial x X^{\frac{5}{2}}$$

$$\int x^2\partial x X^{\frac{5}{2}} = \left(\frac{x}{8c}-\frac{9b}{112c^2}\right)X^3\sqrt{X}+\left(\frac{9b^2}{32c^2}-\frac{a}{8c}\right)\int\partial x X^{\frac{5}{2}}$$

$$\int x^3\partial x X^{\frac{5}{2}} = \left(\frac{x^2}{9c}-\frac{11bx}{144c^2}+\frac{11b^2}{224c^3}-\frac{2a}{63c^2}\right)X^3\sqrt{X}$$
$$-\left(\frac{11b^3}{64c^3}-\frac{3ab}{16c^2}\right)\int\partial x X^{\frac{5}{2}}$$

$$\int x^4\partial x X^{\frac{5}{2}} = \left[\frac{x^3}{10c}-\frac{13bx^2}{180c^2}+\left(\frac{143b^2}{2880c^3}-\frac{3a}{80c^2}\right)x-\frac{143b^3}{4480c^4}\right.$$
$$\left.+\frac{451ab}{10080c^3}\right]X^3\sqrt{X}+\left(\frac{143b^4}{1280c^4}-\frac{33ab^2}{160c^3}+\frac{3a^2}{80c^2}\right)\int\partial x X^{\frac{5}{2}}$$

$$\int x^5\partial x X^{\frac{5}{2}} = \frac{x^4X^3\sqrt{X}}{11c}-\frac{4a}{11c}\int x^3\partial x X^{\frac{5}{2}}-\frac{15b}{22c}\int x^4\partial x X^{\frac{5}{2}}$$

$$\int x^6\partial x X^{\frac{5}{2}} = \left(\frac{x^5}{12c}-\frac{17bx^4}{264c^2}\right)X^3\sqrt{X}+\frac{17ab}{66c^2}\int x^3\partial x X^{\frac{5}{2}}$$
$$+\left(\frac{85b^2}{176c^2}-\frac{5a}{12c}\right)\int x^4\partial x X^{\frac{5}{2}}$$

$$\int x^7\partial x X^{\frac{5}{2}} = \left[\frac{x^6}{13c}-\frac{19bx^5}{312c^2}+\left(\frac{323b^2}{6864c^3}-\frac{6a}{143c^2}\right)x^4\right]X^3\sqrt{X}$$
$$-\left(\frac{323ab^2}{1716c^3}-\frac{24a^2}{143c^2}\right)\int x^3\partial x X^{\frac{5}{2}}-\left(\frac{1615b^3}{4576c^3}-\frac{2125ab}{3432c^2}\right)\int x^4\partial x X^{\frac{5}{2}}$$

$$\int x^8\partial x X^{\frac{5}{2}} = \frac{x^7X^3\sqrt{X}}{14c}-\frac{a}{2c}\int x^6\partial x X^{\frac{5}{2}}-\frac{3b}{4c}\int x^7\partial x X^{\frac{5}{2}}$$

$$\int x^9\partial x X^{\frac{5}{2}} = \left(\frac{x^8}{15c}-\frac{23bx^7}{420c^2}\right)X^3\sqrt{X}+\frac{23ab}{60c^2}\int x^6\partial x X^{\frac{5}{2}}$$
$$+\left(\frac{23b^2}{40c^2}-\frac{8a}{15c}\right)\int x^7\partial x X^{\frac{5}{2}}$$

Taf. LXXVI. $$\int \frac{\partial x(a+bx+cx^2)^{\frac{3}{2}}}{x^m}$$

VZ. $a+bx+cx^2 = X$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x} = \left(\frac{X}{3}+a\right)\sqrt{X} + a^2\int\frac{\partial x}{x\sqrt{X}} + \frac{ab}{2}\int\frac{\partial x}{\sqrt{X}} + \frac{b}{2}\int\partial x\sqrt{X}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^2} = -\frac{X^2\sqrt{X}}{ax} + \frac{3b}{2a}\int\frac{\partial x X^{\frac{3}{2}}}{x} + \frac{4c}{a}\int\partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{b}{4a^2x}\right)X^2\sqrt{X} + \left(\frac{3b^2}{8a^2} + \frac{3c}{2a}\right)\int\frac{\partial x X^{\frac{3}{2}}}{x} + \frac{bc}{a^2}\int\partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^4} = \left[-\frac{1}{3ax^3} + \frac{b}{12a^2x^2} + \left(\frac{b^2}{24a^3} - \frac{2c}{3a^2}\right)\frac{1}{x}\right]X^2\sqrt{X} - \left(\frac{b^3}{16a^3} - \frac{3bc}{4a^2}\right)\int\frac{\partial x X^{\frac{3}{2}}}{x} - \left(\frac{b^2c}{6a^3} - \frac{8c^2}{3a^2}\right)\int\partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^5} = \left[-\frac{1}{4ax^4} + \frac{b}{8a^2x^3} - \left(\frac{b^2}{32a^3} + \frac{c}{8a^2}\right)\frac{1}{x^2} - \left(\frac{b^3}{64a^4} - \frac{3bc}{16a^3}\right)\frac{1}{x}\right]X^2\sqrt{X} + \left(\frac{3b^4}{128a^4} - \frac{3b^2c}{16a^3} + \frac{3c^2}{8a^2}\right)\int\frac{\partial x X^{\frac{3}{2}}}{x} + \left(\frac{b^3c}{16a^4} - \frac{3bc^2}{4a^3}\right)\int\partial x X^{\frac{3}{2}}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^6} = -\frac{X^2\sqrt{X}}{5ax^5} - \frac{b}{2a}\int\frac{\partial x X^{\frac{3}{2}}}{x^5}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{7b}{60a^2x^5}\right)X^2\sqrt{X} + \left(\frac{7b^2}{24a^2} - \frac{c}{6a}\right)\int\frac{\partial x X^{\frac{3}{2}}}{x^5}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^8} = \left[-\frac{1}{7ax^7} + \frac{3b}{28a^2x^6} - \left(\frac{3b^2}{40a^3} - \frac{2c}{35a^2}\right)\frac{1}{x^5}\right]X^2\sqrt{X} - \left(\frac{3b^3}{16a^3} - \frac{bc}{4a^2}\right)\int\frac{\partial x X^{\frac{3}{2}}}{x^5}$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^9} = \left[-\frac{1}{8ax^8} + \frac{11b}{112a^2x^7} - \left(\frac{33b^2}{448a^3} - \frac{c}{16a^2}\right)\frac{1}{x^6} + \left(\frac{33b^3}{640a^4} - \frac{93bc}{1120a^3}\right)\frac{1}{x^5}\right]X^2\sqrt{X} + \left(\frac{33b^4}{256a^4} - \frac{9b^2c}{32a^3} + \frac{c^2}{16a^2}\right)\int\frac{\partial x X^{\frac{3}{2}}}{x^5}$$

$\int x^m \partial x (a+bx+cx^2)^{\frac{7}{2}}$ Taf. LXXIX.

VZ. $a+bx+cx^2=X,\ 4ac-b^2=k$

$$\int \partial x X^{\frac{7}{2}} = \left(\frac{X^3}{16c}+\frac{7kX^2}{384c^2}+\frac{35k^2X}{6144c^3}+\frac{35k^3}{16384c^4}\right)(2cx+b)\sqrt{X} + \frac{35k^4}{32768c^4}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{7}{2}} = \frac{X^4\sqrt{X}}{9c}-\frac{b}{2c}\int\partial x X^{\frac{7}{2}}$$

$$\int x^2\partial x X^{\frac{7}{2}} = \left(\frac{x}{10c}-\frac{11b}{180c^2}\right)X^4\sqrt{X}+\left(\frac{11b^2}{40c^2}-\frac{a}{10c}\right)\int\partial x X^{\frac{7}{2}}$$

$$\int x^3\partial x X^{\frac{7}{2}} = \left(\frac{x^2}{11c}-\frac{13bx}{220c^2}+\frac{13b^2}{360c^3}-\frac{2a}{99c^2}\right)X^4\sqrt{X} - \left(\frac{13b^3}{80c^3}-\frac{3ab}{20c^2}\right)\int\partial x X^{\frac{7}{2}}$$

$$\int x^4\partial x X^{\frac{7}{2}} = \left[\frac{x^3}{12c}-\frac{5bx^2}{88c^2}+\left(\frac{13b^2}{352c^3}-\frac{a}{40c^2}\right)x-\frac{13b^3}{576c^4}+\frac{221ab}{7920c^3}\right]X^4\sqrt{X} + \left(\frac{13b^4}{128c^4}-\frac{13ab^2}{80c^3}+\frac{a^2}{40c^2}\right)\int\partial x X^{\frac{7}{2}}$$

$$\int x^5\partial x X^{\frac{7}{2}} = \frac{x^4X^4\sqrt{X}}{13c}-\frac{4a}{13c}\int x^3\partial x X^{\frac{7}{2}}-\frac{17b}{26c}\int x^4\partial x X^{\frac{7}{2}}$$

$$\int x^6\partial x X^{\frac{7}{2}} = \left(\frac{x^5}{14c}-\frac{19bx^4}{364c^2}\right)X^4\sqrt{X}+\frac{19ab}{91c^2}\int x^3\partial x X^{\frac{7}{2}} + \left(\frac{323b^2}{728c^2}-\frac{5a}{14c}\right)\int x^4\partial x X^{\frac{7}{2}}$$

$$\int x^7\partial x X^{\frac{7}{2}} = \left[\frac{x^6}{15c}-\frac{bx^5}{20c^2}+\left(\frac{19b^2}{520c^3}-\frac{2a}{65c^2}\right)x^4\right]X^4\sqrt{X} - \left(\frac{19ab^2}{130c^3}-\frac{8a^2}{65c^2}\right)\int x^3\partial x X^{\frac{7}{2}}-\left(\frac{323b^3}{1040c^3}-\frac{133ab}{260c^2}\right)\int x^4\partial x X^{\frac{7}{2}}$$

$$\int x^8\partial x X^{\frac{7}{2}} = \frac{x^7X^4\sqrt{X}}{16c}-\frac{7a}{16c}\int x^6\partial x X^{\frac{7}{2}}-\frac{23b}{32c}\int x^7\partial x X^{\frac{7}{2}}$$

$$\int x^9\partial x X^{\frac{7}{2}} = \left(\frac{x^8}{17c}-\frac{25bx^7}{544c^2}\right)X^4\sqrt{X}+\frac{175ab}{544c^2}\int x^6\partial x X^{\frac{7}{2}} + \left(\frac{575b^2}{1088c^2}-\frac{8a}{17c}\right)\int x^7\partial x X^{\frac{7}{2}}$$

Taf. LXXVIII.

$$\int\frac{\partial x(a+bx+cx^2)^{\frac{5}{2}}}{x^m}$$

VZ. $a+bx+cx^2=X$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x}=\left(\frac{X^2}{5}+\frac{aX}{3}+a^2\right)\sqrt{X}+a^3\int\frac{\partial x}{x\sqrt{X}}+\frac{a^2b}{2}\int\frac{\partial x}{\sqrt{X}}$$
$$+\frac{ab}{2}\int\partial x\sqrt{X}+\frac{b}{2}\int\partial x X^{\frac{3}{2}}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^2}=-\frac{X^3\sqrt{X}}{ax}+\frac{5b}{2a}\int\frac{\partial x X^{\frac{5}{2}}}{x}+\frac{6c}{a}\int\partial x X^{\frac{5}{2}}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^3}=\left(-\frac{1}{2ax^2}-\frac{3b}{4a^2x}\right)X^3\sqrt{X}+\left(\frac{15b^2}{8a^2}+\frac{5c}{2a}\right)\int\frac{\partial x X^{\frac{5}{2}}}{x}$$
$$+\frac{9bc}{2a^2}\int\partial x X^{\frac{5}{2}}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^4}=\left[-\frac{1}{3ax^3}-\frac{b}{12a^2x^2}-\left(\frac{b^2}{8a^3}+\frac{4c}{3a^2}\right)\frac{1}{x}\right]X^3\sqrt{X}$$
$$+\left(\frac{5b^3}{16a^3}+\frac{15bc}{4a^2}\right)\int\frac{\partial x X^{\frac{5}{2}}}{x}+\left(\frac{3b^2c}{4a^3}+\frac{8c^2}{a^2}\right)\int\partial x X^{\frac{5}{2}}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^5}=-\frac{X^3\sqrt{X}}{4ax^4}-\frac{b}{8a}\int\frac{\partial x X^{\frac{5}{2}}}{x^4}+\frac{3c}{4a}\int\frac{\partial x X^{\frac{5}{2}}}{x^3}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^6}=\left(-\frac{1}{5ax^5}+\frac{3b}{40a^2x^4}\right)X^3\sqrt{X}+\left(\frac{3b^2}{80a^2}+\frac{2c}{5a}\right)\int\frac{\partial x X^{\frac{5}{2}}}{x^4}$$
$$-\frac{9bc}{40a^2}\int\frac{\partial x X^{\frac{5}{2}}}{x^3}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^7}=\left[-\frac{1}{6ax^6}+\frac{b}{12a^2x^5}-\left(\frac{b^2}{32a^3}+\frac{c}{24a^2}\right)\frac{1}{x^4}\right]X^3\sqrt{X}$$
$$-\left(\frac{b^3}{64a^3}+\frac{3bc}{16a^2}\right)\int\frac{\partial x X^{\frac{5}{2}}}{x^4}+\left(\frac{3b^2c}{32a^3}+\frac{c^2}{8a^2}\right)\int\frac{\partial x X^{\frac{5}{2}}}{x^3}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^8}=-\frac{X^3\sqrt{X}}{7ax^7}-\frac{b}{2a}\int\frac{\partial x X^{\frac{5}{2}}}{x^7}$$

$$\int\frac{\partial x X^{\frac{5}{2}}}{x^9}=\left(-\frac{1}{8ax^8}+\frac{9b}{112a^2x^7}\right)X^3\sqrt{X}+\left(\frac{9b^2}{32a^2}-\frac{c}{8a}\right)\int\frac{\partial x X^{\frac{5}{2}}}{x^7}$$

$\int x^m \partial x (a+bx+cx^2)^{\frac{7}{2}}$ Taf. LXXIX.

VZ. $a+bx+cx^2 = X,\ 4ac-b^2 = k$

$$\int \partial x X^{\frac{7}{2}} = \left(\frac{X^3}{16c}+\frac{7kX^2}{384c^2}+\frac{35k^2X}{6144c^3}+\frac{35k^3}{16384c^4}\right)(2cx+b)\sqrt{X} + \frac{35k^4}{32768c^4}\int\frac{\partial x}{\sqrt{X}}$$

$$\int x\partial x X^{\frac{7}{2}} = \frac{X^4\sqrt{X}}{9c} - \frac{b}{2c}\int \partial x X^{\frac{7}{2}}$$

$$\int x^2\partial x X^{\frac{7}{2}} = \left(\frac{x}{10c}-\frac{11b}{180c^2}\right)X^4\sqrt{X} + \left(\frac{11b^2}{40c^2}-\frac{a}{10c}\right)\int \partial x X^{\frac{7}{2}}$$

$$\int x^3\partial x X^{\frac{7}{2}} = \left(\frac{x^2}{11c}-\frac{13bx}{220c^2}+\frac{13b^2}{360c^3}-\frac{2a}{99c^2}\right)X^4\sqrt{X} - \left(\frac{13b^3}{80c^3}-\frac{3ab}{20c^2}\right)\int \partial x X^{\frac{7}{2}}$$

$$\int x^4\partial x X^{\frac{7}{2}} = \left[\frac{x^3}{12c}-\frac{5bx^2}{88c^2}+\left(\frac{13b^2}{352c^3}-\frac{a}{40c^2}\right)x-\frac{13b^3}{576c^4}+\frac{221ab}{7920c^3}\right]X^4\sqrt{X} + \left(\frac{13b^4}{128c^4}-\frac{13ab^2}{80c^3}+\frac{a^2}{40c^2}\right)\int \partial x X^{\frac{7}{2}}$$

$$\int x^5\partial x X^{\frac{7}{2}} = \frac{x^4X^4\sqrt{X}}{13c} - \frac{4a}{13c}\int x^3\partial x X^{\frac{7}{2}} - \frac{17b}{26c}\int x^4\partial x X^{\frac{7}{2}}$$

$$\int x^6\partial x X^{\frac{7}{2}} = \left(\frac{x^5}{14c}-\frac{19bx^4}{364c^2}\right)X^4\sqrt{X} + \frac{19ab}{91c^2}\int x^3\partial x X^{\frac{7}{2}} + \left(\frac{323b^2}{728c^2}-\frac{5a}{14c}\right)\int x^4\partial x X^{\frac{7}{2}}$$

$$\int x^7\partial x X^{\frac{7}{2}} = \left[\frac{x^6}{15c}-\frac{bx^5}{20c^2}+\left(\frac{19b^2}{520c^3}-\frac{2a}{65c^2}\right)x^4\right]X^4\sqrt{X} - \left(\frac{19ab^2}{130c^3}-\frac{8a^2}{65c^2}\right)\int x^3\partial x X^{\frac{7}{2}} - \left(\frac{323b^3}{1040c^3}-\frac{133ab}{260c^2}\right)\int x^4\partial x X^{\frac{7}{2}}$$

$$\int x^8\partial x X^{\frac{7}{2}} = \frac{x^7X^4\sqrt{X}}{16c} - \frac{7a}{16c}\int x^6\partial x X^{\frac{7}{2}} - \frac{23b}{32c}\int x^7\partial x X^{\frac{7}{2}}$$

$$\int x^9\partial x X^{\frac{7}{2}} = \left(\frac{x^8}{17c}-\frac{25bx^7}{544c^2}\right)X^4\sqrt{X} + \frac{175ab}{544c^2}\int x^6\partial x X^{\frac{7}{2}} + \left(\frac{575b^2}{1088c^2}-\frac{8a}{17c}\right)\int x^7\partial x X^{\frac{7}{2}}$$

T. ſ. LXXX. $\int \frac{\partial x (a + bx + cx^2)^{\frac{7}{2}}}{x^m}$.

VZ. $a + bx + cx^2 = X$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x} = \left(\frac{X^3}{7} + \frac{aX^2}{5} + \frac{a^2 X}{3} + a^3\right)\sqrt{X} + a^4 \int \frac{\partial x}{x\sqrt{X}} + \frac{a^3 b}{2} \int \frac{\partial x}{\sqrt{X}} + \frac{a^2 b}{2} \int \partial x \sqrt{X} + \frac{ab}{2} \int \partial x X^{\frac{3}{2}} + \frac{b}{2} \int \partial x X^{\frac{5}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^2} = -\frac{X^4 \sqrt{X}}{ax} + \frac{7b}{2a} \int \frac{\partial x X^{\frac{7}{2}}}{x} + \frac{8c}{a} \int \partial x X^{\frac{7}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^3} = \left(-\frac{1}{2ax^2} - \frac{5b}{4a^2 x}\right) X^4 \sqrt{X} + \left(\frac{35b^2}{8a^2} + \frac{7c}{2a}\right) \int \frac{\partial x X^{\frac{7}{2}}}{x} + \frac{10bc}{a^2} \int \partial x X^{\frac{7}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^4} = \left[-\frac{1}{3ax^3} - \frac{b}{4a^2 x^2} - \left(\frac{5b^2}{8a^3} + \frac{2c}{a^2}\right)\frac{1}{x}\right] X^4 \sqrt{X} + \left(\frac{35b^3}{16a^3} + \frac{35bc}{4a^2}\right) \int \frac{\partial x X^{\frac{7}{2}}}{x} + \left(\frac{5b^2 c}{a^3} + \frac{16c^2}{a^2}\right) \int \partial x X^{\frac{7}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^5} = \left[-\frac{1}{4ax^4} - \frac{b}{24a^2 x^3} - \left(\frac{b^2}{32a^3} + \frac{5c}{8a^2}\right)\frac{1}{x^2} - \left(\frac{5b^3}{64a^4} + \frac{29bc}{16a^3}\right)\frac{1}{x}\right] X^4 \sqrt{X} + \left(\frac{35b^4}{128a^4} + \frac{105b^2 c}{16a^3} + \frac{35c^2}{8a^2}\right) \int \frac{\partial x X^{\frac{7}{2}}}{x} + \left(\frac{5b^3 c}{8a^4} + \frac{29bc^2}{2a^3}\right) \int \partial x X^{\frac{7}{2}}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^6} = -\frac{X^4 \sqrt{X}}{5ax^5} - \frac{b}{10a} \int \frac{\partial x X^{\frac{7}{2}}}{x^5} + \frac{4c}{5a} \int \frac{\partial x X^{\frac{7}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^7} = \left(-\frac{1}{6ax^6} + \frac{b}{20a^2 x^5}\right) X^4 \sqrt{X} + \left(\frac{b^2}{40a^2} + \frac{c}{2a}\right) \int \frac{\partial x X^{\frac{7}{2}}}{x^5} - \frac{bc}{5a^2} \int \frac{\partial x X^{\frac{7}{2}}}{x^4}$$

$$\int \frac{\partial x X^{\frac{7}{2}}}{x^8} = \left[-\frac{1}{7ax^7} + \frac{5b}{84a^2 x^6} - \left(\frac{b^2}{56a^3} + \frac{2c}{35a^2}\right)\frac{1}{x^5}\right] X^4 \sqrt{X} - \left(\frac{b^3}{112a^3} + \frac{29bc}{140a^2}\right) \int \frac{\partial x X^{\frac{7}{2}}}{x^5} + \left(\frac{b^2 c}{14a^3} + \frac{8c^2}{35a^2}\right) \int \frac{\partial x X^{\frac{7}{2}}}{x^4}$$

$$\int \frac{x^m \partial x \sqrt{x}}{a+bx}, \quad \int \frac{x^m \partial x \sqrt{x}}{(a+bx)^2} \qquad \textit{Taf. LXXXIII.}$$

VZ. $a + bx = X$

$$\int \frac{\partial x \sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x\,\partial x \sqrt{x}}{X} = \left(\frac{x}{3b} - \frac{a}{b^2}\right)2\sqrt{x} + \frac{a^2}{b^2}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^2\partial x \sqrt{x}}{X} = \left(\frac{x^2}{5b} - \frac{ax}{3b^2} + \frac{a^2}{b^3}\right)2\sqrt{x} - \frac{a^3}{b^3}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^3\partial x \sqrt{x}}{X} = \left(\frac{x^3}{7b} - \frac{ax^2}{5b^2} + \frac{a^2x}{3b^3} - \frac{a^3}{b^4}\right)2\sqrt{x} + \frac{a^4}{b^4}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^4\partial x \sqrt{x}}{X} = \left(\frac{x^4}{9b} - \frac{ax^3}{7b^2} + \frac{a^2x^2}{5b^3} - \frac{a^3x}{3b^4} + \frac{a^4}{b^5}\right)2\sqrt{x} - \frac{a^5}{b^5}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^5\partial x \sqrt{x}}{X} = \left(\frac{x^5}{11b} - \frac{ax^4}{9b^2} + \frac{a^2x^3}{7b^3} - \frac{a^3x^2}{5b^4} + \frac{a^4x}{3b^5} - \frac{a^5}{b^6}\right)2\sqrt{x} + \frac{a^6}{b^6}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{\partial x \sqrt{x}}{X^2} = -\frac{\sqrt{x}}{bX} + \frac{1}{2b}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x\,\partial x \sqrt{x}}{X^2} = \frac{2x\sqrt{x}}{bX} - \frac{3a}{b}\int \frac{\partial x\sqrt{x}}{X^2}$$

$$\int \frac{x^2\partial x \sqrt{x}}{X^2} = \left(\frac{x^2}{3b} - \frac{5ax}{3b^2}\right)\frac{2\sqrt{x}}{X} + \frac{5a^2}{b^2}\int \frac{\partial x\sqrt{x}}{X^2}$$

$$\int \frac{x^3\partial x \sqrt{x}}{X^2} = \left(\frac{x^3}{5b} - \frac{7ax^2}{15b^2} + \frac{7a^2x}{3b^3}\right)\frac{2\sqrt{x}}{X} - \frac{7a^3}{b^3}\int \frac{\partial x\sqrt{x}}{X^2}$$

$$\int \frac{x^4\partial x \sqrt{x}}{X^2} = \left(\frac{x^4}{7b} - \frac{9ax^3}{35b^2} + \frac{3a^2x^2}{5b^3} - \frac{3a^3x}{b^4}\right)\frac{2\sqrt{x}}{X} + \frac{9a^4}{b^4}\int \frac{\partial x\sqrt{x}}{X^2}$$

$$\int \frac{x^5\partial x \sqrt{x}}{X^2} = \left(\frac{x^5}{9b} - \frac{11ax^4}{63b^2} + \frac{11a^2x^3}{35b^3} - \frac{11a^3x^2}{15b^4} + \frac{11a^4x}{3b^5}\right)\frac{2\sqrt{x}}{X} - \frac{11a^5}{b^5}\int \frac{\partial x\sqrt{x}}{X^2}$$

$$\int \frac{x^6\partial x \sqrt{x}}{X^2} = \left(\frac{x^6}{11b} - \frac{13ax^5}{99b^2} + \frac{13a^2x^4}{63b^3} - \frac{13a^3x^3}{35b^4} + \frac{13a^4x^2}{15b^5} - \frac{13a^5x}{3b^6}\right)\frac{2\sqrt{x}}{X} + \frac{13a^6}{b^6}\int \frac{\partial x\sqrt{x}}{X^2}$$

Taf. LXXXII. $\int \frac{\partial x}{(a+bx)^n \sqrt{x}}$

VZ. $a + bx = X$

$$\int \frac{\partial x}{X\sqrt{x}} = \left\{ \begin{array}{c} \pm \frac{2}{\sqrt{ab}} \text{Arc Tang} \sqrt{\frac{bx}{a}} \\ \text{oder} \\ \frac{1}{\sqrt{-ab}} \log \frac{a - bx + 2\sqrt{x} . \sqrt{-ab}}{X} \end{array} \right\} {}^{*)} + \text{Const.}$$

$$\int \frac{\partial x}{X^2 \sqrt{x}} = \frac{\sqrt{x}}{aX} + \frac{1}{2a} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{\partial x}{X^3 \sqrt{x}} = \left(\frac{1}{2aX^2} + \frac{3}{4a^2X}\right)\sqrt{x} + \frac{3}{8a^2} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{\partial x}{X^4 \sqrt{x}} = \left(\frac{1}{3aX^3} + \frac{5}{12a^2X^2} + \frac{5}{8a^3X}\right)\sqrt{x} + \frac{5}{16a^3} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{\partial x}{X^5 \sqrt{x}} = \left(\frac{1}{4aX^4} + \frac{7}{24a^2X^3} + \frac{35}{96a^3X^2} + \frac{35}{64a^4X}\right)\sqrt{x} + \frac{35}{128a^4} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{\partial x}{X^6 \sqrt{x}} = \left(\frac{1}{5aX^5} + \frac{9}{40a^2X^4} + \frac{21}{80a^3X^3} + \frac{21}{64a^4X^2} + \frac{63}{128a^5X}\right)\sqrt{x} + \frac{63}{256a^5} \int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{\partial x}{X^7 \sqrt{x}} = \left(\frac{1}{6aX^6} + \frac{11}{60a^2X^5} + \frac{33}{160a^3X^4} + \frac{77}{320a^4X^3} + \frac{77}{256a^5X^2} + \frac{231}{512a^6X}\right)\sqrt{x} + \frac{231}{1024a^6} \int \frac{\partial x}{X\sqrt{x}}$$

*) Der erste Ausdruck wird genommen, wenn a und b dieselben Vorzeichen haben, und alsdann gilt das obere Zeichen für ein positives, das untere für ein negatives a; der zweite Ausdruck hingegen wird genommen, wenn a und b verschiedene Vorzeichen haben. Beide Ausdrücke verschwinden für $x = 0$. Uebrigens ist $\text{Arc Tang} \sqrt{\frac{bx}{a}} = \text{Arc Cot} \sqrt{\frac{a}{bx}} = \text{Arc Sec} \sqrt{\frac{a+bx}{a}} = \text{Arc Cosec} \sqrt{\frac{a+bx}{bx}} = \text{Arc Cos} \sqrt{\frac{a}{a+bx}} = \frac{1}{2} \text{Arc Cos} \frac{a-bx}{a+bx} = \text{Arc Sin} \sqrt{\frac{bx}{a+bx}} = \frac{1}{2} \text{Arc Sin} \frac{2\sqrt{abx}}{a+bx} = \frac{1}{2} \text{Arc Sin vers} \frac{2bx}{a+bx}$.

$$\int \frac{x^m \partial x \sqrt{x}}{a+bx},\quad \int \frac{x^m \partial x \sqrt{x}}{(a+bx)^2}$$ *Taf. LXXXIII.*

VZ. $a + bx = X$

$$\int \frac{\partial x \sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x\,\partial x \sqrt{x}}{X} = \left(\frac{x}{3b} - \frac{a}{b^2}\right)2\sqrt{x} + \frac{a^2}{b^2}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^2 \partial x \sqrt{x}}{X} = \left(\frac{x^2}{5b} - \frac{ax}{3b^2} + \frac{a^2}{b^3}\right)2\sqrt{x} - \frac{a^3}{b^3}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^3 \partial x \sqrt{x}}{X} = \left(\frac{x^3}{7b} - \frac{ax^2}{5b^2} + \frac{a^2x}{3b^3} - \frac{a^3}{b^4}\right)2\sqrt{x} + \frac{a^4}{b^4}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^4 \partial x \sqrt{x}}{X} = \left(\frac{x^4}{9b} - \frac{ax^3}{7b^2} + \frac{a^2x^2}{5b^3} - \frac{a^3x}{3b^4} + \frac{a^4}{b^5}\right)2\sqrt{x} - \frac{a^5}{b^5}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^5 \partial x \sqrt{x}}{X} = \left(\frac{x^5}{11b} - \frac{ax^4}{9b^2} + \frac{a^2x^3}{7b^3} - \frac{a^3x^2}{5b^4} + \frac{a^4x}{3b^5} - \frac{a^5}{b^6}\right)2\sqrt{x} + \frac{a^6}{b^6}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{\partial x \sqrt{x}}{X^2} = -\frac{\sqrt{x}}{bX} + \frac{1}{2b}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x\,\partial x \sqrt{x}}{X^2} = \frac{2x\sqrt{x}}{bX} - \frac{3a}{b}\int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^2 \partial x \sqrt{x}}{X^2} = \left(\frac{x^2}{3b} - \frac{5ax}{3b^2}\right)\frac{2\sqrt{x}}{X} + \frac{5a^2}{b^2}\int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^3 \partial x \sqrt{x}}{X^2} = \left(\frac{x^3}{5b} - \frac{7ax^2}{15b^2} + \frac{7a^2x}{3b^3}\right)\frac{2\sqrt{x}}{X} - \frac{7a^3}{b^3}\int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^4 \partial x \sqrt{x}}{X^2} = \left(\frac{x^4}{7b} - \frac{9ax^3}{35b^2} + \frac{3a^2x^2}{5b^3} - \frac{3a^3x}{b^4}\right)\frac{2\sqrt{x}}{X} + \frac{9a^4}{b^4}\int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^5 \partial x \sqrt{x}}{X^2} = \left(\frac{x^5}{9b} - \frac{11ax^4}{63b^2} + \frac{11a^2x^3}{35b^3} - \frac{11a^3x^2}{15b^4} + \frac{11a^4x}{3b^5}\right)\frac{2\sqrt{x}}{X} - \frac{11a^5}{b^5}\int \frac{\partial x \sqrt{x}}{X^2}$$

$$\int \frac{x^6 \partial x \sqrt{x}}{X^2} = \left(\frac{x^6}{11b} - \frac{13ax^5}{99b^2} + \frac{13a^2x^4}{63b^3} - \frac{13a^3x^3}{35b^4} + \frac{13a^4x^2}{15b^5} - \frac{13a^5x}{3b^6}\right)\frac{2\sqrt{x}}{X} + \frac{13a^6}{b^6}\int \frac{\partial x \sqrt{x}}{X^2}$$

Taf. LXXXIV. $\int \frac{x^m \partial x \sqrt{x}}{(a+bx)^3}$, $\int \frac{x^m \partial x \sqrt{x}}{(a+bx)^4}$

VZ. $a + bx = X$

$$\int \frac{\partial x \sqrt{x}}{X^3} = \left(-\frac{1}{2bX^2} + \frac{1}{4abX}\right)\sqrt{x} + \frac{1}{8ab}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x\partial x \sqrt{x}}{X^3} = -\frac{2x\sqrt{x}}{bX^2} + \frac{3a}{b}\int \frac{\partial x \sqrt{x}}{X^3}$$

$$\int \frac{x^2\partial x \sqrt{x}}{X^3} = \left(\frac{x^2}{b} + \frac{5ax}{b^2}\right)\frac{2\sqrt{x}}{X^2} - \frac{15a^2}{b^2}\int \frac{\partial x \sqrt{x}}{X^3}$$

$$\int \frac{x^3\partial x \sqrt{x}}{X^3} = \left(\frac{x^3}{3b} - \frac{7ax^2}{3b^2} - \frac{35a^2x}{3b^3}\right)\frac{2\sqrt{x}}{X^2} + \frac{35a^3}{b^3}\int \frac{\partial x \sqrt{x}}{X^3}$$

$$\int \frac{x^4\partial x \sqrt{x}}{X^3} = \left(\frac{x^4}{5b} - \frac{3ax^3}{5b^2} + \frac{21a^2x^2}{5b^3} + \frac{21a^3x}{b^4}\right)\frac{2\sqrt{x}}{X^2} - \frac{63a^4}{b^4}\int \frac{\partial x \sqrt{x}}{X^3}$$

$$\int \frac{x^5\partial x \sqrt{x}}{X^3} = \left(\frac{x^5}{7b} - \frac{11ax^4}{35b^2} + \frac{33a^2x^3}{35b^3} - \frac{33a^3x^2}{5b^4} - \frac{33a^4x}{b^5}\right)\frac{2\sqrt{x}}{X^2} + \frac{99a^5}{b^5}\int \frac{\partial x \sqrt{x}}{X^3}$$

$$\int \frac{\partial x \sqrt{x}}{X^4} = \left(-\frac{1}{3bX^3} + \frac{1}{12abX^2} + \frac{1}{8a^2bX}\right)\sqrt{x} + \frac{1}{16a^2b}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x\partial x \sqrt{x}}{X^4} = -\frac{2x\sqrt{x}}{3bX^3} + \frac{a}{b}\int \frac{\partial x \sqrt{x}}{X^4}$$

$$\int \frac{x^2\partial x \sqrt{x}}{X^4} = \left(-\frac{x^2}{b} - \frac{5ax}{3b^2}\right)\frac{2\sqrt{x}}{X^3} + \frac{5a^2}{b^2}\int \frac{\partial x \sqrt{x}}{X^4}$$

$$\int \frac{x^3\partial x \sqrt{x}}{X^4} = \left(\frac{x^3}{b} + \frac{7ax^2}{b^2} + \frac{35a^2x}{3b^3}\right)\frac{2\sqrt{x}}{X^3} - \frac{35a^3}{b^3}\int \frac{\partial x \sqrt{x}}{X^4}$$

$$\int \frac{x^4\partial x \sqrt{x}}{X^4} = \left(\frac{x^4}{3b} - \frac{5ax^3}{b^2} - \frac{21a^2x^2}{b^3} - \frac{35a^3x}{b^4}\right)\frac{2\sqrt{x}}{X^3} + \frac{105a^4}{b^4}\int \frac{\partial x \sqrt{x}}{X^4}$$

$$\int \frac{x^5\partial x \sqrt{x}}{X^4} = \left(\frac{x^5}{5b} - \frac{11ax^4}{15b^2} + \frac{33a^2x^3}{5b^3} + \frac{231a^3x^2}{5b^4} + \frac{77a^4x}{b^5}\right)\frac{2\sqrt{x}}{X^3} - \frac{231a^5}{b^5}\int \frac{\partial x \sqrt{x}}{X^4}$$

$$\int\frac{x^m\partial x\sqrt{x}}{(a+bx^2)^2},\ \int\frac{x^m\partial x\sqrt{x}}{(a+bx^2)^3}$$ *Taf.* LXXXVII.

VZ. $a+bx^2=X$

$$\int\frac{\partial x\sqrt{x}}{X^2}=\frac{x\sqrt{x}}{2aX}+\frac{1}{4a}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x\partial x\sqrt{x}}{X^2}=-\frac{\sqrt{x}}{2bX}+\frac{1}{4b}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{x^2\partial x\sqrt{x}}{X^2}=-\frac{x\sqrt{x}}{2bX}+\frac{3}{4b}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x^3\partial x\sqrt{x}}{X^2}=\left(\frac{2x^2}{b}+\frac{5a}{2b^2}\right)\frac{\sqrt{x}}{X}-\frac{5a}{4b^2}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{x^4\partial x\sqrt{x}}{X^2}=\left(\frac{2x^3}{3b}+\frac{7ax}{6b^2}\right)\frac{\sqrt{x}}{X}-\frac{7a}{4b^2}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x^5\partial x\sqrt{x}}{X^2}=\left(\frac{2x^4}{5b}-\frac{18ax^2}{5b^2}-\frac{9a^2}{2b^3}\right)\frac{\sqrt{x}}{X}+\frac{9a^2}{4b^3}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{x^6\partial x\sqrt{x}}{X^2}=\left(\frac{2x^5}{7b}-\frac{22ax^3}{21b^2}-\frac{11a^2x}{6b^3}\right)\frac{\sqrt{x}}{X}+\frac{11a^2}{4b^3}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x^7\partial x\sqrt{x}}{X^2}=\left(\frac{2x^6}{9b}-\frac{26ax^4}{45b^2}+\frac{26a^2x^3}{5b^3}+\frac{13a^3}{2b^4}\right)\frac{\sqrt{x}}{X}-\frac{13a^3}{4b^4}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{\partial x\sqrt{x}}{X^3}=\left(\frac{1}{4aX^2}+\frac{5}{16a^2X}\right)x\sqrt{x}+\frac{5}{32a^2}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x\partial x\sqrt{x}}{X^3}=\frac{(bx^2-3a)\sqrt{x}}{16abX^2}+\frac{3}{32ab}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{x^2\partial x\sqrt{x}}{X^3}=-\frac{2x\sqrt{x}}{5bX^2}+\frac{3a}{5b}\int\frac{\partial x\sqrt{x}}{X^3}$$

$$\int\frac{x^3\partial x\sqrt{x}}{X^3}=-\frac{2x^2\sqrt{x}}{3bX^2}+\frac{5a}{3b}\int\frac{x\partial x\sqrt{x}}{X^3}$$

$$\int\frac{x^4\partial x\sqrt{x}}{X^3}=\left(-\frac{x^3}{b}-\frac{7ax}{5b^2}\right)\frac{2\sqrt{x}}{X^2}+\frac{21a^2}{5b^2}\int\frac{\partial x\sqrt{x}}{X^3}$$

$$\int\frac{x^5\partial x\sqrt{x}}{X^3}=\left(\frac{x^4}{b}+\frac{3ax^2}{b^2}\right)\frac{2\sqrt{x}}{X^2}-\frac{15a^2}{b^2}\int\frac{x\partial x\sqrt{x}}{X^3}$$

$$\int\frac{x^6\partial x\sqrt{x}}{X^3}=\left(\frac{x^5}{3b}+\frac{11ax^3}{3b^2}+\frac{77a^2x}{15b^3}\right)\frac{2\sqrt{x}}{X^2}-\frac{77a^3}{5b^3}\int\frac{\partial x\sqrt{x}}{X^3}$$

Taf. LXXXVI. $\int \frac{x^m \partial x \sqrt{x}}{a+bx^2}$

VZ. $a + bx^2 = X$

$$\int \frac{\partial x \sqrt{x}}{X} = \int \frac{\partial x \sqrt{x}}{X} \; {}^{*)}$$

$$\int \frac{x \partial x \sqrt{x}}{X} = \frac{2\sqrt{x}}{b} - \frac{a}{b}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^2 \partial x \sqrt{x}}{X} = \frac{2x\sqrt{x}}{3b} - \frac{a}{b}\int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x^3 \partial x \sqrt{x}}{X} = \left(\frac{x^2}{5b} - \frac{a}{b^2}\right)2\sqrt{x} + \frac{a^2}{b^2}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^4 \partial x \sqrt{x}}{X} = \left(\frac{x^3}{7b} - \frac{ax}{3b^2}\right)2\sqrt{x} + \frac{a^2}{b^2}\int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x^5 \partial x \sqrt{x}}{X} = \left(\frac{x^4}{9b} - \frac{ax^2}{5b^2} + \frac{a^2}{b^3}\right)2\sqrt{x} - \frac{a^3}{b^3}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^6 \partial x \sqrt{x}}{X} = \left(\frac{x^5}{11b} - \frac{ax^3}{7b^2} + \frac{a^2x}{3b^3}\right)2\sqrt{x} - \frac{a^3}{b^3}\int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x^7 \partial x \sqrt{x}}{X} = \left(\frac{x^6}{13b} - \frac{ax^4}{9b^2} + \frac{a^2x^2}{5b^3} - \frac{a^3}{b^4}\right)2\sqrt{x} + \frac{a^4}{b^4}\int \frac{\partial x}{X\sqrt{x}}$$

$$\int \frac{x^8 \partial x \sqrt{x}}{X} = \left(\frac{x^7}{15b} - \frac{ax^5}{11b^2} + \frac{a^2x^3}{7b^3} - \frac{a^3x}{3b^4}\right)2\sqrt{x} + \frac{a^4}{b^4}\int \frac{\partial x \sqrt{x}}{X}$$

$$\int \frac{x^9 \partial x \sqrt{x}}{X} = \left(\frac{x^8}{17b} - \frac{ax^6}{13b^2} + \frac{a^2x^4}{9b^3} - \frac{a^3x^2}{5b^4} + \frac{a^4}{b^5}\right)2\sqrt{x} - \frac{a^5}{b^5}\int \frac{\partial x}{X\sqrt{x}}$$

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*) Haben a und b dieselben Vorzeichen, so ist

$$\int \frac{\partial x \sqrt{x}}{X} = \frac{1}{bk\sqrt{2}}\left[-\log \frac{x + k^2 + k\sqrt{2x}}{\sqrt{X}} + \operatorname{Arc\,Tang} \frac{k\sqrt{2x}}{k^2 - x}\right]$$

wo alsdann $k = \sqrt[4]{\frac{a}{b}}$.

Haben a und b verschiedene Vorzeichen, so ist

$$\int \frac{\partial x \sqrt{x}}{X} = \frac{1}{2bk}\left[\log \frac{k - \sqrt{x}}{k + \sqrt{x}} + 2\operatorname{Arc\,Tang} \frac{\sqrt{x}}{k}\right]$$

und es ist $k = \sqrt[4]{-\frac{a}{b}}$.

$$\int\frac{x^m\partial x\sqrt{x}}{(a+bx^2)^2},\ \int\frac{x^m\partial x\sqrt{x}}{(a+bx^2)^3}$$ *Taf. LXXXVII.*

VZ. $a+bx^2=X$

$$\int\frac{\partial x\sqrt{x}}{X^2}=\frac{x\sqrt{x}}{2aX}+\frac{1}{4a}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x\partial x\sqrt{x}}{X^2}=-\frac{\sqrt{x}}{2bX}+\frac{1}{4b}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{x^2\partial x\sqrt{x}}{X^2}=-\frac{x\sqrt{x}}{2bX}+\frac{3}{4b}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x^3\partial x\sqrt{x}}{X^2}=\left(\frac{2x^2}{b}+\frac{5a}{2b^2}\right)\frac{\sqrt{x}}{X}-\frac{5a}{4b^2}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{x^4\partial x\sqrt{x}}{X^2}=\left(\frac{2x^3}{3b}+\frac{7ax}{6b^2}\right)\frac{\sqrt{x}}{X}-\frac{7a}{4b^2}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x^5\partial x\sqrt{x}}{X^2}=\left(\frac{2x^4}{5b}-\frac{18ax^2}{5b^2}-\frac{9a^2}{2b^3}\right)\frac{\sqrt{x}}{X}+\frac{9a^2}{4b^3}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{x^6\partial x\sqrt{x}}{X^2}=\left(\frac{2x^5}{7b}-\frac{22ax^3}{21b^2}-\frac{11a^2x}{6b^3}\right)\frac{\sqrt{x}}{X}+\frac{11a^2}{4b^3}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x^7\partial x\sqrt{x}}{X^2}=\left(\frac{2x^6}{9b}-\frac{26ax^4}{45b^2}+\frac{26a^2x^3}{5b^3}+\frac{13a^3}{2b^4}\right)\frac{\sqrt{x}}{X}-\frac{13a^3}{4b^4}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{\partial x\sqrt{x}}{X^3}=\left(\frac{1}{4aX^2}+\frac{5}{16a^2X}\right)x\sqrt{x}+\frac{5}{32a^2}\int\frac{\partial x\sqrt{x}}{X}$$

$$\int\frac{x\partial x\sqrt{x}}{X^3}=\frac{(bx^2-3a)\sqrt{x}}{16abX^2}+\frac{3}{32ab}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{x^2\partial x\sqrt{x}}{X^3}=-\frac{2x\sqrt{x}}{5bX^2}+\frac{3a}{5b}\int\frac{\partial x\sqrt{x}}{X^3}$$

$$\int\frac{x^3\partial x\sqrt{x}}{X^3}=-\frac{2x^2\sqrt{x}}{3bX^2}+\frac{5a}{3b}\int\frac{x\partial x\sqrt{x}}{X^3}$$

$$\int\frac{x^4\partial x\sqrt{x}}{X^3}=\left(-\frac{x^3}{b}-\frac{7ax}{5b^2}\right)\frac{2\sqrt{x}}{X^2}+\frac{21a^2}{5b^2}\int\frac{\partial x\sqrt{x}}{X^3}$$

$$\int\frac{x^5\partial x\sqrt{x}}{X^3}=\left(\frac{x^4}{b}+\frac{3ax^2}{b^2}\right)\frac{2\sqrt{x}}{X^2}-\frac{15a^2}{b^2}\int\frac{x\partial x\sqrt{x}}{X^3}$$

$$\int\frac{x^6\partial x\sqrt{x}}{X^3}=\left(\frac{x^5}{3b}+\frac{11ax^3}{3b^2}+\frac{77a^2x}{15b^3}\right)\frac{2\sqrt{x}}{X^2}-\frac{77a^3}{5b^3}\int\frac{\partial x\sqrt{x}}{X^3}$$

Taf. LXXXVIII. $\int\frac{\partial x}{(a+bx)x^m\sqrt{x}},\ \int\frac{\partial x}{(a+bx)^2x^m\sqrt{x}}$

VZ. $a+bx=X$

$$\int\frac{\partial x}{Xx\sqrt{x}}=-\frac{2}{a\sqrt{x}}-\frac{b}{a}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{\partial x}{Xx^2\sqrt{x}}=\left(-\frac{1}{3ax}+\frac{b}{a^2}\right)\frac{2}{\sqrt{x}}+\frac{b^2}{a^2}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{\partial x}{Xx^3\sqrt{x}}=\left(-\frac{1}{5ax^2}+\frac{b}{3a^2x}-\frac{b^2}{a^3}\right)\frac{2}{\sqrt{x}}-\frac{b^3}{a^3}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{\partial x}{Xx^4\sqrt{x}}=\left(-\frac{1}{7ax^3}+\frac{b}{5a^2x^2}-\frac{b^2}{3a^3x}+\frac{b^3}{a^4}\right)\frac{2}{\sqrt{x}}+\frac{b^4}{a^4}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{\partial x}{Xx^5\sqrt{x}}=\left(-\frac{1}{9ax^4}+\frac{b}{7a^2x^3}-\frac{b^2}{5a^3x^2}+\frac{b^3}{3a^4x}-\frac{b^4}{a^5}\right)\frac{2}{\sqrt{x}}-\frac{b^5}{a^5}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{\partial x}{Xx^6\sqrt{x}}=\left(-\frac{1}{11ax^5}+\frac{b}{9a^2x^4}-\frac{b^2}{7a^3x^3}+\frac{b^3}{5a^4x^2}-\frac{b^4}{3a^5x}+\frac{b^5}{a^6}\right)\frac{2}{\sqrt{x}}$$
$$+\frac{b^6}{a^6}\int\frac{\partial x}{X\sqrt{x}}$$

$$\int\frac{\partial x}{X^2x\sqrt{x}}=-\frac{2}{aX\sqrt{x}}-\frac{3b}{a}\int\frac{\partial x}{X^2\sqrt{x}}$$

$$\int\frac{\partial x}{X^2x^2\sqrt{x}}=\left(-\frac{1}{3ax}+\frac{5b}{3a^2}\right)\frac{2}{X\sqrt{x}}+\frac{5b^2}{a^2}\int\frac{\partial x}{X^2\sqrt{x}}$$

$$\int\frac{\partial x}{X^2x^3\sqrt{x}}=\left(-\frac{1}{5ax^2}+\frac{7b}{15a^2x}-\frac{7b^2}{3a^3}\right)\frac{2}{X\sqrt{x}}-\frac{7b^3}{a^3}\int\frac{\partial x}{X^2\sqrt{x}}$$

$$\int\frac{\partial x}{X^2x^4\sqrt{x}}=\left(-\frac{1}{7ax^3}+\frac{9b}{35a^2x^2}-\frac{3b^2}{5a^3x}+\frac{3b^3}{a^4}\right)\frac{2}{X\sqrt{x}}+\frac{9b^4}{a^4}\int\frac{\partial x}{X^2\sqrt{x}}$$

$$\int\frac{\partial x}{X^2x^5\sqrt{x}}=\left(-\frac{1}{9ax^4}+\frac{11b}{63a^2x^3}-\frac{11b^2}{35a^3x^2}+\frac{11b^3}{15a^4x}-\frac{11b^4}{3a^5}\right)\frac{2}{X\sqrt{x}}$$
$$-\frac{11b^5}{a^5}\int\frac{\partial x}{X^2\sqrt{x}}$$

$$\int\frac{\partial x}{X^2x^6\sqrt{x}}=\left(-\frac{1}{11ax^5}+\frac{13b}{99a^2x^4}-\frac{13b^2}{63a^3x^3}+\frac{13b^3}{35a^4x^2}-\frac{13b^4}{15a^5x}\right.$$
$$\left.+\frac{13b^5}{3a^6}\right)\frac{2}{X\sqrt{x}}+\frac{13b^6}{a^6}\int\frac{\partial x}{X^2\sqrt{x}}$$

$$\int\frac{\partial x}{(a+bx)^3x^m\sqrt{x}},\ \int\frac{\partial x}{(a+bx)^4x^m\sqrt{x}}$$ *Taf. LXXXIX.*

VZ: $a+bx=X$

$$\int\frac{\partial x}{X^3x\sqrt{x}}=-\frac{2}{aX^2\sqrt{x}}-\frac{5b}{a}\int\frac{\partial x}{X^3\sqrt{x}}$$

$$\int\frac{\partial x}{X^3x^2\sqrt{x}}=\left(-\frac{1}{3ax}+\frac{7b}{3a^2}\right)\frac{2}{X^2\sqrt{x}}+\frac{35b^2}{3a^2}\int\frac{\partial x}{X^3\sqrt{x}}$$

$$\int\frac{\partial x}{X^3x^3\sqrt{x}}=\left(-\frac{1}{5ax^2}+\frac{3b}{5a^2x}-\frac{21b^2}{5a^3}\right)\frac{2}{X^2\sqrt{x}}+\frac{21b^3}{a^3}\int\frac{\partial x}{X^3\sqrt{x}}$$

$$\int\frac{\partial x}{X^3x^4\sqrt{x}}=\left(-\frac{1}{7ax^3}+\frac{11b}{35a^2x^2}-\frac{33b^2}{35a^3x}+\frac{33b^3}{5a^4}\right)\frac{2}{X^2\sqrt{x}}+\frac{33b^4}{a^4}\int\frac{\partial x}{X^3\sqrt{x}}$$

$$\int\frac{\partial x}{X^3x^5\sqrt{x}}=\left(-\frac{1}{9ax^4}+\frac{13b}{63a^2x^3}-\frac{143b^2}{315a^3x^2}+\frac{143b^3}{105a^4x}-\frac{143b^4}{15a^5}\right)\frac{2}{X^2\sqrt{x}}$$
$$-\frac{143b^5}{3a^5}\int\frac{\partial x}{X^3\sqrt{x}}$$

$$\int\frac{\partial x}{X^3x^6\sqrt{x}}=\left(-\frac{1}{11ax^5}+\frac{5b}{33a^2x^4}-\frac{65b^2}{231a^3x^3}+\frac{13b^3}{21a^4x^2}-\frac{13b^4}{7a^5x}\right.$$
$$\left.+\frac{13b^5}{a^6}\right)\frac{2}{X^2\sqrt{x}}+\frac{65b^6}{a^6}\int\frac{\partial x}{X^3\sqrt{x}}$$

$$\int\frac{\partial x}{X^4x\sqrt{x}}=-\frac{2}{aX^3\sqrt{x}}-\frac{7b}{a}\int\frac{\partial x}{X^4\sqrt{x}}$$

$$\int\frac{\partial x}{X^4x^2\sqrt{x}}=\left(-\frac{1}{3ax}+\frac{3b}{a^2}\right)\frac{2}{X^3\sqrt{x}}+\frac{21b^2}{a^2}\int\frac{\partial x}{X^4\sqrt{x}}$$

$$\int\frac{\partial x}{X^4x^3\sqrt{x}}=\left(-\frac{1}{5ax^2}+\frac{11b}{15a^2x}-\frac{33b^2}{5a^3}\right)\frac{2}{X^3\sqrt{x}}-\frac{231b^3}{5a^3}\int\frac{\partial x}{X^4\sqrt{x}}$$

$$\int\frac{\partial x}{X^4x^4\sqrt{x}}=\left(-\frac{1}{7ux^3}+\frac{13b}{35a^2x^2}-\frac{143b^2}{105a^3x}+\frac{429b^3}{35a^4}\right)\frac{2}{X^3\sqrt{x}}$$
$$+\frac{429b^4}{5a^4}\int\frac{\partial x}{X^4\sqrt{x}}$$

$$\int\frac{\partial x}{X^4x^5\sqrt{x}}=\left(-\frac{1}{9ax^3}+\frac{5b}{21a^2x^3}-\frac{13b^2}{21a^3x^2}+\frac{143b^3}{63a^4x}-\frac{143b^4}{7a^5}\right)\frac{2}{X^3\sqrt{x}}$$
$$-\frac{143b^5}{a^5}\int\frac{\partial x}{X^4\sqrt{x}}$$

Taf. XC.

$$\int \frac{\partial x}{(f+gx)^n \sqrt{(a+bx)}}$$

VZ. $f+gx=X,\ a+bx=X',\ bf-ag=k$

$$\int \frac{\partial x}{X\sqrt{X'}} = \left\{ \begin{array}{c} \pm \frac{2}{\sqrt{gk}} \operatorname{ArcTang} \sqrt{\frac{gX'}{k}} \\ \text{oder} \\ \frac{1}{\sqrt{-gk}} \log \frac{bf-2ag-bgx+2\sqrt{-gk}.\sqrt{X'}}{X} \end{array} \right\} {}^{*)}$$

$$\int \frac{\partial x}{X^2\sqrt{X'}} = \frac{\sqrt{X'}}{kX} + \frac{b}{2k}\int \frac{\partial x}{X\sqrt{X'}}$$

$$\int \frac{\partial x}{X^3\sqrt{X'}} = \left(\frac{1}{2kX^2} + \frac{3b}{4k^2X}\right)\sqrt{X'} + \frac{3b^2}{8k^2}\int \frac{\partial x}{X\sqrt{X'}}$$

$$\int \frac{\partial x}{X^4\sqrt{X'}} = \left(\frac{1}{3kX^3} + \frac{5b}{12k^2X^2} + \frac{5b^2}{8k^3X}\right)\sqrt{X'} + \frac{5b^3}{16k^3}\int \frac{\partial x}{X\sqrt{X'}}$$

$$\int \frac{\partial x}{X^5\sqrt{X'}} = \left(\frac{1}{4kX^4} + \frac{7b}{24k^2X^3} + \frac{35b^2}{96k^3X^2} + \frac{35b^3}{64k^4X}\right)\sqrt{X'} + \frac{35b^4}{128k^4}\int \frac{\partial x}{X\sqrt{X'}}$$

$$\int \frac{\partial x}{X^6\sqrt{X'}} = \left(\frac{1}{5kX^5} + \frac{9b}{40k^2X^4} + \frac{21b^2}{80k^3X^3} + \frac{21b^3}{64k^4X^2} + \frac{63b^4}{128k^5X}\right)\sqrt{X'} + \frac{63b^5}{256k^5}\int \frac{\partial x}{X\sqrt{X'}}$$

$$\int \frac{\partial x}{X^7\sqrt{X'}} = \left(\frac{1}{6kX^6} + \frac{11b}{60k^2X^5} + \frac{33b^2}{160k^3X^4} + \frac{77b^3}{320k^4X^3} + \frac{77b^4}{256k^5X^2} + \frac{231b^5}{512k^6X}\right)\sqrt{X'} + \frac{231b^6}{1024k^6}\int \frac{\partial x}{X\sqrt{X'}}$$

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*) Der erste Ausdruck mit dem Vorzeichen + wird genommen, wenn g und k zugleich positiv sind, und mit dem Vorzeichen —, wenn g und k zugleich negativ sind. Der zweite Ausdruck wird genommen, wenn g und k verschiedene Vorzeichen haben. Wenn $k=0$ wird, so gehet $\int \frac{\partial x}{X\sqrt{X'}}$ in $\frac{b}{g}\int \frac{\partial x}{(a+bx)^{\frac{3}{2}}} = -\frac{2}{g\sqrt{(a+bx)}}$ über.

$$\int\frac{x^m\partial x}{(f+gx)\sqrt{(a+bx)}},\ \int\frac{x^m\partial x}{(f+gx)^2\sqrt{(a+bx)}}$$ *Taf. XCI.*

VZ. $f+gx=X,\ a+bx=X'$

$$\int\frac{x\partial x}{X\sqrt{X'}}=\frac{1}{g}\int\frac{\partial x}{\sqrt{X'}}-\frac{f}{g}\int\frac{\partial x}{X\sqrt{X'}}$$

$$\int\frac{x^2\partial x}{X\sqrt{X'}}=\frac{1}{g}\int\frac{x\partial x}{\sqrt{X'}}-\frac{f}{g^2}\int\frac{\partial x}{\sqrt{X'}}+\frac{f^2}{g^2}\int\frac{\partial x}{X\sqrt{X'}}$$

$$\int\frac{x^3\partial x}{X\sqrt{X'}}=\frac{1}{g}\int\frac{x^2\partial x}{\sqrt{X'}}-\frac{f}{g^2}\int\frac{x\partial x}{\sqrt{X'}}+\frac{f^2}{g^3}\int\frac{\partial x}{\sqrt{X'}}-\frac{f^3}{g^3}\int\frac{\partial x}{X\sqrt{X'}}$$

$$\int\frac{x^4\partial x}{X\sqrt{X'}}=\frac{1}{g}\int\frac{x^3\partial x}{\sqrt{X'}}-\frac{f}{g^2}\int\frac{x^2\partial x}{\sqrt{X'}}+\frac{f^2}{g^3}\int\frac{x\partial x}{\sqrt{X'}}-\frac{f^3}{g^4}\int\frac{\partial x}{\sqrt{X'}}+\frac{f^4}{g^4}\int\frac{\partial x}{X\sqrt{X'}}$$

$$\int\frac{x^5\partial x}{X\sqrt{X'}}=\frac{1}{g}\int\frac{x^4\partial x}{\sqrt{X'}}-\frac{f}{g^2}\int\frac{x^3\partial x}{\sqrt{X'}}+\frac{f^2}{g^3}\int\frac{x^2\partial x}{\sqrt{X'}}-\frac{f^3}{g^4}\int\frac{x\partial x}{\sqrt{X'}}+\frac{f^4}{g^5}\int\frac{\partial x}{\sqrt{X'}}-\frac{f^5}{g^5}\int\frac{\partial x}{X\sqrt{X'}}$$

$$\int\frac{x\partial x}{X^2\sqrt{X'}}=\frac{1}{g}\int\frac{\partial x}{X\sqrt{X'}}-\frac{f}{g}\int\frac{\partial x}{X^2\sqrt{X'}}$$

$$\int\frac{x^2\partial x}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{\partial x}{\sqrt{X'}}-\frac{2f}{g^2}\int\frac{\partial x}{X\sqrt{X'}}+\frac{f^2}{g^2}\int\frac{\partial x}{X^2\sqrt{X'}}$$

$$\int\frac{x^3\partial x}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{x\partial x}{\sqrt{X'}}-\frac{2f}{g^3}\int\frac{\partial x}{\sqrt{X'}}+\frac{3f^2}{g^3}\int\frac{\partial x}{X\sqrt{X'}}-\frac{f^3}{g^3}\int\frac{\partial x}{X^2\sqrt{X'}}$$

$$\int\frac{x^4\partial x}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{x^2\partial x}{\sqrt{X'}}-\frac{2f}{g^3}\int\frac{x\partial x}{\sqrt{X'}}+\frac{3f^2}{g^4}\int\frac{\partial x}{\sqrt{X'}}-\frac{4f^3}{g^4}\int\frac{\partial x}{X\sqrt{X'}}+\frac{f^4}{g^4}\int\frac{\partial x}{X^2\sqrt{X'}}$$

$$\int\frac{x^5\partial x}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{x^3\partial x}{\sqrt{X'}}-\frac{2f}{g^3}\int\frac{x^2\partial x}{\sqrt{X'}}+\frac{3f^2}{g^4}\int\frac{x\partial x}{\sqrt{X'}}-\frac{4f^3}{g^5}\int\frac{\partial x}{\sqrt{X'}}+\frac{5f^4}{g^5}\int\frac{\partial x}{X\sqrt{X'}}-\frac{f^5}{g^5}\int\frac{\partial x}{X^2\sqrt{X'}}$$

$$\int\frac{x^6\partial x}{X^2\sqrt{X'}}=\frac{1}{g^2}\int\frac{x^4\partial x}{\sqrt{X'}}-\frac{2f}{g^3}\int\frac{x^3\partial x}{\sqrt{X'}}+\frac{3f^2}{g^4}\int\frac{x^2\partial x}{\sqrt{X'}}-\frac{4f^3}{g^5}\int\frac{x\partial x}{\sqrt{X'}}+\frac{5f^4}{g^6}\int\frac{\partial x}{\sqrt{X'}}-\frac{6f^5}{g^6}\int\frac{\partial x}{X\sqrt{X'}}+\frac{f^6}{g^6}\int\frac{\partial x}{X^2\sqrt{X'}}$$

Taf. XCII. $\int\frac{x^m\partial x}{(f+gx)^3\sqrt{(a+bx)}},\ \int\frac{x^m\partial x}{(f+gx)^4\sqrt{(a+bx)}}$

VZ. $f+gx=X,\ a+bx=X'$

$$\int\frac{x\partial x}{X^3\sqrt{X'}}=\frac{1}{g}\int\frac{\partial x}{X^2\sqrt{X'}}-\frac{f}{g}\int\frac{\partial x}{X^3\sqrt{X'}}$$

$$\int\frac{x^2\partial x}{X^3\sqrt{X'}}=\frac{1}{g^2}\int\frac{\partial x}{X\sqrt{X'}}-\frac{2f}{g^2}\int\frac{\partial x}{X^2\sqrt{X'}}+\frac{f^2}{g^2}\int\frac{\partial x}{X^3\sqrt{X'}}$$

$$\int\frac{x^3\partial x}{X^3\sqrt{X'}}=\frac{1}{g^3}\int\frac{\partial x}{\sqrt{X'}}-\frac{3f}{g^3}\int\frac{\partial x}{X\sqrt{X'}}+\frac{3f^2}{g^3}\int\frac{\partial x}{X^2\sqrt{X'}}-\frac{f^3}{g^3}\int\frac{\partial x}{X^3\sqrt{X'}}$$

$$\int\frac{x^4\partial x}{X^3\sqrt{X'}}=\frac{1}{g^3}\int\frac{x\partial x}{\sqrt{X'}}-\frac{3f}{g^4}\int\frac{\partial x}{\sqrt{X'}}+\frac{6f^2}{g^4}\int\frac{\partial x}{X\sqrt{X'}}-\frac{4f^3}{g^4}\int\frac{\partial x}{X^2\sqrt{X'}}$$
$$+\frac{f^4}{g^4}\int\frac{\partial x}{X^3\sqrt{X'}}$$

$$\int\frac{x^5\partial x}{X^3\sqrt{X'}}=\frac{1}{g^3}\int\frac{x^2\partial x}{\sqrt{X'}}-\frac{3f}{g^4}\int\frac{x\partial x}{\sqrt{X'}}+\frac{6f^2}{g^5}\int\frac{\partial x}{\sqrt{X'}}-\frac{10f^3}{g^5}\int\frac{\partial x}{X\sqrt{X'}}$$
$$+\frac{5f^4}{g^5}\int\frac{\partial x}{X^2\sqrt{X'}}-\frac{f^5}{g^5}\int\frac{\partial x}{X^3\sqrt{X'}}$$

$$\int\frac{x\partial x}{X^4\sqrt{X'}}=\frac{1}{g}\int\frac{\partial x}{X^3\sqrt{X'}}-\frac{f}{g}\int\frac{\partial x}{X^4\sqrt{X'}}$$

$$\int\frac{x^2\partial x}{X^4\sqrt{X'}}=\frac{1}{g^2}\int\frac{\partial x}{X^2\sqrt{X'}}-\frac{2f}{g^2}\int\frac{\partial x}{X^3\sqrt{X'}}+\frac{f^2}{g^2}\int\frac{\partial x}{X^4\sqrt{X'}}$$

$$\int\frac{x^3\partial x}{X^4\sqrt{X'}}=\frac{1}{g^3}\int\frac{\partial x}{X\sqrt{X'}}-\frac{3f}{g^3}\int\frac{\partial x}{X^2\sqrt{X'}}+\frac{3f^2}{g^3}\int\frac{\partial x}{X^3\sqrt{X'}}$$
$$-\frac{f^3}{g^3}\int\frac{\partial x}{X^4\sqrt{X'}}$$

$$\int\frac{x^4\partial x}{X^4\sqrt{X'}}=\frac{1}{g^4}\int\frac{\partial x}{\sqrt{X'}}-\frac{4f}{g^4}\int\frac{\partial x}{X\sqrt{X'}}+\frac{6f^2}{g^4}\int\frac{\partial x}{X^2\sqrt{X'}}-\frac{4f^3}{g^4}\int\frac{\partial x}{X^3\sqrt{X'}}$$
$$+\frac{f^4}{g^4}\int\frac{\partial x}{X^4\sqrt{X'}}$$

$$\int\frac{x^5\partial x}{X^4\sqrt{X'}}=\frac{1}{g^4}\int\frac{x\partial x}{\sqrt{X'}}-\frac{4f}{g^5}\int\frac{\partial x}{\sqrt{X'}}+\frac{10f^2}{g^5}\int\frac{\partial x}{X\sqrt{X'}}-\frac{10f^3}{g^5}\int\frac{\partial x}{X^2\sqrt{X'}}$$
$$+\frac{5f^4}{g^5}\int\frac{\partial x}{X^3\sqrt{X'}}-\frac{f^5}{g^5}\int\frac{\partial x}{X^4\sqrt{X'}}$$

$$\int \frac{x^m \partial x}{(f+gx)\sqrt{(a+bx^2)}}$$

Taf. XCIII.

VZ. $a+bx^2=X,\ f+gx=X',\ ag^2+bf^2=k$

$$\int \frac{\partial x}{X'\sqrt{X}} = \left\{ \begin{array}{c} \pm \frac{1}{\sqrt{k}} \log \frac{ag-bfx \mp \sqrt{k}.\sqrt{X}}{X'} \\ \text{oder} \\ \frac{1}{\sqrt{-k}} \text{ Arc Tang } \frac{ag-bfx}{\sqrt{-k}.\sqrt{X}} \end{array} \right\} {}^{*)}$$

$$\int \frac{x\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{\partial x}{\sqrt{X}} - \frac{f}{g}\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^2\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x\partial x}{\sqrt{X}} - \frac{f}{g^2}\int \frac{\partial x}{\sqrt{X}} + \frac{f^2}{g^2}\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^2\partial x}{\sqrt{X}} - \frac{f}{g^2}\int \frac{x\partial x}{\sqrt{X}} + \frac{f^2}{g^3}\int \frac{\partial x}{\sqrt{X}} - \frac{f^3}{g^3}\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^3\partial x}{\sqrt{X}} - \frac{f}{g^2}\int \frac{x^2\partial x}{\sqrt{X}} + \frac{f^2}{g^3}\int \frac{x\partial x}{\sqrt{X}} - \frac{f^3}{g^4}\int \frac{\partial x}{\sqrt{X}} + \frac{f^4}{g^4}\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^4\partial x}{\sqrt{X}} - \frac{f}{g^2}\int \frac{x^3\partial x}{\sqrt{X}} + \frac{f^2}{g^3}\int \frac{x^2\partial x}{\sqrt{X}} - \frac{f^3}{g^4}\int \frac{x\partial x}{\sqrt{X}} + \frac{f^4}{g^5}\int \frac{\partial x}{\sqrt{X}} - \frac{f^5}{g^5}\int \frac{\partial x}{X'\sqrt{X}}$$

*) Der erste Ausdruck wird reell, wenn k positiv, der zweite, wenn k negativ ist. Von den Vorzeichen $\pm$ und $\mp$, welche in dem ersten Ausdruck vorkommen, gehören die oberen zusammen, und eben so die unteren; sonst ist es gleichgültig, welche man braucht. Uebrigens ist

$$\text{Arc Tang } \frac{ag-bfx}{\sqrt{-k}.\sqrt{X}} = \text{Arc Sin } \frac{ag-bfx}{(f+gx)\sqrt{-ab}}$$

$$= \text{Arc Cos } \frac{\sqrt{-k}.\sqrt{X}}{(f+gx)\sqrt{-ab}} = \text{etc.}$$

Der Factor $\sqrt{-ab}$, welcher hier im Sinus und Cosinus vorkommt, wird nothwendig reell, weil a und b weder zugleich positiv, noch zugleich negativ seyn können: denn im ersten Falle würde k gewiſs positiv werden, und also der logarithmische Ausdruck gelten, im zweiten Falle würde $\sqrt{(a+bx^2)}$ nothwendig imaginär werden.

Taf. XCIV. $\int \frac{x^m \partial x}{(f+gx^2)\sqrt{(a+bx^2)}}$, $\int \frac{x^m \partial x \sqrt{(a+bx^2)}}{f+gx^2}$

VZ. $a+bx^2 = X,\ f+gx^2 = X'$

$$\left.\begin{aligned}\int \frac{\partial x}{X'\sqrt{X}} &= \int \frac{\partial x}{X'\sqrt{X}}\\ \int \frac{x\partial x}{X'\sqrt{X}} &= \int \frac{x\partial x}{X'\sqrt{X}}\end{aligned}\right\}$$ (Man s. die folgende Seite.)

$$\int \frac{x^2\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{\partial x}{\sqrt{X}} - \frac{f}{g}\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^3\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x\partial x}{\sqrt{X}} - \frac{f}{g}\int \frac{x\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^4\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^2\partial x}{\sqrt{X}} - \frac{f}{g^2}\int \frac{\partial x}{\sqrt{X}} + \frac{f^2}{g^2}\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^5\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^3\partial x}{\sqrt{X}} - \frac{f}{g^2}\int \frac{x\partial x}{\sqrt{X}} + \frac{f^2}{g^2}\int \frac{x\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^6\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^4\partial x}{\sqrt{X}} - \frac{f}{g^2}\int \frac{x^2\partial x}{\sqrt{X}} + \frac{f^2}{g^3}\int \frac{\partial x}{\sqrt{X}} - \frac{f^3}{g^3}\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^7\partial x}{X'\sqrt{X}} = \frac{1}{g}\int \frac{x^5\partial x}{\sqrt{X}} - \frac{f}{g^2}\int \frac{x^3\partial x}{\sqrt{X}} + \frac{f^2}{g^3}\int \frac{x\partial x}{\sqrt{X}} + \frac{f^3}{g^3}\int \frac{x\partial x}{X'\sqrt{X}}$$

$$\int \frac{\partial x\sqrt{X}}{X'} = \frac{b}{g}\int \frac{\partial x}{\sqrt{X}} + \left(a - \frac{bf}{g}\right)\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x\partial x\sqrt{X}}{X'} = \frac{b}{g}\int \frac{x\partial x}{\sqrt{X}} + \left(a - \frac{bf}{g}\right)\int \frac{x\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^2\partial x\sqrt{X}}{X'} = \frac{b}{g}\int \frac{x^2\partial x}{\sqrt{X}} + \left(\frac{a}{g} - \frac{bf}{g^2}\right)\int \frac{\partial x}{\sqrt{X}} - \left(\frac{af}{g} - \frac{bf^2}{g^2}\right)\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^3\partial x\sqrt{X}}{X'} = \frac{b}{g}\int \frac{x^3\partial x}{\sqrt{X}} + \left(\frac{a}{g} - \frac{bf}{g^2}\right)\int \frac{x\partial x}{\sqrt{X}} - \left(\frac{af}{g} - \frac{bf^2}{g^2}\right)\int \frac{x\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^4\partial x\sqrt{X}}{X'} = \frac{b}{g}\int \frac{x^4\partial x}{\sqrt{X}} + \left(\frac{a}{g} - \frac{bf}{g^2}\right)\int \frac{x^2\partial x}{\sqrt{X}} - \left(\frac{af}{g^2} - \frac{bf^2}{g^3}\right)\int \frac{\partial x}{\sqrt{X}}$$
$$+ \left(\frac{af^2}{g^2} - \frac{bf^3}{g^3}\right)\int \frac{\partial x}{X'\sqrt{X}}$$

$$\int \frac{x^5\partial x\sqrt{X}}{X'} = \frac{b}{g}\int \frac{x^5\partial x}{\sqrt{X}} + \left(\frac{a}{g} - \frac{bf}{g^2}\right)\int \frac{x^3\partial x}{\sqrt{X}} - \left(\frac{af}{g^2} - \frac{bf^2}{g^3}\right)\int \frac{x\partial x}{\sqrt{X}}$$
$$+ \left(\frac{af^2}{g^2} - \frac{bf^3}{g^3}\right)\int \frac{x\partial x}{X'\sqrt{X}}$$

Anmerkung zur vorhergehenden Tafel.

I. $\int \frac{\partial x}{X' \sqrt{X}}$

Es ist im Allgemeinen, was auch a, b, f, g für Vorzeichen haben mögen,

$$\int \frac{\partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(bf^2 - afg)}} \log \frac{f\sqrt{(a+bx^2)} + x\sqrt{(bf^2 - afg)}}{\sqrt{(f+gx^2)}}$$

$$\text{oder} \int \frac{\partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(afg - bf^2)}} \text{Arc Tang} \frac{x\sqrt{(afg - bf^2)}}{f\sqrt{(a+bx^2)}}.$$

Die erste Form wird reell, wenn $bf^2 - afg$ eine positive, die zweite, wenn $bf^2 - afg$ eine negative Gröſse ist. Für $\sqrt{(f+gx^2)}$ in der ersten Form kann man unbeschadet auch $\sqrt{-(f+gx^2)}$ setzen, wenn f und g negativ seyn sollten. Uebrigens ist

$$\text{Arc Tang} \frac{x\sqrt{(afg - bf^2)}}{f\sqrt{(a+bx^2)}} = \text{Arc Cos} \sqrt{\frac{af + bfx^2}{af + agx^2}}$$

$$= \text{Arc Sin } x\sqrt{\frac{ag - bf}{af + agx^2}} = \text{etc.}$$

II. $\int \frac{x\partial x}{X' \sqrt{X}}$

Für jedes a, b, f, g, ist entweder

$$\int \frac{x\partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(ag^2 - bfg)}} \log \frac{g\sqrt{(a+bx^2)} - \sqrt{(ag^2 - bfg)}}{\sqrt{(f+gx^2)}}$$

$$\text{oder} \int \frac{x\partial x}{X' \sqrt{X}} = \frac{1}{\sqrt{(bfg - ag^2)}} \text{Arc Tang} \frac{g\sqrt{(a+bx^2)}}{\sqrt{(bfg - ag^2)}}.$$

Die erste Form wird reell, wenn $ag^2 - bfg$ eine positive, die zweite, wenn $ag^2 - bfg$ eine negative Gröſse ist. Wegen $\sqrt{(f+gx^2)}$ in der ersten Form die nämliche Bemerkung wie oben. Uebrigens ist

$$\text{Arc Tang} \frac{g\sqrt{(a+bx^2)}}{\sqrt{(bfg - ag^2)}} = \text{Arc Cos} \sqrt{\frac{bf - ag}{bf + bgx^2}}$$

$$= \text{Arc Sin} \sqrt{\frac{ag + bgx^2}{bf + bgx^2}} = \text{etc.}$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^n = X$

$$\int x^m \partial x X^p = \frac{x^{m+1} X^p}{m+1} - \frac{pnb}{m+1} \int x^{m+n} \partial x X^{p-1}$$

$$\int \frac{x^m \partial x}{X^p} = - \frac{x^{m-n+1}}{(p-1)nbX^{p-1}} + \frac{m-n+1}{(p-1)nb} \int \frac{x^{m-n} \partial x}{X^{p-1}}$$

$$\int x^m \partial x X^p = \frac{x^{m-n+1} X^{p+1}}{(m+np+1)b} - \frac{(m-n+1)a}{(m+np+1)b} \int x^{m-n} \partial x X^p$$

$$\int \frac{x^m \partial x}{X^p} = \frac{x^{m-n+1}}{(m-np+1)bX^{p-1}} - \frac{(m-n+1)a}{(m-np+1)b} \int \frac{x^{m-n} \partial x}{X^p}$$

$$\int x^m \partial x X^p = \frac{x^{m+1} X^p}{m+np+1} + \frac{pna}{m+np+1} \int x^m \partial x X^{p-1}$$

$$\int \frac{\partial x X^p}{x^m} = - \frac{X^p}{(m-np-1)x^{m-1}} - \frac{pna}{m-np-1} \int \frac{\partial x X^{p-1}}{x^m}$$

$$\int \frac{\partial x X^p}{x^m} = - \frac{X^{p+1}}{(m-1)ax^{m-1}} - \frac{(m-n-np-1)b}{(m-1)a} \int \frac{\partial x X^p}{x^{m-n}}$$

$$\int \frac{\partial x}{x^m X^p} = - \frac{1}{(m-1)ax^{m-1}X^{p-1}} - \frac{(m-n+np-1)b}{(m-1)a} \int \frac{\partial x}{x^{m-n} X^p}$$

$$\int \frac{x^m \partial x}{X^p} = \frac{x^{m+1}}{(p-1)naX^{p-1}} - \frac{m+n-np+1}{(p-1)na} \int \frac{x^m \partial x}{X^{p-1}}$$

$$\int \frac{\partial x}{x^m X^p} = \frac{1}{(p-1)nax^{m-1}X^{p-1}} + \frac{m-n+np-1}{(p-1)na} \int \frac{\partial x}{x^m X^{p-1}}$$

$$\int \frac{\partial x}{X^p} = \frac{x}{(p-1)naX^{p-1}} + \frac{np-n-1}{(p-1)na} \int \frac{\partial x}{X^{p-1}}$$

$$\int \partial x X^p = \frac{x X^p}{np+1} + \frac{pna}{np+1} \int \partial x X^{p-1}$$

$$\int \frac{x^m \partial x}{(f+gx)\sqrt{(a+bx+cx^2)}}$$ Taf. XCVI.

VZ. $a+bx+cx^2=X,\ f+gx=Z$
$ag^2-bfg+cf^2=k$

$$\int \frac{\partial x}{Z\sqrt{X}} = \begin{cases} \pm \frac{1}{\sqrt{k}} \log \frac{2ag-bf+(bg-2cf)x \mp 2\sqrt{k}.\sqrt{X}}{f+gx} \\ \text{oder} \\ \frac{1}{\sqrt{-k}} \operatorname{Arc\,Tang} \frac{2ag-bf+(bg-2cf)x}{2\sqrt{-k}.\sqrt{X}} \end{cases} \text{*)}$$

$$\int \frac{x\partial x}{Z\sqrt{X}} = \frac{1}{g}\int\frac{\partial x}{\sqrt{X}} - \frac{f}{g}\int\frac{\partial x}{Z\sqrt{X}}$$

$$\int \frac{x^2\partial x}{Z\sqrt{X}} = \frac{1}{g}\int\frac{x\partial x}{\sqrt{X}} - \frac{f}{g^2}\int\frac{\partial x}{\sqrt{X}} + \frac{f^2}{g^2}\int\frac{\partial x}{Z\sqrt{X}}$$

$$\int \frac{x^3\partial x}{Z\sqrt{X}} = \frac{1}{g}\int\frac{x^2\partial x}{\sqrt{X}} - \frac{f}{g^2}\int\frac{x\partial x}{\sqrt{X}} + \frac{f^2}{g^3}\int\frac{\partial x}{\sqrt{X}} - \frac{f^3}{g^3}\int\frac{\partial x}{Z\sqrt{X}}$$

$$\int \frac{x^4\partial x}{Z\sqrt{X}} = \frac{1}{g}\int\frac{x^3\partial x}{\sqrt{X}} - \frac{f}{g^2}\int\frac{x^2\partial x}{\sqrt{X}} + \frac{f^2}{g^3}\int\frac{x\partial x}{\sqrt{X}} - \frac{f^3}{g^4}\int\frac{\partial x}{\sqrt{X}} + \frac{f^4}{g^4}\int\frac{\partial x}{Z\sqrt{X}}$$

$$\int \frac{x^5\partial x}{Z\sqrt{X}} = \frac{1}{g}\int\frac{x^4\partial x}{\sqrt{X}} - \frac{f}{g^2}\int\frac{x^3\partial x}{\sqrt{X}} + \frac{f^2}{g^3}\int\frac{x^2\partial x}{\sqrt{X}} - \frac{f^3}{g^4}\int\frac{x\partial x}{\sqrt{X}} + \frac{f^4}{g^5}\int\frac{\partial x}{\sqrt{X}} - \frac{f^5}{g^5}\int\frac{\partial x}{Z\sqrt{X}}$$

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*) Der erste Ausdruck wird reell, wenn k positiv, der zweite, wenn k negativ ist. Von den Zeichen $\pm$ $\mp$ in dem ersten Ausdrucke gehören die oberen zusammen, und eben so die unteren; sonst ist es gleichgültig, welche man braucht. Uebrigens ist

$$\operatorname{Arc\,Tang} \frac{2ag-bf+(bg-2cf)x}{2\sqrt{-k}.\sqrt{X}} = \operatorname{Arc\,Cos} \frac{2\sqrt{-k}.\sqrt{X}}{(f+gx)\sqrt{(b^2-4ac)}}$$
$$= \operatorname{Arc\,Sin} \frac{2ag-bf+(bg-2cf)x}{(f+gx)\sqrt{(b^2-4ac)}} = \text{etc.}$$

Die Wurzelgröſse $\sqrt{(b^2-4ac)}$, welche hier im Sinus und Cosinus vorkommt, wird gewiſs reell, wenn $ag^2-bfg+cf^2$ eine negative Gröſse ist, weil sonst $\sqrt{(a+bx+cx^2)}$ nicht reell seyn könnte.

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^n = X$

$$\int x^m \partial x X^p = \frac{x^{m+1}X^p}{m+1} - \frac{pnb}{m+1}\int x^{m+n}\partial x X^{p-1}$$

$$\int \frac{x^m \partial x}{X^p} = -\frac{x^{m-n+1}}{(p-1)nbX^{p-1}} + \frac{m-n+1}{(p-1)nb}\int \frac{x^{m-n}\partial x}{X^{p-1}}$$

$$\int x^m \partial x X^p = \frac{x^{m-n+1}X^{p+1}}{(m+np+1)b} - \frac{(m-n+1)a}{(m+np+1)b}\int x^{m-n}\partial x X^p$$

$$\int \frac{x^m \partial x}{X^p} = \frac{x^{m-n+1}}{(m-np+1)bX^{p-1}} - \frac{(m-n+1)a}{(m-np+1)b}\int \frac{x^{m-n}\partial x}{X^p}$$

$$\int x^m \partial x X^p = \frac{x^{m+1}X^p}{m+np+1} + \frac{pna}{m+np+1}\int x^m \partial x X^{p-1}$$

$$\int \frac{\partial x X^p}{x^m} = -\frac{X^p}{(m-np-1)x^{m-1}} - \frac{pna}{m-np-1}\int \frac{\partial x X^{p-1}}{x^m}$$

$$\int \frac{\partial x X^p}{x^m} = -\frac{X^{p+1}}{(m-1)ax^{m-1}} - \frac{(m-n-np-1)b}{(m-1)a}\int \frac{\partial x X^p}{x^{m-n}}$$

$$\int \frac{\partial x}{x^m X^p} = -\frac{1}{(m-1)ax^{m-1}X^{p-1}} - \frac{(m-n+np-1)b}{(m-1)a}\int \frac{\partial x}{x^{m-n}X^p}$$

$$\int \frac{x^m \partial x}{X^p} = \frac{x^{m+1}}{(p-1)naX^{p-1}} - \frac{m+n-np+1}{(p-1)na}\int \frac{x^m \partial x}{X^{p-1}}$$

$$\int \frac{\partial x}{x^m X^p} = \frac{1}{(p-1)nax^{m-1}X^{p-1}} + \frac{m-n+np-1}{(p-1)na}\int \frac{\partial x}{x^m X^{p-1}}$$

$$\int \frac{\partial x}{X^p} = \frac{x}{(p-1)naX^{p-1}} + \frac{np-n-1}{(p-1)na}\int \frac{\partial x}{X^{p-1}}$$

$$\int \partial x X^p = \frac{xX^p}{np+1} + \frac{pna}{np+1}\int \partial x X^{p-1}$$

Tafel

einiger allgemeineren Formeln.

VZ. $ax^k + bx^{k+n} = X$

$$\int x^m \partial x X^p = \frac{x^{m+1} X^p}{m+pk+1} - \frac{pnb}{m+pk+1} \int x^{m+k+n} \partial x X^{p-1}$$

$$\int \frac{x^m \partial x}{X^p} = -\frac{x^{m-k-n+1}}{(p-1)nbX^{p-1}} + \frac{m-pk-n+1}{(p-1)nb} \int \frac{x^{m-k-n} \partial x}{X^{p-1}}$$

$$\int x^m \partial x X^p = \frac{x^{m-k-n+1} X^{p+1}}{(m+pk+np+1)b} - \frac{(m+pk-n+1)a}{(m+pk+np+1)b} \int x^{m-n} \partial x X^p$$

$$\int \frac{x^m \partial x}{X^p} = \frac{x^{m-k-n+1}}{(m-pk-np+1)bX^{p-1}} - \frac{(m-pk-n+1)a}{(m-pk-np+1)b} \int \frac{x^{m-n} \partial x}{X^p}$$

$$\int x^m \partial x X^p = \frac{x^{m+1} X^p}{m+pk+np+1} + \frac{pna}{m+pk+np+1} \int x^{m+k} \partial x X^{p-1}$$

$$\int \frac{\partial x X^p}{x^m} = -\frac{X^p}{(m-pk-np-1)x^{m-1}} - \frac{pna}{m-pk-np-1} \int \frac{\partial x X^{p-1}}{x^{m-k}}$$

$$\int \frac{\partial x X^p}{x^m} = -\frac{X^{p+1}}{(m-pk-1)ax^{m+k-1}} - \frac{(m-n-pk-np-1)b}{(m-pk-1)a} \int \frac{\partial x X^p}{x^{m-n}}$$

$$\int \frac{\partial x}{x^m X^p} = -\frac{1}{(m+pk-1)ax^{m+k-1}X^{p-1}} - \frac{(m-n+pk+np-1)b}{(m+pk-1)a} \int \frac{\partial x}{x^{m-n} X^p}$$

$$\int \frac{x^m \partial x}{X^p} = \frac{x^{m-k+1}}{(p-1)naX^{p-1}} - \frac{m+n-pk-np+1}{(p-1)na} \int \frac{x^{m-k} \partial x}{X^{p-1}}$$

$$\int \frac{\partial x}{x^m X^p} = \frac{1}{(p-1)nax^{m+k-1}X^{p-1}} + \frac{m-n+pk+np-1}{(p-1)na} \int \frac{\partial x}{x^{m+k} X^{p-1}}$$

$$\int \frac{\partial x}{X^p} = \frac{1}{(p-1)nax^{k-1}X^{p-1}} + \frac{pk+np-n-1}{(p-1)na} \int \frac{\partial x}{x^k X^{p-1}}$$

$$\int \partial x X^p = \frac{xX^p}{pk+np+1} + \frac{pna}{pk+np+1} \int x^k \partial x X^{p-1}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$\int \frac{x^m \partial x}{\sqrt{X}} = \Big(\frac{X^m}{2m+1} - \frac{{}^m\mathfrak{A} a X^{m-1}}{2m-1} + \frac{{}^m\mathfrak{B} a^2 X^{m-2}}{2m-3} - \frac{{}^m\mathfrak{C} a^3 X^{m-3}}{2m-5} + \ldots\ldots$$

$$\ldots\ldots \pm \frac{{}^m\overset{-2}{\mathfrak{M}} a^{m-2} X^2}{5} \mp \frac{{}^m\overset{-1}{\mathfrak{M}} a^{m-1} X}{3} \pm \frac{{}^m\mathfrak{M} a^m}{1}\Big)\frac{2\sqrt{X}}{b^{m+1}}$$

$$\int \frac{x^m \partial x}{X^{\frac{3}{2}}} = \Big(\frac{X^m}{2m-1} - \frac{{}^m\mathfrak{A} a X^{m-1}}{2m-3} + \frac{{}^m\mathfrak{B} a^2 X^{m-2}}{2m-5} - \frac{{}^m\mathfrak{C} a^3 X^{m-3}}{2m-7} + \ldots\ldots$$

$$\ldots\ldots \pm \frac{{}^m\overset{-2}{\mathfrak{M}} a^{m-2} X^2}{3} \mp \frac{{}^m\overset{-1}{\mathfrak{M}} a^{m-1} X}{1} \pm \frac{{}^m\mathfrak{M} a^m}{-1}\Big)\frac{2}{b^{m+1}\sqrt{X}}$$

$$\int \frac{x^m \partial x}{X^{\frac{5}{2}}} = \Big(\frac{X^m}{2m-3} - \frac{{}^m\mathfrak{A} a X^{m-1}}{2m-5} + \frac{{}^m\mathfrak{B} a^2 X^{m-2}}{2m-7} - \frac{{}^m\mathfrak{C} a^3 X^{m-3}}{2m-9} + \ldots\ldots$$

$$\ldots\ldots \pm \frac{{}^m\overset{-2}{\mathfrak{M}} a^{m-2} X^2}{1} \mp \frac{{}^m\overset{-1}{\mathfrak{M}} a^{m-1} X}{-1} \pm \frac{{}^m\mathfrak{M} a^m}{-3}\Big)\frac{2}{b^{m+1} X\sqrt{X}}$$

$$\int \frac{x^m \partial x}{X^{\frac{7}{2}}} = \Big(\frac{X^m}{2m-5} - \frac{{}^m\mathfrak{A} a X^{m-1}}{2m-7} + \frac{{}^m\mathfrak{B} a^2 X^{m-2}}{2m-9} - \frac{{}^m\mathfrak{C} a^3 X^{m-3}}{2m-11} + \ldots\ldots$$

$$\ldots\ldots \pm \frac{{}^m\overset{-2}{\mathfrak{M}} a^{m-2} X^2}{-1} \mp \frac{{}^m\overset{-1}{\mathfrak{M}} a^{m-1} X}{-3} \pm \frac{{}^m\mathfrak{M} a^m}{-5}\Big)\frac{2}{b^{m+1} X^2\sqrt{X}}$$

$$\int \frac{x^m \partial x}{X^{\frac{n}{2}}} = \Big(\frac{X^m}{2m-n+2} - \frac{{}^m\mathfrak{A} a X^{m-1}}{2m-n} + \frac{{}^m\mathfrak{B} a^2 X^{m-2}}{2m-n-2} - \ldots\ldots$$

$$\ldots\ldots \pm \frac{{}^m\overset{-2}{\mathfrak{M}} a^{m-2} X^2}{-(n-6)} \mp \frac{{}^m\overset{-1}{\mathfrak{M}} a^{m-1} X}{-(n-4)} \pm \frac{{}^m\mathfrak{M} a^m}{-(n-2)}\Big)\frac{2}{b^{m+1} X^{\frac{n-2}{2}}}$$

$$\int x^m \partial x \sqrt{X} = \Big(\frac{X^m}{2m+3} - \frac{{}^m\mathfrak{A} a X^{m-1}}{2m+1} + \frac{{}^m\mathfrak{B} a^2 X^{m-2}}{2m-1} - \frac{{}^m\mathfrak{C} a^3 X^{m-3}}{2m-3} + \ldots\ldots$$

$$\ldots\ldots \pm \frac{{}^m\overset{-2}{\mathfrak{M}} a^{m-2} X^2}{7} \mp \frac{{}^m\overset{-1}{\mathfrak{M}} a^{m-1} X}{5} \pm \frac{{}^m\mathfrak{M} a^m}{3}\Big)\frac{2X\sqrt{X}}{b^{m+1}}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$\int x^m \partial x X^{\frac{3}{2}} = \left(\frac{X^m}{2m+5} - \frac{{}^m\mathfrak{A}aX^{m-1}}{2m+3} + \frac{{}^m\mathfrak{B}a^2X^{m-2}}{2m+1} - \frac{{}^m\mathfrak{C}a^3X^{m-3}}{2m-1} + \ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{\overset{-2}{{}^m\mathfrak{M}}a^{m-2}X^2}{9} \mp \frac{\overset{-1}{{}^m\mathfrak{M}}a^{m-1}X}{7} \pm \frac{{}^m\mathfrak{M}a^m}{5}\right)\frac{2X^2\sqrt{X}}{b^{m+1}}$$

$$\int x^m \partial x X^{\frac{5}{2}} = \left(\frac{X^m}{2m+7} - \frac{{}^m\mathfrak{A}aX^{m-1}}{2m+5} + \frac{{}^m\mathfrak{B}a^2X^{m-2}}{2m+3} - \frac{{}^m\mathfrak{C}a^3X^{m-3}}{2m+1} + \ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{\overset{-2}{{}^m\mathfrak{M}}a^{m-2}X^2}{11} \mp \frac{\overset{-1}{{}^m\mathfrak{M}}a^{m-1}X}{9} \pm \frac{{}^m\mathfrak{M}a^m}{7}\right)\frac{2X^3\sqrt{X}}{b^{m+1}}$$

$$\int x^m \partial x X^{\frac{n}{2}} = \left(\frac{X^m}{2m+n+2} - \frac{{}^m\mathfrak{A}aX^{m-1}}{2m+n} + \frac{{}^m\mathfrak{B}a^2X^{m-2}}{2m+n-2} - \frac{{}^m\mathfrak{C}a^3X^{m-3}}{2m+n-4} + \ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{\overset{-2}{{}^m\mathfrak{M}}a^{m-2}X^2}{n+6} \mp \frac{\overset{-1}{{}^m\mathfrak{M}}a^{m-1}X}{n+4} \pm \frac{{}^m\mathfrak{M}a^m}{n+2}\right)\frac{2X^{\frac{n+2}{2}}}{b^{m+1}}$$

$$\int \frac{x^m \partial x}{X^{\frac{p}{q}}} = \left(\frac{X^m}{qm-p+q} - \frac{{}^m\mathfrak{A}aX^{m-1}}{qm-p} + \frac{{}^m\mathfrak{B}a^2X^{m-2}}{qm-p-q} - \frac{{}^m\mathfrak{C}a^3X^{m-3}}{qm-p-2q}\right.$$

$$+ \frac{{}^m\mathfrak{D}a^4X^{m-4}}{qm-p-3q} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{\overset{-2}{{}^m\mathfrak{M}}a^{m-2}X^2}{-(p-3q)} \mp \frac{\overset{-1}{{}^m\mathfrak{M}}a^{m-1}X}{-(p-2q)} \pm \frac{{}^m\mathfrak{M}a^m}{-(p-q)}\right)\frac{q}{b^{m+1}X^{\frac{p}{q}-1}}$$

$$\int x^m \partial x X^{\frac{p}{q}} = \left(\frac{X^m}{qm+p+q} - \frac{{}^m\mathfrak{A}aX^{m-1}}{qm+p} + \frac{{}^m\mathfrak{B}a^2X^{m-2}}{qm+p-q} - \frac{{}^m\mathfrak{C}a^3X^{m-3}}{qm+p-2q}\right.$$

$$+ \frac{{}^m\mathfrak{D}a^4X^{m-4}}{qm+p-3q} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{\overset{-2}{{}^m\mathfrak{M}}a^{m-2}X^2}{p+3q} \mp \frac{\overset{-1}{{}^m\mathfrak{M}}a^{m-1}X}{p+2q} \pm \frac{{}^m\mathfrak{M}a^m}{p+q}\right)\frac{qX^{\frac{p}{q}+1}}{b^{m+1}}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$\int \frac{\partial x}{x^m \sqrt{X}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \cdots\cdots\right.$$

$$\left.\cdots\cdots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\sqrt{X} \mp \frac{Lb}{2}\int \frac{\partial x}{x\sqrt{X}}.$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-3)b}{(2m-4)a}A,\ C = \frac{(2m-5)b}{(2m-6)a}B,$$

$$D = \frac{(2m-7)b}{(2m-8)a}C,\ E = \frac{(2m-9)b}{(2m-10)a}D,\ \cdots\cdots L = \frac{3b}{2a}K.$$

$$\int \frac{\partial x}{x^m X^{\frac{3}{2}}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \cdots\cdots\right.$$

$$\left.\cdots\cdots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{\sqrt{X}} \mp \frac{3Lb}{2}\int \frac{\partial x}{xX^{\frac{3}{2}}}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-1)b}{(2m-4)a}A,\ C = \frac{(2m-3)b}{(2m-6)a}B,$$

$$D = \frac{(2m-5)b}{(2m-8)a}C,\ E = \frac{(2m-7)b}{(2m-10)a}D,\ \cdots\cdots L = \frac{5b}{2a}K.$$

$$\int \frac{\partial x}{x^m X^{\frac{5}{2}}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \cdots\cdots\right.$$

$$\left.\cdots\cdots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{X\sqrt{X}} \mp \frac{5bL}{2}\int \frac{\partial x}{xX^{\frac{5}{2}}}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m+1)b}{(2m-4)a}A,\ C = \frac{(2m-1)b}{(2m-6)a}B,$$

$$D = \frac{(2m-3)b}{(2m-8)a}C,\ E = \frac{(2m-5)b}{(2m-10)a}D,\ \cdots\cdots L = \frac{7b}{2a}K.$$

$$\int \frac{\partial x}{x^m X^{\frac{n}{2}}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \cdots\cdots\right.$$

$$\left.\cdots\cdots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{X^{\frac{n-2}{2}}} \mp \frac{nbL}{2}\int \frac{\partial x}{xX^{\frac{n}{2}}}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m+n-4)b}{(2m-4)a}A,\ C = \frac{(2m+n-6)b}{(2m-6)a}B,$$
$$D = \frac{(2m+n-8)b}{(2m-8)a}C,\ E = \frac{(2m+n-10)b}{(2m-10)a}D, \ldots\ldots L = \frac{(n+2)b}{2a}K.$$

$$\int\frac{\partial x}{x^m X^{\frac{p}{q}}} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots\ldots\right.$$
$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{X^{\frac{p-q}{q}}} \mp \frac{pbL}{q}\int\frac{\partial x}{xX^{\frac{p}{q}}}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(qm+p-2q)b}{(m-2)qa}A,\ C = \frac{(qm+p-3q)b}{(m-3)qa}B,$$
$$D = \frac{(qm+p-4q)b}{(m-4)qa}C,\ E = \frac{(qm+p-5q)b}{(m-5)qa}D, \ldots\ldots L = \frac{(p+q)b}{qa}K.$$

$$\int\frac{\partial x\sqrt{X}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots\ldots\right.$$
$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)X\sqrt{X} \pm \frac{bL}{2}\int\frac{\partial x\sqrt{X}}{x}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-5)b}{(2m-4)a}A,\ C = \frac{(2m-7)b}{(2m-6)a}B,$$
$$D = \frac{(2m-9)b}{(2m-8)a}C,\ E = \frac{(2m-11)b}{(2m-10)a}D, \ldots\ldots L = \frac{b}{2a}K.$$

$$\int\frac{\partial x X^{\frac{3}{2}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots\ldots\right.$$
$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)X^2\sqrt{X} \pm \frac{3bL}{2}\int\frac{\partial x X^{\frac{3}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-7)b}{(2m-4)a}A,\ C = \frac{(2m-9)b}{(2m-6)a}B,$$
$$D = \frac{(2m-11)b}{(2m-8)a}C,\ E = \frac{(2m-13)b}{(2m-10)a}D, \ldots\ldots L = \frac{-b}{2a}K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$\int \frac{\partial x X^{\frac{5}{2}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right) X^3 \sqrt{X} \pm \frac{5bL}{2} \int \frac{\partial x X^{\frac{5}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-9)b}{(2m-4)a} A,\ C = \frac{(2m-11)b}{(2m-6)a} B,$$

$$D = \frac{(2m-13)b}{(2m-8)a} C,\ E = \frac{(2m-15)b}{(2m-10)a} D,\ \ldots\ldots L = \frac{-3b}{2a} K.$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right) X^{\frac{n+2}{2}} \pm \frac{nbL}{2} \int \frac{\partial x X^{\frac{n}{2}}}{x}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(2m-n-4)b}{(2m-4)a} A,\ C = \frac{(2m-n-6)b}{(2m-6)a} B,$$

$$D = \frac{(2m-n-8)b}{(2m-8)a} C,\ \ldots\ldots\ldots\ldots L = \frac{-(n-2)b}{2a} K.$$

$$\int \frac{\partial x X^{\frac{p}{q}}}{x^m} = \left(\frac{A}{x^{m-1}} - \frac{B}{x^{m-2}} + \frac{C}{x^{m-3}} - \frac{D}{x^{m-4}} + \frac{E}{x^{m-5}} - \ldots\ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right) X^{\frac{p+q}{q}} \pm \frac{pbL}{q} \int \frac{\partial x X^{\frac{p}{q}}}{x}$$

$$A = -\frac{1}{(m-1)a},\ B = \frac{(qm-p-2q)b}{(m-2)qa} A,\ C = \frac{(qm-p-3q)b}{(m-3)qa} B,$$

$$D = \frac{(qm-p-4q)b}{(m-4)qa} C,\ E = \frac{(qm-p-5q)b}{(m-5)qa} D,\ \ldots\ldots L = \frac{(q-p)b}{qa} K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx = X$

$$\int \frac{\partial x X^{\frac{2n+1}{2}}}{x} = \left(\frac{X^n}{2n+1} + \frac{aX^{n-1}}{2n-1} + \frac{a^2X^{n-2}}{2n-3} + \frac{a^3X^{n-3}}{2n-5} + \ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots + \frac{a^{n-2}X^2}{5} + \frac{a^{n-1}X}{3} + \frac{a^n}{1}\right) 2\sqrt{X} + a^{n+1} \int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x X^{\frac{p}{q}}}{x} = \frac{qX^{\frac{p}{q}}}{p} + \frac{qaX^{\frac{p}{q}-1}}{p-q} + \frac{qa^2X^{\frac{p}{q}-2}}{p-2q} + \frac{qa^3X^{\frac{p}{q}-3}}{p-3q} + \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ldots + \frac{qa^{i-1}X^{\frac{p}{q}-i+1}}{p-(i-1)q} + a^i \int \frac{\partial x X^{\frac{p}{q}-i}}{x}$$

$$\int \frac{\partial x}{xX^{\frac{2n+1}{2}}} = \left[\frac{1}{(2n-1)aX^{n-1}} + \frac{1}{(2n-3)a^2X^{n-2}} + \frac{1}{(2n-5)a^3X^{n-3}} + \ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots + \frac{1}{5a^{n-2}X^2} + \frac{1}{3a^{n-1}X} + \frac{1}{a^n}\right]\frac{2}{\sqrt{X}} + \frac{1}{a^n}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{xX^{\frac{p}{q}}} = \frac{q}{(p-q)aX^{\frac{p}{q}-1}} + \frac{q}{(p-2q)a^2X^{\frac{p}{q}-2}} + \frac{q}{(p-3q)a^3X^{\frac{p}{q}-3}} + \ldots\ldots$$

$$\ldots\ldots\ldots\ldots + \frac{q}{(p-iq)a^iX^{\frac{p}{q}-i}} + \frac{1}{a^i}\int \frac{\partial x}{xX^{\frac{p}{q}-i}}$$

$$\int \frac{x^m \partial x \sqrt{x}}{X^n} = \frac{2x^m\sqrt{x}}{(2m-2n+3)bX^{n-1}} - \frac{(2m+1)a}{(2m-2n+3)b}\int \frac{x^{m-1}\partial x\sqrt{x}}{X^n}$$

$$\int \frac{x^m \partial x \sqrt{x}}{X^n} = \left(Ax^m - Bx^{m-1} + Cx^{m-2} - Dx^{m-3} + \ldots\ldots\ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm Kx^2 \mp Lx\right)\frac{2\sqrt{x}}{X^{n-1}} \pm \frac{3aL}{2}\int \frac{\partial x\sqrt{x}}{X^n}$$

$$A = \frac{1}{(2m-2n+3)b},\; B = \frac{(2m+1)a}{(2m-2n+1)b}A,\; C = \frac{(2m-1)a}{(2m-2n-1)b}B,$$

$$D = \frac{(2m-3)a}{(2m-2n-3)b}C,\; \ldots\ldots\ldots\ldots\; L = \frac{5a}{(5-2n)b}K.$$

Tafel
einiger allgemeineren Formeln.

VZ. $a+bx=X$, $ad-bc=k$

$$\int\frac{\partial x}{X^p\sqrt{(c+dx)}}=\frac{\sqrt{(c+dx)}}{(p-1)kX^{p-1}}+\frac{(2p-3)d}{(2p-2)k}\int\frac{\partial x}{X^{p-1}\sqrt{(c+dx)}}$$

$$\int\frac{\partial x}{X^p\sqrt{(c+dx)}}=\Big(\frac{A}{X^{p-1}}+\frac{B}{X^{p-2}}+\frac{C}{X^{p-3}}+\frac{D}{X^{p-4}}+\frac{E}{X^{p-5}}+\ldots\ldots\ldots$$

$$\ldots\ldots\ldots+\frac{K}{X^2}+\frac{L}{X}\Big)\sqrt{(c+dx)}+\frac{dL}{2}\int\frac{\partial x}{X\sqrt{(c+dx)}}$$

$$A=\frac{1}{(p-1)k},\quad B=\frac{(2p-3)d}{(2p-4)k}A,\quad C=\frac{(2p-5)d}{(2p-6)k}B,$$

$$D=\frac{(2p-7)d}{(2p-8)k}C,\quad E=\frac{(2p-9)d}{(2p-10)k}D,\quad \ldots\ldots L=\frac{3d}{2k}K.$$

VZ. $a+bx^2=X$

$$\int x^m\partial x X^{\frac{n}{2}}=\frac{x^{m+1}X^{\frac{n}{2}}}{m+1}-\frac{nb}{m+1}\int x^{m+2}\partial x X^{\frac{n}{2}-1}$$

$$\int\frac{x^m\partial x}{X^{\frac{n}{2}}}=-\frac{x^{m-1}}{(n-2)bX^{\frac{n}{2}-1}}+\frac{m-1}{(n-2)b}\int\frac{x^{m-2}\partial x}{X^{\frac{n}{2}-1}}$$

$$\int x^m\partial x X^{\frac{n}{2}}=\frac{x^{m-1}X^{\frac{n}{2}+1}}{(m+n+1)b}-\frac{(m-1)a}{(m+n+1)b}\int x^{m-2}\partial x X^{\frac{n}{2}}$$

$$\int\frac{x^m\partial x}{X^{\frac{n}{2}}}=\frac{x^{m-1}}{(m-n+1)bX^{\frac{n}{2}-1}}-\frac{(m-1)a}{(m-n+1)b}\int\frac{x^{m-2}\partial x}{X^{\frac{n}{2}}}$$

$$\int x^m\partial x X^{\frac{n}{2}}=\frac{x^{m+1}X^{\frac{n}{2}}}{m+n+1}+\frac{na}{m+n+1}\int x^m\partial x X^{\frac{n}{2}-1}$$

$$\int\frac{\partial x X^{\frac{n}{2}}}{x^m}=-\frac{X^{\frac{n}{2}}}{(m-n-1)x^{m-1}}-\frac{na}{m-n-1}\int\frac{\partial x X^{\frac{n}{2}-1}}{x^m}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x^m} = -\frac{X^{\frac{n}{2}+1}}{(m-1)ax^{m-1}} - \frac{(m-n-3)b}{(m-1)a}\int \frac{\partial x X^{\frac{n}{2}}}{x^{m-2}}$$

$$\int \frac{\partial x}{x^m X^{\frac{n}{2}}} = -\frac{1}{(m-1)ax^{m-1}X^{\frac{n}{2}-1}} - \frac{(m+n-3)b}{(m-1)a}\int \frac{\partial x}{x^{m-2}X^{\frac{n}{2}}}$$

$$\int \frac{x^m \partial x}{X^{\frac{n}{2}}} = \frac{x^{m+1}}{(n-2)aX^{\frac{n}{2}-1}} - \frac{m-n+3}{(n-2)a}\int \frac{x^m \partial x}{X^{\frac{n}{2}-1}}$$

$$\int \frac{\partial x}{x^m X^{\frac{n}{2}}} = \frac{1}{(n-2)ax^{m-1}X^{\frac{n}{2}-1}} + \frac{m+n-3}{(n-2)a}\int \frac{\partial x}{x^m X^{\frac{n}{2}-1}}$$

$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \frac{x}{(n-2)aX^{\frac{n}{2}-1}} + \frac{n-3}{(n-2)a}\int \frac{\partial x}{X^{\frac{n}{2}-1}}$$

$$\int \partial x X^{\frac{n}{2}} = \frac{xX^{\frac{n}{2}}}{n+1} + \frac{na}{n+1}\int \partial x X^{\frac{n}{2}-1}$$

$$\int \frac{\partial x}{xX^{\frac{n}{2}}} = \frac{1}{(n-2)aX^{\frac{n}{2}-1}} + \frac{1}{a}\int \frac{\partial x}{xX^{\frac{n}{2}-1}}$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x} = \frac{X^{\frac{n}{2}}}{n} + a\int \frac{\partial x X^{\frac{n}{2}-1}}{x}$$

$$\int \frac{\partial x}{X^{\frac{2n+1}{2}}} = \left(\frac{A}{X^{n-1}} + \frac{B}{X^{n-2}} + \frac{C}{X^{n-3}} + \cdots\cdots + \frac{K}{X} + L\right)\frac{x}{\sqrt{X}}$$

$$A = \frac{1}{(2n-1)a},\quad B = \frac{2n-2}{(2n-3)a}A,\quad C = \frac{2n-4}{(2n-5)a}B,$$

$$D = \frac{2n-6}{(2n-7)a}C,\quad E = \frac{2n-8}{(2n-9)a}D,\quad \cdots\cdots L = \frac{2}{a}K.$$

$$\int \frac{x\partial x}{X^n} = -\frac{1}{(n-1)2bX^{n-1}}$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$\int \partial x X^{\frac{2n+1}{2}} = \Big(AX^n + BX^{n-1} + CX^{n-2} + DX^{n-3} + \ldots\ldots \\ \ldots\ldots + KX + L\Big)x\sqrt{X} + La\int \frac{\partial x}{\sqrt{X}}$$

$$A = \frac{1}{2n+2},\quad B = \frac{(2n+1)a}{2n}A,\quad C = \frac{(2n-1)a}{2n-2}B,$$

$$D = \frac{(2n-3)a}{2n-4}C,\quad E = \frac{(2n-5)a}{2n-6}D,\quad \ldots\ldots L = \frac{3a}{2}K.$$

$$\int x\partial x X^n = \frac{X^{n+1}}{(n+1)2b}$$

$$\int \frac{\partial x X^{\frac{2n+1}{2}}}{x} = \Big(\frac{X^n}{2n+1} + \frac{aX^{n-1}}{2n-1} + \frac{a^2X^{n-2}}{2n-3} + \frac{a^3X^{n-3}}{2n-5} + \ldots\ldots \\ \ldots\ldots + \frac{a^{n-2}X^2}{5} + \frac{a^{n-1}X}{3} + \frac{a^n}{1}\Big)\sqrt{X} + a^{n+1}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{\partial x}{xX^{\frac{2n+1}{2}}} = \Big[\frac{1}{(2n-1)aX^{n-1}} + \frac{1}{(2n-3)a^2X^{n-2}} + \frac{1}{(2n-5)a^3X^{n-3}} + \ldots \\ \ldots\ldots + \frac{1}{5a^{n-2}X^2} + \frac{1}{3a^{n-1}X} + \frac{1}{a^n}\Big)\frac{1}{\sqrt{X}} + \frac{1}{a^n}\int \frac{\partial x}{x\sqrt{X}}$$

$$\int \frac{x^m\partial x}{X^{\frac{n}{2}}} = \Big(Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \ldots\ldots \\ \ldots\ldots \pm Kx^{m-2i+3} \mp Lx^{m-2i+1}\Big)X^{-\frac{n}{2}+1} \pm (m-2i+1)aL\int \frac{x^{m-2i}\partial x}{X^{\frac{n}{2}}}$$

$$A = \frac{1}{(m-n+1)b},\quad B = \frac{(m-1)a}{(m-n-1)b}A,\quad C = \frac{(m-3)a}{(m-n-3)b}B,$$

$$D = \frac{(m-5)a}{(m-n-5)b}C;\quad E = \frac{(m-7)a}{(m-n-7)b}D, \ldots\ldots L = \frac{(m-2i+3)a}{(m-n-2i+3)b}K.$$

$$\int \frac{x^{2m+1}\partial x}{\sqrt{X}} = \Big(Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + \ldots\ldots \\ \ldots\ldots \pm Kx^4 \mp Lx^2\Big)\sqrt{X} \pm 2aL\int \frac{x\partial x}{\sqrt{X}}$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$A = \frac{1}{2m+1},\quad B = \frac{2ma}{(2m-1)b}A,\quad C = \frac{(2m-2)a}{(2m-3)b}B,$$
$$D = \frac{(2m-4)a}{(2m-5)b}C,\quad E = \frac{(2m-6)a}{(2m-7)b}D,\ \ldots\ldots\ L = \frac{4a}{3b}K.$$

$$\int \frac{x^{2m}\partial x}{\sqrt{X}} = \Big(Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \ldots\ldots$$
$$\ldots\ldots \pm Kx^3 \mp Lx\Big)\sqrt{X} \pm aL\int\frac{\partial x}{\sqrt{X}}$$

$$A = \frac{1}{2mb},\quad B = \frac{(2m-1)a}{(2m-2)b}A,\quad C = \frac{(2m-3)a}{(2m-4)b}B,$$
$$D = \frac{(2m-5)a}{(2m-6)b}C,\quad E = \frac{(2m-7)a}{(2m-8)b}D,\ \ldots\ldots\ L = \frac{3a}{2b}K.$$

$$\int \frac{x^{2m+1}\partial x}{X^{\frac{3}{2}}} = \Big(Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \ldots\ldots$$
$$\ldots\ldots \pm Kx^4 \mp Lx^2\Big)\frac{1}{\sqrt{X}} \pm 2aL\int\frac{x\partial x}{X^{\frac{3}{2}}}$$

$$A = \frac{1}{(2m-1)b},\quad B = \frac{2ma}{(2m-3)b}A,\quad C = \frac{(2m-2)a}{(2m-5)b}B,$$
$$D = \frac{(2m-4)a}{(2m-7)b}C,\quad E = \frac{(2m-6)a}{(2m-9)b}D,\ \ldots\ldots\ L = \frac{4a}{b}K.$$

$$\int \frac{x^{2m}\partial x}{X^{\frac{3}{2}}} = \Big(Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \ldots\ldots$$
$$\ldots\ldots \pm Kx^5 \mp Lx^3\Big)\frac{1}{\sqrt{X}} \pm 3aL\int\frac{x^2\partial x}{X^{\frac{3}{2}}}$$

$$A = \frac{1}{(2m-2)b},\quad B = \frac{(2m-1)a}{(2m-4)b}A,\quad C = \frac{(2m-3)a}{(2m-6)b}B,$$
$$D = \frac{(2m-5)a}{(2m-8)b}C,\quad E = \frac{(2m-7)a}{(2m-10)b}D,\ \ldots\ldots\ L = \frac{5a}{2b}K.$$

$$\int \frac{\partial x}{x^m X^{\frac{n}{2}}} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \ldots\ldots$$
$$\ldots\ldots \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\Big)X^{-\frac{n}{2}+1} \mp (m+n-2i-1)bL\int\frac{\partial x}{x^{m-2i}X^{\frac{n}{2}}}$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$A = -\frac{1}{(m-1)a}, \quad B = \frac{(m+n-3)b}{(m-3)a}A, \quad C = \frac{(m+n-5)b}{(m-5)a}B,$$

$$D = \frac{(m+n-7)b}{(m-7)a}C, \quad E = \frac{(m+n-9)b}{(m-9)a}D, \ldots\ldots L = \frac{(m+n-2i+1)b}{(m-2i+1)a}K.$$

$$\int \frac{\partial x}{x^{2m+1}\sqrt{X}} = \left(\frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \ldots\ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^4} \mp \frac{L}{x^2}\right)\sqrt{X} \mp bL\int \frac{\partial x}{x\sqrt{X}}$$

$$A = -\frac{1}{2ma}, \quad B = \frac{(2m-1)b}{(2m-2)a}A, \quad C = \frac{(2m-3)b}{(2m-4)a}B,$$

$$D = \frac{(2m-5)b}{(2m-6)a}C, \quad E = \frac{(2m-7)b}{(2m-8)a}D, \ldots\ldots L = \frac{3b}{2a}K.$$

$$\int \frac{\partial x}{x^{2m}\sqrt{X}} = \left(\frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \ldots\ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^3} \mp \frac{L}{x}\right)\sqrt{X}$$

$$A = -\frac{1}{(2m-1)a}, \quad B = \frac{(2m-2)b}{(2m-3)a}A, \quad C = \frac{(2m-4)b}{(2m-5)a}B,$$

$$D = \frac{(2m-6)b}{(2m-7)a}C, \quad E = \frac{(2m-8)b}{(2m-9)a}D, \ldots\ldots\ldots L = \frac{2b}{a}K.$$

$$\int \frac{\partial x}{x^{2m+1}X^{\frac{3}{2}}} = \left(\frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \ldots\ldots\ldots\ldots\ldots\right.$$

$$\left.\ldots\ldots\ldots\ldots \pm \frac{K}{x^4} \mp \frac{L}{x^2}\right)\frac{1}{\sqrt{X}} \mp 3bL\int \frac{\partial x}{xX^{\frac{3}{2}}}$$

$$A = -\frac{1}{2ma}, \quad B = \frac{(2m+1)b}{(2m-2)a}A, \quad C = \frac{(2m-1)b}{(2m-4)a}B,$$

$$D = \frac{(2m-3)b}{(2m-6)a}C, \quad E = \frac{(2m-5)b}{(2m-8)a}D, \ldots\ldots L = \frac{5b}{2a}K.$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$\int \frac{\partial x}{x^{2m} X^{\frac{3}{2}}} = \left(\frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \dots\dots \right.$$

$$\left. \dots\dots \pm \frac{K}{x^3} \mp \frac{L}{x} \right) \frac{1}{\sqrt{X}} \mp 2bL \int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$A = -\frac{1}{(2m-1)a},\; B = \frac{2mb}{(2m-3)a} A,\; C = \frac{(2m-2)b}{(2m-5)a} B,$$

$$D = \frac{(2m-4)b}{(2m-7)a} C,\; E = \frac{(2m-6)b}{(2m-9)a} D,\; \dots\dots L = \frac{4b}{a} K.$$

$$\int x^m \partial x X^{\frac{n}{2}} = \left(Ax^{m-1} - Bx^{m-3} + Cx^{m-5} - Dx^{m-7} + Ex^{m-9} - \dots\dots \right.$$

$$\left. \dots \pm Kx^{m-2i+3} \mp Lx^{m-2i+1} \right) X^{\frac{n}{2}+1} \pm (m-2i+1)aL \int x^{m-2i} \partial x X^{\frac{n}{2}}$$

$$A = \frac{1}{(m+n+1)b},\; B = \frac{(m-1)a}{(m+n-1)b} A,\; C = \frac{(m-3)a}{(m+n-3)b} B,$$

$$D = \frac{(m-5)a}{(m+n-5)b} C,\; E = \frac{(m-7)a}{(m+n-7)b} D, \dots\dots L = \frac{(m-2i+3)a}{(m+n-2i+3)b} K.$$

$$\int x^{2m+1} \partial x \sqrt{X} = \left(Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \dots\dots \right.$$

$$\left. \dots\dots \pm Kx^4 \mp Lx^2 \right) X\sqrt{X} \pm 2aL \int x \partial x \sqrt{X}$$

$$A = \frac{1}{(2m+3)b},\; B = \frac{2ma}{(2m+1)b} A,\; C = \frac{(2m-2)a}{(2m-1)b} B,$$

$$D = \frac{(2m-4)a}{(2m-3)b} C,\; E = \frac{(2m-6)a}{(2m-5)b} D,\; \dots\dots L = \frac{4a}{5b} K.$$

$$\int x^{2m} \partial x \sqrt{X} = \left(Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \dots\dots \right.$$

$$\left. \dots\dots \pm Kx^3 \mp Lx \right) X\sqrt{X} \pm aL \int \partial x \sqrt{X}$$

$$A = \frac{1}{(2m+2)b},\; B = \frac{(2m-1)a}{2mb} A,\; C = \frac{(2m-3)a}{(2m-2)b} B,$$

$$D = \frac{(2m-5)a}{(2m-4)b} C,\; E = \frac{(2m-7)a}{(2m-6)b} D,\; \dots\dots L = \frac{3a}{4b} K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$\int x^{2m+1}X^{\frac{3}{2}} = \Big(Ax^{2m} - Bx^{2m-2} + Cx^{2m-4} - Dx^{2m-6} + Ex^{2m-8} - \ldots\ldots$$

$$\ldots\ldots\ldots \pm Kx^4 \mp Lx^2\Big)X^2\sqrt{X} \pm 2aL\int x\partial x X^{\frac{3}{2}}$$

$$A = \frac{1}{(2m+5)a},\quad B = \frac{2ma}{(2m+3)b}A,\quad C = \frac{(2m-2)a}{(2m+1)b}B,$$

$$D = \frac{(2m-4)a}{(2m-1)b}C,\quad E = \frac{(2m-6)a}{(2m-3)b}D,\quad \ldots\ldots\ldots L = \frac{4a}{7b}K.$$

$$\int x^{2m}X^{\frac{3}{2}} = \Big(Ax^{2m-1} - Bx^{2m-3} + Cx^{2m-5} - Dx^{2m-7} + Ex^{2m-9} - \ldots\ldots$$

$$\ldots\ldots\ldots \pm Kx^3 \mp Lx\Big)X^2\sqrt{X} \pm aL\int \partial x X^{\frac{3}{2}}$$

$$A = \frac{1}{(2m+4)b},\quad B = \frac{(2m-1)a}{(2m+2)b}A,\quad C = \frac{(2m-3)a}{2mb}B,$$

$$D = \frac{(2m-5)a}{(2m-2)b}C,\quad E = \frac{(2m-7)a}{(2m-4)b}D,\quad \ldots\ldots\ldots L = \frac{3a}{6b}K.$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x^m} = \Big(\frac{A}{x^{m-1}} - \frac{B}{x^{m-3}} + \frac{C}{x^{m-5}} - \frac{D}{x^{m-7}} + \frac{E}{x^{m-9}} - \ldots\ldots\ldots$$

$$\ldots\ldots\ldots \pm \frac{K}{x^{m-2i+3}} \mp \frac{L}{x^{m-2i+1}}\Big)X^{\frac{n}{2}+1} \mp (m-n-2i-1)bL\int \frac{\partial x X^{\frac{n}{2}}}{x^{m-2i}}$$

$$A = -\frac{1}{(m-1)a},\quad B = \frac{(m-n-3)b}{(m-3)a}A,\quad C = \frac{(m-n-5)b}{(m-5)a}B,$$

$$D = \frac{(m-n-7)b}{(m-7)a}C,\quad E = \frac{(m-n-9)b}{(m-9)a}D,\quad \ldots\ldots L = \frac{(m-n-2i+1)b}{(m-2i+1)a}K.$$

$$\int \frac{\partial x\sqrt{X}}{x^{2m+1}} = \Big(\frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \ldots\ldots\ldots$$

$$\ldots\ldots\ldots \pm \frac{K}{x^4} \mp \frac{L}{x^2}\Big)X\sqrt{X} \pm bL\int \frac{\partial x\sqrt{X}}{x}$$

Tafel
einiger allgemeineren Formeln.

VZ. $a + bx^2 = X$

$$A = -\frac{1}{2ma},\quad B = \frac{(2m-3)b}{(2m-2)a}A,\quad C = \frac{(2m-5)b}{(2m-4)a}B,$$
$$D = \frac{(2m-7)b}{(2m-6)a}C,\quad E = \frac{(2m-9)b}{(2m-8)a}D,\ \ldots\ldots L = \frac{b}{2a}K.$$

$$\int \frac{\partial x \sqrt{X}}{x^{2m}} = \left(\frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \ldots\ldots\ldots\ldots \right.$$
$$\left. \ldots\ldots\ldots\ldots \pm \frac{K}{x^5} \mp \frac{L}{x^3}\right) X\sqrt{X}$$

$$A = -\frac{1}{(2m-1)a},\quad B = \frac{(2m-4)b}{(2m-3)a}A,\quad C = \frac{(2m-6)b}{(2m-5)a}B,$$
$$D = \frac{(2m-8)b}{(2m-7)a}C,\quad E = \frac{(2m-10)b}{(2m-9)a}D,\ \ldots\ldots L = \frac{2b}{3a}K.$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^{2m+1}} = \left(\frac{A}{x^{2m}} - \frac{B}{x^{2m-2}} + \frac{C}{x^{2m-4}} - \frac{D}{x^{2m-6}} + \frac{E}{x^{2m-8}} - \ldots\ldots\ldots\ldots \right.$$
$$\left. \ldots\ldots\ldots\ldots \pm \frac{K}{x^4} \mp \frac{L}{x^2}\right) X^2\sqrt{X} \pm 3bL \int \frac{\partial x X^{\frac{3}{2}}}{x}$$

$$A = -\frac{1}{2ma},\quad B = \frac{(2m-5)b}{(2m-2)a}A,\quad C = \frac{(2m-7)b}{(2m-4)a}B,$$
$$D = \frac{(2m-9)b}{(2m-6)a}C,\quad E = \frac{(2m-11)b}{(2m-8)a}D,\ \ldots\ldots L = \frac{-b}{2a}K.$$

$$\int \frac{\partial x X^{\frac{3}{2}}}{x^{2m}} = \left(\frac{A}{x^{2m-1}} - \frac{B}{x^{2m-3}} + \frac{C}{x^{2m-5}} - \frac{D}{x^{2m-7}} + \frac{E}{x^{2m-9}} - \ldots\ldots\ldots\ldots \right.$$
$$\left. \ldots\ldots\ldots\ldots \pm \frac{K}{x^3} \mp \frac{L}{x}\right) X^2\sqrt{X} \pm 4bL \int \partial x X^{\frac{3}{2}}$$

$$A = -\frac{1}{(2m-1)a},\quad B = \frac{(2m-6)b}{(2m-3)a}A,\quad C = \frac{(2m-8)b}{(2m-5)a}B,$$
$$D = \frac{(2m-10)b}{(2m-7)a}C,\quad E = \frac{(2m-12)b}{(2m-9)a}D,\ \ldots\ldots L = \frac{-2b}{a}K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $ax + bx^2 = X$

$$\int x^m \partial x X^{\frac{n}{2}} = \frac{2x^{m+1}X^{\frac{n}{2}}}{2m+n+2} - \frac{nb}{2m+n+2}\int x^{m+2}\partial x X^{\frac{n}{2}-1}$$

$$\int \frac{x^m \partial x}{X^{\frac{n}{2}}} = -\frac{2x^{m-1}}{(n-2)bX^{\frac{n}{2}-1}} + \frac{2m-n}{(n-2)b}\int \frac{x^{m-2}\partial x}{X^{\frac{n}{2}-1}}$$

$$\int x^m \partial x X^{\frac{n}{2}} = \frac{x^{m-1}X^{\frac{n}{2}-1}}{(m+n+1)b} - \frac{(2m+n)a}{(m+n+1)2b}\int x^{m-1}\partial x X^{\frac{n}{2}}$$

$$\int \frac{x^m \partial x}{X^{\frac{n}{2}}} = \frac{x^{m-1}}{(m-n+1)bX^{\frac{n}{2}}} - \frac{(2m-n)a}{(m-n+1)2b}\int \frac{x^{m-1}\partial x}{X^{\frac{n}{2}}}$$

$$\int x^m \partial x X^{\frac{n}{2}} = \frac{x^{m+1}X^{\frac{n}{2}}}{m+n+1} + \frac{na}{2(m+n+1)}\int x^{m+1}\partial x X^{\frac{n}{2}-1}$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x^m} = -\frac{X^{\frac{n}{2}}}{(m-n-1)x^{m-1}} - \frac{na}{2(m-n-1)}\int \frac{\partial x X^{\frac{n}{2}-1}}{x^{m-1}}$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x^m} = -\frac{2X^{\frac{n}{2}+1}}{(2m-n-2)ax^m} - \frac{(m-n-2)2b}{(2m-n-2)a}\int \frac{\partial x X^{\frac{n}{2}}}{x^{m-1}}$$

$$\int \frac{\partial x}{x^m X^{\frac{n}{2}}} = -\frac{2}{(2m+n-2)ax^m X^{\frac{n}{2}-1}} - \frac{(m+n-2)2b}{(2m+n-2)a}\int \frac{\partial x}{x^{m-1}X^{\frac{n}{2}}}$$

$$\int \frac{x^m \partial x}{X^{\frac{n}{2}}} = \frac{2x^m}{(n-2)aX^{\frac{n}{2}-1}} - \frac{2(m-n+2)}{(n-2)a}\int \frac{x^{m-1}\partial x}{X^{\frac{n}{2}-1}}$$

$$\int \frac{\partial x}{x^m X^{\frac{n}{2}}} = \frac{2}{(n-2)ax^m X^{\frac{n}{2}-1}} + \frac{2(m+n-2)}{(n-2)a}\int \frac{\partial x}{x^{m+1}X^{\frac{n}{2}-1}}$$

$$\int \frac{\partial x}{X^{\frac{n}{2}}} = -\frac{2(2bx+a)}{(n-2)a^2X^{\frac{n}{2}-1}} - \frac{(n-3)4b}{(n-2)a^2}\int \frac{\partial x}{X^{\frac{n}{2}-1}}$$

$$\int \partial x X^{\frac{n}{2}} = \frac{(2bx+a)X^{\frac{n}{2}}}{(n+1)2b} - \frac{na^2}{(n+1)4b}\int \partial x X^{\frac{n}{2}-1}$$

Tafel
einiger allgemeineren Formeln.

VZ. $ax + bx^2 = X$

$$\int x^m \partial x X^{\frac{n}{2}} = \Big(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots\dots$$

$$\dots\dots \pm Kx^{m-i+1} \mp Lx^{m-i}\Big) X^{\frac{n}{2}+1} \pm \left(m + \frac{n}{2} - i + 1\right) aL \int x^{m-i} \partial x X^{\frac{n}{2}}$$

$$A = \frac{1}{(m+n+1)b},\ B = \frac{(2m+n)a}{(m+n)2b} A,\ C = \frac{(2m+n-2)a}{(m+n-1)2b} B,$$

$$D = \frac{(2m+n-4)a}{(m+n-2)2b} C,\ E = \frac{(2m+n-6)a}{(m+n-3)2b} D,\ \dots\dots$$

$$\dots\dots L = \frac{(2m+n-2i+4)a}{(m+n-i+2)2b} K.$$

$$\int x^m \partial x \sqrt{X} = \Big(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots\dots$$

$$\dots\dots \pm Kx \mp L\Big) X\sqrt{X} \pm \frac{3aL}{2} \int \partial x \sqrt{X}$$

$$A = \frac{1}{(m+2)b},\ B = \frac{(2m+1)a}{(m+1)2b} A,\ C = \frac{(2m-1)a}{2mb} B,$$

$$D = \frac{(2m-3)a}{(m-1)2b} C,\ E = \frac{(2m-5)a}{(m-2)2b} D,\ \dots\dots L = \frac{5a}{6b} K.$$

$$\int x^m \partial x X^{\frac{3}{2}} = \Big(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \dots\dots$$

$$\dots\dots \pm Kx \mp L\Big) X^2 \sqrt{X} \pm \frac{5aL}{2} \int \partial x X^{\frac{3}{2}}$$

$$A = \frac{1}{(m+4)b},\ B = \frac{(2m+3)a}{(m+3)2b} A,\ C = \frac{(2m+1)a}{(m+2)2b} B,$$

$$D = \frac{(2m-1)a}{(m+1)2b} C,\ E = \frac{(2m-3)a}{2mb} D,\ \dots\dots L = \frac{7a}{10b} K.$$

$$\int \frac{\partial x X^{\frac{n}{2}}}{x^m} = \Big(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \dots\dots$$

$$\dots\dots \pm \frac{K}{x^{m-i+2}} \mp \frac{L}{x^{m-i+1}}\Big) X^{\frac{n}{2}+1} \mp (m-n-i-1) bL \int \frac{\partial x X^{\frac{n}{2}}}{x^{m-i}}$$

Tafel
einiger allgemeineren Formeln.

VZ. $ax + bx^2 = X$

$$A = -\frac{2}{(2m-n-2)a},\ B = \frac{(m-n-2)2b}{(2m-n-4)a}A,\ C = \frac{(m-n-3)2b}{(2m-n-6)a}B,$$

$$D = \frac{(m-n-4)2b}{(2m-n-8)a}C,\ E = \frac{(m-n-5)2b}{(2m-n-10)a}D,\ \ldots\ldots\ldots$$

$$\ldots\ldots\ L = \frac{(m-n-i)2b}{(2m-n-2i)a}K.$$

$$\int\frac{\partial x\sqrt{X}}{x^m} = \Big(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots\ldots$$

$$\ldots\ldots\ldots \pm \frac{K}{x^5} \mp \frac{L}{x^4}\Big)X\sqrt{X} \mp bL\int\frac{\partial x\sqrt{X}}{x^3}$$

$$A = -\frac{2}{(2m-3)a},\ B = \frac{(m-3)2b}{(2m-5)a}A,\ C = \frac{(m-4)2b}{(2m-7)a}B,$$

$$D = \frac{(m-5)2b}{(2m-9)a}C,\ E = \frac{(m-6)2b}{(2m-11)a}D,\ \ldots\ldots\ L = \frac{4b}{5a}K.$$

$$\int\frac{\partial x X^{\frac{3}{2}}}{x^m} = \Big(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots\ldots$$

$$\ldots\ldots\ldots \pm \frac{K}{x^7} \mp \frac{L}{x^6}\Big)X^2\sqrt{X} \mp bL\int\frac{\partial x X^{\frac{3}{2}}}{x^5}$$

$$A = -\frac{2}{(2m-5)a},\ B = \frac{(m-5)2b}{(2m-7)a}A,\ C = \frac{(m-6)2b}{(2m-9)a}B,$$

$$D = \frac{(m-7)2b}{(2m-11)a}C,\ E = \frac{(m-8)2b}{(2m-13)a}D,\ \ldots\ldots\ L = \frac{4b}{7a}K.$$

$$\int\frac{x^m\partial x}{X^{\frac{n}{2}}} = \Big(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\ldots\ldots$$

$$\ldots\ldots \pm Kx^{m-i+1} \mp Lx^{m-i}\Big)X^{-\frac{n}{2}+1} \pm \left(m - \frac{n}{2} - i + 1\right)aL\int\frac{x^{m-i}\partial x}{X^{\frac{n}{2}}}$$

$$A = \frac{1}{(m-n+1)b},\ B = \frac{(2m-n)a}{(m-n)2b}A,\ C = \frac{(2m-n-2)a}{(m-n-1)2b}B,$$

$$D = \frac{(2m-n-4)a}{(m-n-2)2b}C,\ E = \frac{(2m-n-6)a}{(m-n-3)2b}D,\ \ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ L = \frac{(2m-n-2i+4)a}{(m-n-i+2)2b}K.$$

Tafel
einiger allgemeineren Formeln.

VZ. $ax + bx^2 = X$

$$\int\frac{x^m\partial x}{\sqrt{X}} = \Big(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\ldots\ldots$$

$$\ldots\ldots\ldots \pm Kx \mp L\Big)\sqrt{X} \pm \frac{aL}{2}\int\frac{\partial x}{\sqrt{X}}$$

$$A = \frac{1}{mb},\ B = \frac{(2m-1)a}{(m-1)2b}A,\ C = \frac{(2m-3)a}{(m-2)2b}B,$$

$$D = \frac{(2m-5)a}{(m-3)2b}C,\ E = \frac{(2m-7)a}{(m-4)2b}D,\ \ldots\ldots\ L = \frac{3a}{2b}K.$$

$$\int\frac{x^m\partial x}{X^{\frac{3}{2}}} = \Big(Ax^{m-1} - Bx^{m-2} + Cx^{m-3} - Dx^{m-4} + Ex^{m-5} - \ldots\ldots\ldots$$

$$\ldots\ldots\ldots \pm Kx^3 \mp Lx^2\Big)\frac{1}{\sqrt{X}} \pm \frac{3aL}{2}\int\frac{x^2\partial x}{X^{\frac{3}{2}}}$$

$$A = \frac{1}{(m-2)b},\ B = \frac{(2m-3)a}{(m-3)2b}A,\ C = \frac{(2m-5)a}{(m-4)2b}B,$$

$$D = \frac{(2m-7)a}{(m-5)2b}C,\ E = \frac{(2m-9)a}{(m-6)2b}D,\ \ldots\ldots\ L = \frac{5a}{2b}K.$$

$$\int\frac{\partial x}{x^m X^{\frac{n}{2}}} = \Big(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots\ldots$$

$$\ldots\ldots \pm \frac{K}{x^{m-i+2}} \mp \frac{L}{x^{m-i+1}}\Big)X^{-\frac{n}{2}+1} \mp (m+n-i-1)bL\int\frac{\partial x}{x^{m-i}X^{\frac{n}{2}}}$$

$$A = -\frac{2}{(2m+n-2)a},\ B = \frac{(m+n-2)2b}{(2m+n-4)a}A,\ C = \frac{(m+n-3)2b}{(2m+n-6)a}B,$$

$$D = \frac{(m+n-4)2b}{(2m+n-8)a}C,\ E = \frac{(m+n-5)2b}{(2m+n-10)a}D,\ \ldots\ldots\ldots$$

$$\ldots\ldots\ldots L = \frac{(m+n-i)2b}{(2m+n-2i)a}K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $ax + bx^2 = X$

$$\int \frac{\partial x}{x^m \sqrt{X}} = \left(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\sqrt{X}$$

$$A = -\frac{2}{(2m-1)a},\ B = \frac{(m-1)2b}{(2m-3)a}A,\ C = \frac{(m-2)2b}{(2m-5)a}B,$$

$$D = \frac{(m-3)2b}{(2m-7)a}C,\ E = \frac{(m-4)2b}{(2m-9)a}D,\ \ldots\ldots L = \frac{2b}{a}K.$$

$$\int \frac{\partial x}{x^m X^{\frac{3}{2}}} = \left(\frac{A}{x^m} - \frac{B}{x^{m-1}} + \frac{C}{x^{m-2}} - \frac{D}{x^{m-3}} + \frac{E}{x^{m-4}} - \ldots\ldots \pm \frac{K}{x^2} \mp \frac{L}{x}\right)\frac{1}{\sqrt{X}} \mp 2bL\int \frac{\partial x}{X^{\frac{3}{2}}}$$

$$A = -\frac{2}{(2m+1)a},\ B = \frac{(m+1)2b}{(2m-1)a}A,\ C = \frac{2mb}{(2m-3)a}B,$$

$$D = \frac{(m-1)2b}{(2m-5)a}C,\ E = \frac{(m-2)2b}{(2m-7)a}D,\ \ldots\ldots L = \frac{6b}{3a}K.$$

VZ. $ax + bx^2 = X,\ 2bx + a = U$

$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \left(\frac{A}{X^{\frac{n-3}{2}}} - \frac{B}{X^{\frac{n-5}{2}}} + \frac{C}{X^{\frac{n-7}{2}}} - \frac{D}{X^{\frac{n-9}{2}}} + \frac{E}{X^{\frac{n-11}{2}}} - \ldots\ldots \pm \frac{K}{X^{\frac{n-2i+1}{2}}} \mp \frac{L}{X^{\frac{n-2i-1}{2}}}\right)\frac{2U}{\sqrt{X}} \mp (n-2i-1)4bL\int \frac{\partial x}{X^{\frac{n}{2}-i}}$$

$$A = -\frac{1}{(n-2)a^2},\ B = \frac{(n-3)4b}{(n-4)a^2}A,\ C = \frac{(n-5)4b}{(n-6)a^2}B,$$

$$D = \frac{(n-7)4b}{(n-8)a^2}C,\ E = \frac{(n-9)4b}{(n-10)a^2}D,\ \ldots\ldots L = \frac{(n-2i+1)4b}{(n-2i)a^2}K.$$

Tafel

einiger allgemeineren Formeln.

VZ. $ax+bx^2=X,\ 2bx+a=U$

$$\int\frac{\partial x}{X^{\frac{n}{2}}}=\Big(\frac{A}{X^{\frac{n-3}{2}}}-\frac{B}{X^{\frac{n-5}{2}}}+\frac{C}{X^{\frac{n-7}{2}}}-\frac{D}{X^{\frac{n-9}{2}}}+\frac{E}{X^{\frac{n-11}{2}}}-\ldots\ldots$$

$$\ldots\ldots\pm\frac{K}{X^2}\mp\frac{L}{X}\Big)\frac{2U}{\sqrt{X}}\mp 8bL\int\frac{\partial x}{X^{\frac{3}{2}}}$$

$$A=-\frac{1}{(n-2)a^2},\ B=\frac{(n-3)4b}{(n-4)a^2}A,\ C=\frac{(n-5)4b}{(n-6)a^2}B,$$

$$D=\frac{(n-7)4b}{(n-8)a^2}C,\ E=\frac{(n-9)4b}{(n-10)a^2}D,\ \ldots\ldots L=\frac{4\cdot 4b}{3a^2}K.$$

VZ. $a+bx+cx^2=X$

$$\int x^m\partial x X^p=\frac{x^{m+1}X^p}{m+1}-\frac{pb}{m+1}\int x^{m+1}\partial x X^{p-1}-\frac{2pc}{m+1}\int x^{m+2}\partial x X^{p-1}$$

$$\int x^m\partial x X^p=\frac{x^{m-1}X^{p+1}}{(m+2p+1)c}-\frac{(m-1)a}{(m+2p+1)c}\int x^{m-2}\partial x X^p$$

$$-\frac{(m+p)b}{(m+2p+1)c}\int x^{m-1}\partial x X^p$$

$$\int\frac{x^m\partial x}{X^p}=\frac{x^{m-1}}{(m-2p+1)cX^{p-1}}-\frac{(m-1)a}{(m-2p+1)c}\int\frac{x^{m-2}\partial x}{X^p}$$

$$-\frac{(m-p)b}{(m-2p+1)c}\int\frac{x^{m-1}\partial x}{X^p}$$

$$\int\frac{\partial x X^p}{x^m}=-\frac{X^p}{(m-1)x^{m-1}}+\frac{pb}{m-1}\int\frac{\partial x X^{p-1}}{x^{m-1}}+\frac{2pc}{m-1}\int\frac{\partial x X^{p-1}}{x^{m-2}}$$

$$\int x^m\partial x X^p=\frac{x^{m+1}X^p}{m+2p+1}+\frac{2pa}{m+2p+1}\int x^m\partial x X^{p-1}$$

$$+\frac{pb}{m+2p+1}\int x^{m+1}\partial x X^{p-1}$$

$$\int\frac{\partial x X^p}{x^m}=-\frac{X^p}{(m-2p-1)x^{m-1}}-\frac{2pa}{m-2p-1}\int\frac{\partial x X^{p-1}}{x^m}$$

$$-\frac{pb}{m-2p-1}\int\frac{\partial x X^{p-1}}{x^{m-1}}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a + bx + cx^2 = X$, $4ac - b^2 = k$

$$\int \frac{\partial x X^p}{x^m} = -\frac{X^{p+1}}{(m-1)ax^{m-1}} - \frac{(m-p-2)b}{(m-1)a}\int \frac{\partial x X^p}{x^{m-1}}$$
$$- \frac{(m-2p-3)c}{(m-1)a}\int \frac{\partial x X^p}{x^{m-2}}$$

$$\int \frac{\partial x}{x^m X^p} = -\frac{1}{(m-1)ax^{m-1}X^{p-1}} - \frac{(m+p-2)b}{(m-1)a}\int \frac{\partial x}{x^{m-1}X^p}$$
$$- \frac{(m+2p-3)c}{(m-1)a}\int \frac{\partial x}{x^{m-2}X^p}$$

$$\int \frac{\partial x}{X^p} = \frac{2cx+b}{(p-1)kX^{p-1}} + \frac{(2p-3)2c}{(p-1)k}\int \frac{\partial x}{X^{p-1}}$$

$$\int \partial x X^p = \frac{(2cx+b)X^p}{(2p+1)2c} + \frac{pk}{(2p+1)2c}\int \partial x X^{p-1}$$

$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \left(\frac{A}{X^{\frac{n-3}{2}}} + \frac{B}{X^{\frac{n-5}{2}}} + \frac{C}{X^{\frac{n-7}{2}}} + \frac{D}{X^{\frac{n-9}{2}}} + \frac{E}{X^{\frac{n-11}{2}}} + \cdots\cdots\right.$$
$$\left.\cdots\cdots + \frac{K}{X^{\frac{n-2i+1}{2}}} + \frac{L}{X^{\frac{n-2i-1}{2}}}\right)\frac{2(2cx+b)}{\sqrt{X}}$$
$$+ (n-2i-1)4cL\int \frac{\partial x}{X^{\frac{n}{2}-i}}$$

$$A = \frac{1}{(n-2)k},\quad B = \frac{(n-3)4c}{(n-4)k}A,\quad C = \frac{(n-5)4c}{(n-6)k}B,$$
$$D = \frac{(n-7)4c}{(n-8)k}C,\quad E = \frac{(n-9)4c}{(n-10)k}D, \cdots\cdots L = \frac{(n-2i+1)4c}{(n-2i)k}K.$$

$$\int \frac{\partial x}{X^{\frac{n}{2}}} = \left(\frac{A}{X^{\frac{n-3}{2}}} + \frac{B}{X^{\frac{n-5}{2}}} + \frac{C}{X^{\frac{n-7}{2}}} + \frac{D}{X^{\frac{n-9}{2}}} + \frac{E}{X^{\frac{n-11}{2}}} + \cdots\cdots\right.$$
$$\left.\cdots\cdots + \frac{K}{X^2} + \frac{L}{X}\right)\frac{2(2cx+b)}{\sqrt{X}} + 8cL\int \frac{\partial x}{X^{\frac{3}{2}}}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a+bx+cx^2=X,\ 4ac-b^2=k$

$$A=\frac{1}{(n-2)k},\ B=\frac{(n-3)4c}{(n-4)k}A,\ C=\frac{(n-5)4c}{(n-6)k}B,$$

$$D=\frac{(n-7)4c}{(n-8)k}C,\ E=\frac{(n-9)4c}{(n-10)k}D,\ \ldots\ldots\ L=\frac{4\cdot 4c}{3k}K.$$

$$\int \partial x X^{\frac{n}{2}}=\Big(AX^{\frac{n-1}{2}}+BX^{\frac{n-3}{2}}+CX^{\frac{n-5}{2}}+DX^{\frac{n-7}{2}}+EX^{\frac{n-9}{2}}+\ldots\ldots$$

$$\ldots\ldots\ldots\ldots+KX^{\frac{n-2i+3}{2}}+LX^{\frac{n-2i+1}{2}}\Big)(2cx+b)\sqrt{X}$$

$$+\frac{n-2i+2}{2}kL\int \partial x X^{\frac{n}{2}-i}$$

$$A=\frac{1}{(n+1)2c},\ B=\frac{nk}{(n-1)4c}A,\ C=\frac{(n-2)k}{(n-3)4c}B,$$

$$D=\frac{(n-4)k}{(n-5)4c}C,\ E=\frac{(n-6)k}{(n-7)4c}D,\ \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ L=\frac{(n-2i+4)k}{(n-2i+3)4c}K.$$

$$\int \partial x X^{\frac{n}{2}}=\Big(AX^{\frac{n-1}{2}}+BX^{\frac{n-3}{2}}+CX^{\frac{n-5}{2}}+DX^{\frac{n-7}{2}}+EX^{\frac{n-9}{2}}+\ldots\ldots$$

$$\ldots\ldots+KX^2+LX\Big)(2cx+b)\sqrt{X}+\frac{3kL}{2}\int \partial x\sqrt{X}$$

$$A=\frac{1}{(n+1)2c},\ B=\frac{nk}{(n-1)4c}A,\ C=\frac{(n-2)k}{(n-3)4c}B,$$

$$D=\frac{(n-4)k}{(n-5)4c}C,\ E=\frac{(n-6)k}{(n-7)4c}D,\ \ldots\ldots\ L=\frac{5k}{4\cdot 4c}K.$$

$$\int\frac{x\partial x}{X^{\frac{n}{2}}}=-\frac{1}{(n-2)cX^{\frac{n}{2}-1}}-\frac{b}{2c}\int\frac{\partial x}{X^{\frac{n}{2}}}$$

$$\int x\partial x X^{\frac{n}{2}}=\frac{X^{\frac{n}{2}+1}}{(n+2)c}-\frac{b}{2c}\int \partial x X^{\frac{n}{2}}$$

Tafel

einiger allgemeineren Formeln.

VZ. $a+bx+cx^2=X$

$$\int\frac{\partial x}{xX^{\frac{n}{2}}}=\frac{1}{(n-2)aX^{\frac{n-2}{2}}}+\frac{1}{a}\int\frac{\partial x}{xX^{\frac{n-2}{2}}}-\frac{b}{2a}\int\frac{\partial x}{X^{\frac{n}{2}}}$$

$$\int\frac{\partial xX^{\frac{n}{2}}}{x}=\frac{X^{\frac{n}{2}}}{n}+a\int\frac{\partial xX^{\frac{n-2}{2}}}{x}+\frac{b}{2}\int\partial xX^{\frac{n-2}{2}}$$

$$\int\frac{\partial x}{xX^{\frac{2n+1}{2}}}=\left[\frac{1}{(2n-1)aX^{n-1}}+\frac{1}{(2n-3)a^2X^{n-2}}+\frac{1}{(2n-5)a^3X^{n-3}}+\ldots\ldots\right.$$
$$\left.\ldots\ldots+\frac{1}{5a^{n-2}X^2}+\frac{1}{3a^{n-1}X}+\frac{1}{a^n}\right]\frac{1}{\sqrt{X}}$$
$$-\frac{b}{2a}\int\frac{\partial x}{X^{\frac{2n+1}{2}}}-\frac{b}{2a^2}\int\frac{\partial x}{X^{\frac{2n-1}{2}}}-\frac{b}{2a^3}\int\frac{\partial x}{X^{\frac{2n-3}{2}}}-\ldots\ldots$$
$$\ldots\ldots-\frac{b}{2a^{n-1}}\int\frac{\partial x}{X^{\frac{5}{2}}}-\frac{b}{2a^n}\int\frac{\partial x}{X^{\frac{3}{2}}}+\frac{1}{a^n}\int\frac{\partial x}{x\sqrt{X}}$$

$$\int\frac{\partial xX^{\frac{2n+1}{2}}}{x}=\left(\frac{X^n}{2n+1}+\frac{aX^{n-1}}{2n-1}+\frac{a^2X^{n-2}}{2n-3}+\frac{a^3X^{n-3}}{2n-5}+\ldots\ldots\right.$$
$$\left.\ldots\ldots+\frac{a^{n-2}X^2}{5}+\frac{a^{n-1}X}{3}+\frac{a^n}{1}\right)\sqrt{X}$$
$$+\frac{b}{2}\int\partial xX^{\frac{2n-1}{2}}+\frac{ab}{2}\int\partial xX^{\frac{2n-3}{2}}+\frac{a^2b}{2}\int\partial xX^{\frac{2n-5}{2}}+\ldots\ldots$$
$$\ldots\ldots+\frac{a^{n-2}b}{2}\int\partial xX^{\frac{3}{2}}+\frac{a^{n-1}b}{2}\int\partial x\sqrt{X}$$
$$+\frac{a^nb}{2}\int\frac{\partial x}{\sqrt{X}}+a^{n+1}\int\frac{\partial x}{x\sqrt{X}}$$

Werthe der bestimmten Integrale

$$\int' \frac{x^m \partial x}{\sqrt{(a^2-x^2)}}, \quad \int' x^m \partial x \sqrt{(a^2-x^2)},$$

von $x=0$ bis $x=a$.

VZ. $a^2-x^2=X$, $\pi=3,14159\cdots\cdots$

$$\int' \frac{\partial x}{\sqrt{X}} = \frac{\pi}{2} \qquad \int' \frac{x \partial x}{\sqrt{X}} = a$$

$$\int' \frac{x^2 \partial x}{\sqrt{X}} = \frac{1}{2} \cdot \frac{\pi a^2}{2} \qquad \int' \frac{x^3 \partial x}{\sqrt{X}} = \frac{2}{3} \cdot a^3$$

$$\int' \frac{x^4 \partial x}{\sqrt{X}} = \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{\pi a^4}{2} \qquad \int' \frac{x^5 \partial x}{\sqrt{X}} = \frac{2 \cdot 4}{3 \cdot 5} \cdot a^5$$

$$\int' \frac{x^6 \partial x}{\sqrt{X}} = \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{\pi a^6}{2} \qquad \int' \frac{x^7 \partial x}{\sqrt{X}} = \frac{2 \cdot 4 \cdot 6}{3 \cdot 5 \cdot 7} \cdot a^7$$

$$\int' \frac{x^8 \partial x}{\sqrt{X}} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{\pi a^8}{2} \qquad \int' \frac{x^9 \partial x}{\sqrt{X}} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{3 \cdot 5 \cdot 7 \cdot 9} \cdot a^9$$

$$\int' \frac{x^{10} \partial x}{\sqrt{X}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \cdot \frac{\pi a^{10}}{2} \qquad \int' \frac{x^{11} \partial x}{\sqrt{X}} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10}{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11} \cdot a^{11}$$

$$\int' \frac{x^{2r} \partial x}{\sqrt{X}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots\cdots (2r-3)(2r-1)}{2 \cdot 4 \cdot 6 \cdot 8 \cdots\cdots\cdots\cdots (2r-2)2r} \cdot \frac{\pi a^{2r}}{2}$$

$$\int' \frac{x^{2r+1} \partial x}{\sqrt{X}} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdots\cdots\cdots\cdots (2r-2)2r}{3 \cdot 5 \cdot 7 \cdot 9 \cdots\cdots (2r-1)(2r+1)} \cdot a^{2r+1}$$

$$\int' \partial x \sqrt{X} = \frac{\pi a^2}{4} \qquad \int' x \partial x \sqrt{X} = \frac{a^3}{3}$$

$$\int' x^2 \partial x \sqrt{X} = \frac{1}{4} \cdot \frac{\pi a^4}{4} \qquad \int' x^3 \partial x \sqrt{X} = \frac{2}{5} \cdot \frac{a^5}{3}$$

$$\int' x^4 \partial x \sqrt{X} = \frac{1 \cdot 3}{4 \cdot 6} \cdot \frac{\pi a^6}{4} \qquad \int' x^5 \partial x \sqrt{X} = \frac{2 \cdot 4}{5 \cdot 7} \cdot \frac{a^7}{3}$$

$$\int' x^6 \partial x \sqrt{X} = \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} \cdot \frac{\pi a^8}{4} \qquad \int' x^7 \partial x \sqrt{X} = \frac{2 \cdot 4 \cdot 6}{5 \cdot 7 \cdot 9} \cdot \frac{a^9}{3}$$

$$\int' x^8 \partial x \sqrt{X} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{4 \cdot 6 \cdot 8 \cdot 10} \cdot \frac{\pi a^{10}}{4} \qquad \int' x^9 \partial x \sqrt{X} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{5 \cdot 7 \cdot 9 \cdot 11} \cdot \frac{a^{11}}{3}$$

$$\int' x^{2r} \partial x \sqrt{X} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots\cdots (2r-3)(2r-1)}{4 \cdot 6 \cdot 8 \cdot 10 \cdots\cdots\cdots\cdots 2r(2r+2)} \cdot \frac{\pi a^{2r+2}}{4}$$

$$\int' x^{2r+1} \partial x \sqrt{X} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdots\cdots\cdots\cdots (2r-2)\, 2r}{5 \cdot 7 \cdot 9 \cdot 11 \cdots\cdots (2r+1)(2r+3)} \cdot \frac{a^{2r+3}}{3}$$

Werthe der bestimmten Integrale

$$\int' x^m \partial x (a^2 - x^2)^{\frac{3}{2}}, \quad \int' x^m \partial x (a^2 - x^2)^{\frac{5}{2}},$$

von $x = 0$ bis $x = a$.

VZ. $a^2 - x^2 = X$, $\pi = 3{,}14159 \cdots\cdots$

$$\int' \partial x X^{\frac{3}{2}} = \frac{3\pi a^4}{16}$$

$$\int' x^2 \partial x X^{\frac{3}{2}} = \frac{1}{6} \cdot \frac{3\pi a^6}{16}$$

$$\int' x^4 \partial x X^{\frac{3}{2}} = \frac{1 \cdot 3}{6 \cdot 8} \cdot \frac{3\pi a^8}{16}$$

$$\int' x^6 \partial x X^{\frac{3}{2}} = \frac{1 \cdot 3 \cdot 5}{6 \cdot 8 \cdot 10} \cdot \frac{3\pi a^{10}}{16}$$

$$\int' x^8 \partial x X^{\frac{3}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{6 \cdot 8 \cdot 10 \cdot 12} \cdot \frac{3\pi a^{12}}{16}$$

$$\int' x \partial x X^{\frac{3}{2}} = \frac{a^5}{5}$$

$$\int' x^3 \partial x X^{\frac{3}{2}} = \frac{2}{7} \cdot \frac{a^7}{5}$$

$$\int' x^5 \partial x X^{\frac{3}{2}} = \frac{2 \cdot 4}{7 \cdot 9} \cdot \frac{a^9}{5}$$

$$\int' x^7 \partial x X^{\frac{3}{2}} = \frac{2 \cdot 4 \cdot 6}{7 \cdot 9 \cdot 11} \cdot \frac{a^{11}}{5}$$

$$\int' x^9 \partial x X^{\frac{3}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{7 \cdot 9 \cdot 11 \cdot 13} \cdot \frac{a^{13}}{5}$$

$$\int' x^{2r} \partial x X^{\frac{3}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots\cdots (2r-3)(2r-1)}{6 \cdot 8 \cdot 10 \cdot 12 \cdots\cdots (2r+2)(2r+4)} \cdot \frac{3\pi a^{2}}{16}$$

$$\int' x^{2r+1} \partial x X^{\frac{3}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdots\cdots (2r-2)\, 2r}{7 \cdot 9 \cdot 11 \cdot 13 \cdots\cdots (2r+3)(2r+5)} \cdot \frac{a^{2r+5}}{5}$$

$$\int' \partial x X^{\frac{5}{2}} = \frac{5\pi a^6}{32}$$

$$\int' x^2 \partial x X^{\frac{5}{2}} = \frac{1}{8} \cdot \frac{5\pi a^8}{32}$$

$$\int' x^4 \partial x X^{\frac{5}{2}} = \frac{1 \cdot 3}{8 \cdot 10} \cdot \frac{5\pi a^{10}}{32}$$

$$\int' x^6 \partial x X^{\frac{5}{2}} = \frac{1 \cdot 3 \cdot 5}{8 \cdot 10 \cdot 12} \cdot \frac{5\pi a^{12}}{32}$$

$$\int' x^8 \partial x X^{\frac{5}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{8 \cdot 10 \cdot 12 \cdot 14} \cdot \frac{5\pi a^{14}}{32}$$

$$\int' x \partial x X^{\frac{5}{2}} = \frac{a^7}{7}$$

$$\int' x^3 \partial x X^{\frac{5}{2}} = \frac{2}{9} \cdot \frac{a^9}{7}$$

$$\int' x^5 \partial x X^{\frac{5}{2}} = \frac{2 \cdot 4}{9 \cdot 11} \cdot \frac{a^{11}}{7}$$

$$\int' x^7 \partial x X^{\frac{5}{2}} = \frac{2 \cdot 4 \cdot 6}{9 \cdot 11 \cdot 13} \cdot \frac{a^{13}}{7}$$

$$\int' x^9 \partial x X^{\frac{5}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8}{9 \cdot 11 \cdot 13 \cdot 15} \cdot \frac{a^{1}}{7}$$

$$\int' x^{2r} \partial x X^{\frac{5}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots\cdots (2r-3)(2r-1)}{8 \cdot 10 \cdot 12 \cdot 14 \cdots\cdots (2r+4)(2r+6)} \cdot \frac{5\pi a}{3}$$

$$\int' x^{2r+1} \partial x X^{\frac{5}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdots\cdots (2r-2)\, 2r}{9 \cdot 11 \cdot 13 \cdot 15 \cdots\cdots (2r+5)(2r+7)} \cdot \frac{a^{2r}}{7}$$

Werthe der bestimmten Integrale

$$\int' \partial x (a^2 - x^2)^{\frac{n}{2}}, \quad \int' x^m \partial x (a^2 - x^2)^{\frac{n}{2}}$$

von $x = 0$ bis $x = a$.

VZ. $a^2 - x^2 = X$, $\pi = 3{,}14159\cdots\cdots$

$$\int' \partial x \sqrt{X} = \frac{1}{2} \cdot \frac{\pi a^2}{2}$$

$$\int' \partial x X^{\frac{3}{2}} = \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{\pi a^4}{2}$$

$$\int' \partial x X^{\frac{5}{2}} = \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{\pi a^6}{2}$$

$$\int' \partial x X^{\frac{7}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{\pi a^8}{2}$$

$$\int' \partial x X^{\frac{9}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \cdot \frac{\pi a^{10}}{2}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\int' \partial x X^{\frac{n}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots\cdots\cdots\cdots (n-2) \cdot n}{2 \cdot 4 \cdot 6 \cdot 8 \cdots\cdots (n-1)(n+1)} \cdot \frac{\pi a^{n+1}}{2}$$

$$\int' x^2 \partial x X^{\frac{n}{2}} = \frac{1}{n+3} \cdot a^2 \int' \partial x X^{\frac{n}{2}}$$

$$\int' x^4 \partial x X^{\frac{n}{2}} = \frac{1 \cdot 3}{(n+3)(n+5)} \cdot a^4 \int' \partial x X^{\frac{n}{2}}$$

$$\int' x^6 \partial x X^{\frac{n}{2}} = \frac{1 \cdot 3 \cdot 5}{(n+3)(n+5)(n+7)} \cdot a^6 \int' \partial x X^{\frac{n}{2}}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\int' x \partial x X^{\frac{n}{2}} = \frac{a^{n+2}}{n+2}$$

$$\int' x^3 \partial x X^{\frac{n}{2}} = \frac{2}{n+4} \cdot \frac{a^{n+4}}{n+2}$$

$$\int' x^5 \partial x X^{\frac{n}{2}} = \frac{2 \cdot 4}{(n+4)(n+6)} \cdot \frac{a^{n+6}}{n+2}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\int' x^{2r} \partial x X^{\frac{n}{2}} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdots\cdots\cdots (2r-1)}{(n+3)(n+5)(n+7)\cdots\cdots(n+2r+1)} \cdot a^{2r} \int' \partial x X^{\frac{n}{2}}$$

$$\int' x^{2r+1} \partial x X^{\frac{n}{2}} = \frac{2 \cdot 4 \cdot 6 \cdot 8 \cdots\cdots\cdots\cdots 2r}{(n+4)(n+6)(n+8)\cdots\cdots(n+2r+2)} \cdot \frac{a^{n+2r+2}}{n+2}$$

Werthe der bestimmten Integrale

$$\int' \frac{x^m \partial x}{\sqrt{(a^4 - x^4)}}, \quad \int' x^m \partial x (a^4 - x^4)^{\frac{n}{2}},$$

von $x = 0$ bis $x = a$.

VZ. $a^2 - x^2 = X$, $\pi = 3{,}14159\ldots$

$$\int' \frac{\partial x}{\sqrt{(a^4 - x^4)}} = \int' \frac{\partial x (a^2 + x^2)^{-\frac{1}{2}}}{\sqrt{X}} = \frac{1}{a} \int' \frac{\partial x}{\sqrt{X}} + \frac{\overset{-\frac{1}{2}}{\mathfrak{A}}}{a^3} \int' \frac{x^2 \partial x}{\sqrt{X}}$$

$$+ \frac{\overset{-\frac{1}{2}}{\mathfrak{B}}}{a^5} \int' \frac{x^4 \partial x}{\sqrt{X}} + \frac{\overset{-\frac{1}{2}}{\mathfrak{C}}}{a^7} \int' \frac{x^6 \partial x}{\sqrt{X}} + \frac{\overset{-\frac{1}{2}}{\mathfrak{D}}}{a^9} \int' \frac{x^8 \partial x}{\sqrt{X}} + \text{etc.}$$

$$= \frac{\pi}{2a} \Big[1 - \left(\frac{1}{2}\right)^2 + \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^2 - \left(\frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\right)^2 + \left(\frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\right)^2$$

$$- \left(\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10}\right)^2 + \left(\frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9 \cdot 11}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10 \cdot 12}\right)^2 - \text{etc.} \Big]$$

$$\int' \partial x \sqrt{(a^4 - x^4)} = \int' \partial x (a^2 + x^2)^{\frac{1}{2}} \sqrt{X} = a \int' \partial x \sqrt{X} + \frac{\overset{\frac{1}{2}}{\mathfrak{A}}}{a} \int' x^2 \partial x \sqrt{X}$$

$$+ \frac{\overset{\frac{1}{2}}{\mathfrak{B}}}{a^3} \int' x^4 \partial x \sqrt{X} + \frac{\overset{\frac{1}{2}}{\mathfrak{C}}}{a^5} \int' x^6 \partial x \sqrt{X} + \text{etc.}$$

$$= \frac{\pi a^3}{4} \Big[1 + \frac{1}{2} \cdot \frac{1}{4} - \frac{1 \cdot 1}{2 \cdot 4} \cdot \frac{1 \cdot 3}{4 \cdot 6} + \frac{1 \cdot 1 \cdot 3}{2 \cdot 4 \cdot 6} \cdot \frac{1 \cdot 3 \cdot 5}{4 \cdot 6 \cdot 8} - \frac{1 \cdot 1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 8} \cdot \frac{1 \cdot 3 \cdot 5 \cdot 7}{4 \cdot 6 \cdot 8 \cdot 10}$$

$$+ \frac{1 \cdot 1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} \cdot \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{4 \cdot 6 \cdot 8 \cdot 10 \cdot 12} - \text{etc.} \Big]$$

$$\int' \frac{x^m \partial x}{\sqrt{(a^4 - x^4)}} = \frac{1}{a} \int' \frac{x^m \partial x}{\sqrt{X}} + \frac{\overset{-\frac{1}{2}}{\mathfrak{A}}}{a^3} \int' \frac{x^{m+2} \partial x}{\sqrt{X}} + \frac{\overset{-\frac{1}{2}}{\mathfrak{B}}}{a^5} \int' \frac{x^{m+4} \partial x}{\sqrt{X}}$$

$$+ \frac{\overset{-\frac{1}{2}}{\mathfrak{C}}}{a^7} \int' \frac{x^{m+6} \partial x}{\sqrt{X}} + \frac{\overset{-\frac{1}{2}}{\mathfrak{D}}}{a^9} \int' \frac{x^{m+8} \partial x}{\sqrt{X}} + \text{etc.}$$

$$\int' x^m \partial x (a^4 - x^4)^{\frac{n}{2}} = a^n \int' x^m \partial x X^{\frac{n}{2}} + \overset{\frac{n}{2}}{\mathfrak{A}} a^{n-2} \int' x^{m+2} \partial x X^{\frac{n}{2}}$$

$$+ \overset{\frac{n}{2}}{\mathfrak{B}} a^{n-4} \int' x^{m+4} \partial x X^{\frac{n}{2}} + \overset{\frac{n}{2}}{\mathfrak{C}} a^{n-6} \int' x^{m+6} \partial x X^{\frac{n}{2}} + \text{etc.}$$

Werthe der bestimmten Integrale

$$\int'\frac{x^m\partial x(1+cx^h)^{\frac{p}{q}}}{\sqrt{(a^2-x^2)}},\quad \int' x^m\partial x(1+cx^h)^{\frac{p}{q}}(a^2-x^2)^{\frac{n}{2}}$$

von $x=0$ bis $x=a$.

VZ. $a^2-x^2=X$

$$\int'\frac{\partial x(1+cx^h)^{\frac{p}{q}}}{\sqrt{X}}=\int'\frac{\partial x}{\sqrt{X}}+\tfrac{p}{q}\mathfrak{A}c\int'\frac{x^h\partial x}{\sqrt{X}}+\tfrac{p}{q}\mathfrak{B}c^2\int'\frac{x^{2h}\partial x}{\sqrt{X}}$$
$$+\tfrac{p}{q}\mathfrak{C}c^3\int'\frac{x^{3h}\partial x}{\sqrt{X}}+\tfrac{p}{q}\mathfrak{D}c^4\int'\frac{x^{4h}\partial x}{\sqrt{X}}+\text{etc.}$$

$$\int'(1+cx^h)^{\frac{p}{q}}X^{\frac{n}{2}}=\int'\partial xX^{\frac{n}{2}}+\tfrac{p}{q}\mathfrak{A}c\int'x^h\partial xX^{\frac{n}{2}}+\tfrac{p}{q}\mathfrak{B}c^2\int'x^{2h}\partial xX^{\frac{n}{2}}$$
$$+\tfrac{p}{q}\mathfrak{C}c^3\int'x^{3h}\partial xX^{\frac{n}{2}}+\tfrac{p}{q}\mathfrak{D}c^4\int'x^{4h}\partial xX^{\frac{n}{2}}+\text{etc.}$$

$$\int'\frac{\partial x}{(1+cx^h)^{\frac{p}{q}}\sqrt{X}}=\int'\frac{\partial x}{\sqrt{X}}+\tfrac{-p}{q}\mathfrak{A}c\int'\frac{x^h\partial x}{\sqrt{X}}+\tfrac{-p}{q}\mathfrak{B}c^2\int'\frac{x^{2h}\partial x}{\sqrt{X}}$$
$$+\tfrac{-p}{q}\mathfrak{C}c^3\int'\frac{x^{3h}\partial x}{\sqrt{X}}+\tfrac{-p}{q}\mathfrak{D}c^4\int'\frac{x^{4h}\partial x}{\sqrt{X}}+\text{etc.}$$

$$\int'\frac{\partial xX^{\frac{n}{2}}}{(1+cx^h)^{\frac{p}{q}}}=\int'\partial xX^{\frac{n}{2}}+\tfrac{-p}{q}\mathfrak{A}c\int'x^h\partial xX^{\frac{n}{2}}+\tfrac{-p}{q}\mathfrak{B}c^2\int'x^{2h}\partial xX^{\frac{n}{2}}$$
$$+\tfrac{-p}{q}\mathfrak{C}c^3\int'x^{3h}\partial xX^{\frac{n}{2}}+\tfrac{-p}{q}\mathfrak{D}c^4\int'x^{4h}\partial xX^{\frac{n}{2}}+\text{etc.}$$

$$\int'\frac{x^m\partial x(1+cx^h)^{\frac{p}{q}}}{\sqrt{X}}=\int'\frac{x^m\partial x}{\sqrt{X}}+\tfrac{p}{q}\mathfrak{A}c\int'\frac{x^{m+h}\partial x}{\sqrt{X}}+\tfrac{p}{q}\mathfrak{B}c^2\int'\frac{x^{m+2h}\partial x}{\sqrt{X}}$$
$$+\tfrac{p}{q}\mathfrak{C}c^3\int'\frac{x^{m+3h}\partial x}{\sqrt{X}}+\tfrac{p}{q}\mathfrak{D}c^4\int'\frac{x^{m+4h}\partial x}{\sqrt{X}}+\text{etc.}$$

$$\int'x^m\partial x(1+cx^h)^{\frac{p}{q}}X^{\frac{n}{2}}=\int'x^m\partial xX^{\frac{n}{2}}+\tfrac{p}{q}\mathfrak{A}c\int'x^{m+h}\partial xX^{\frac{n}{2}}$$
$$\tfrac{p}{q}\mathfrak{B}c^2\int'x^{m+2h}\partial xX^{\frac{n}{2}}+\tfrac{p}{q}\mathfrak{C}c^3\int'x^{m+3h}\partial xX^{\frac{n}{2}}+\text{etc.}$$

$$\int'\frac{x^m\partial x}{(1+cx^h)^{\frac{p}{q}}\sqrt{X}}=\int'\frac{x^m\partial x}{\sqrt{X}}+\tfrac{-p}{q}\mathfrak{A}c\int'\frac{x^{m+h}\partial x}{\sqrt{X}}+\tfrac{-p}{q}\mathfrak{B}c^2\int'\frac{x^{m+2h}\partial x}{\sqrt{X}}$$
$$+\tfrac{-p}{q}\mathfrak{C}c^3\int'\frac{x^{m+3h}\partial x}{\sqrt{X}}+\tfrac{-p}{q}\mathfrak{D}c^4\int'\frac{x^{m+4h}\partial x}{\sqrt{X}}+\text{etc.}$$

$$\int'\frac{x^m\partial xX^{\frac{n}{2}}}{(1+cx^h)^{\frac{p}{q}}}=\int'x^m\partial xX^{\frac{n}{2}}+\tfrac{-p}{q}\mathfrak{A}c\int'x^{m+h}\partial xX^{\frac{n}{2}}$$
$$+\tfrac{-p}{q}\mathfrak{B}c^2\int'x^{m+2h}\partial xX^{\frac{n}{2}}+\tfrac{-p}{q}\mathfrak{C}c^3\int'x^{m+3h}\partial xX^{\frac{n}{2}}+\text{etc.}$$

Relationen zwischen den Werthen der bestimmten Integrale.

VZ. $1 - x^n = X$, $\pi = 3{,}14159\cdots\cdots$
m, n, p, r, beliebige positive Zahlen.
Die Integrale von $x = 0$ bis $x = 1$ genommen.

$$\int^{1} \frac{x^m \partial x}{\sqrt{(x - x^2)}} = 2\int^{1} \frac{x^{2m} \partial x}{\sqrt{(1 - x^2)}}$$

$$\int^{1} \frac{x^r \partial x}{\sqrt{(1 - x^2)}} \times \int^{1} \frac{x^{r+1} \partial x}{\sqrt{(1 - x^2)}} = \frac{1}{r+1} \cdot \frac{\pi}{2}$$

$$\int^{1} \frac{x^r \partial x}{\sqrt{(1 - x^4)}} \times \int^{1} \frac{x^{r+2} \partial x}{\sqrt{(1 - x^4)}} = \frac{1}{2(r+1)} \cdot \frac{\pi}{2}$$

$$\int^{1} \frac{x^r \partial x}{\sqrt{(1 - x^{2n})}} \times \int^{1} \frac{x^{r+n} \partial x}{\sqrt{(1 - x^{2n})}} = \frac{1}{n(r+1)} \cdot \frac{\pi}{2}$$

$$\int^{1} \frac{\partial x}{\sqrt{(1 - x^{2n})}} \times \int^{1} \frac{x^n \partial x}{\sqrt{(1 - x^{2n})}} = \frac{1}{n} \cdot \frac{\pi}{2}$$

$$\int^{1} x^{m-1} \partial x X^{\frac{p-n}{n}} = \int^{1} x^{p-1} \partial x X^{\frac{m-n}{n}}$$

$$\int^{1} x^{m-1} \partial x X^{\frac{p-n}{n}} = \frac{(m+p)(m+p+n)(m+p+2n)\cdots(m+p+in)}{m \quad (m+n) \quad (m+2u)\cdots\cdots\cdots\cdots(m+in)} \times \int^{1} x^{m+(i+1)n-1} \partial x X^{\frac{p-n}{n}}$$

$$\frac{\int^{1} x^{m-1} \partial x X^{\frac{p-n}{n}}}{\int^{1} x^{r-1} \partial x X^{\frac{p-n}{n}}} = \frac{(m+p)(m+p+n)(m+p+2n) \text{ in inf.}}{m \quad (m+n) \quad (m+2n) \quad \text{in inf.}} \times \frac{r \quad (r+n) \quad (r+2n) \quad \text{in inf.}}{(r+p)(r+p+n)(r+p+2n) \text{ in inf.}}$$

$$\frac{\int^{1} x^{m-1} \partial x X^{\frac{p-n}{n}}}{\int^{1} x^{m+r-1} \partial x X^{\frac{p-n}{n}}} = \frac{\int^{1} x^{m-1} \partial x X^{\frac{r-n}{n}}}{\int^{1} x^{m+p-1} \partial x X^{\frac{r-n}{n}}}$$

$$\int^{1} \frac{x^{m-1} \partial x}{1 + x^n} = \frac{\pi}{n \operatorname{Sin} \frac{m\pi}{n}}$$

[Dieser und der folgende Werth gilt nur so lange als $m - 1 < n$.]

$$\int^{1} \frac{x^{m-1} \partial x}{\sqrt[n]{X^m}} = \int^{1} x^{n-m-1} \partial x X^{\frac{m-n}{n}} = \frac{\pi}{n \operatorname{Sin} \frac{m\pi}{n}}$$

Entwickelung der Integralformel $\int x^m \partial x (a+bx^n)^p$ in Reihen.

VZ. $a + bx^n = X$

$$\int x^m \partial x X^p = a^p x^{m+1}\left(A + Bx^n + Cx^{2n} + Dx^{3n} + Ex^{4n} + \text{etc.}\right)$$

$$A = \frac{1}{m+1},\quad B = \frac{p\mathfrak{A}}{m+n+1}\cdot\frac{b}{a},\quad C = \frac{p\mathfrak{B}}{m+2n+1}\cdot\frac{b^2}{a^2},$$

$$D = \frac{p\mathfrak{C}}{m+3n+1}\cdot\frac{b^3}{a^3},\quad E = \frac{p\mathfrak{D}}{m+4n+1}\cdot\frac{b^4}{a^4} + \text{etc.}$$

$$\int x^m \partial x X^p = b^p x^{m+np+1}\left(A + \frac{B}{x^n} + \frac{C}{x^{2n}} + \frac{D}{x^{3n}} + \frac{E}{x^{4n}} + \text{etc.}\right)$$

$$A = \frac{1}{m+np+1},\quad B = \frac{p\mathfrak{A}}{m+(p-1)n+1}\cdot\frac{a}{b},$$

$$C = \frac{p\mathfrak{B}}{m+(p-2)n+1}\cdot\frac{a^2}{b^2},\quad D = \frac{p\mathfrak{C}}{m+(p-3)n+1}\cdot\frac{a^3}{b^3},$$

$$E = \frac{p\mathfrak{D}}{m+(p-4)n+1}\cdot\frac{a^4}{b^4},\ \text{etc.}$$

$$\int x^m \partial x X^p = x^{m+1} X^{p+1}\left(\frac{A}{x^n} - \frac{B}{x^{2n}} + \frac{C}{x^{3n}} - \frac{D}{x^{4n}} + \frac{E}{x^{5n}} - \text{etc.}\right)$$

$$A = \frac{1}{(m+np+1)b},\quad B = \frac{(m-n+1)a}{(m-n+np+1)b}A,$$

$$C = \frac{(m-2n+1)a}{(m-2n+np+1)b}B,\quad D = \frac{(m-3n+1)a}{(m-3n+np+1)b}C,$$

$$E = \frac{(m-4n+1)a}{(m-4n+np+1)b}D,\ \text{etc.}$$

$$\int x^m \partial x X^p = x^{m+1} X^{p+1}\left(A - Bx^n + Cx^{2n} - Dx^{3n} + Ex^{4n} - \text{etc.}\right)$$

$$A = \frac{1}{(m+1)a},\quad B = \frac{(m+n+np+1)b}{(m+n+1)a}A,\quad C = \frac{(m+2n+np+1)b}{(m+2n+1)a}B,$$

$$D = \frac{(m+3n+np+1)b}{(m+3n+1)a}C,\quad E = \frac{(m+4n+np+1)b}{(m+4n+1)a}D,\ \text{etc.}$$

Entwickelung der Integralformel $\int x^m \partial x (a + bx^n)^p$ in Reihen.

VZ. $a + bx^n = X$

$$\int x^m \partial x X^p = - x^{m+1} X^{p+1} \left(A + BX + CX^2 + DX^3 + \text{etc.} \right)$$

$$A = \frac{1}{(p+1)na},\ B = \frac{m+n+np+1}{(p+2)na} A,\ C = \frac{m+2n+np+1}{(p+3)na} B,$$

$$D = \frac{m+3n+np+1}{(p+4)na} C,\ E = \frac{m+4n+np+1}{(p+5)na} D,\ \text{etc.}$$

$$\int x^m \partial x X^p = x^{m+1} X^p \left(A + \frac{B}{X} + \frac{C}{X^2} + \frac{D}{X^3} + \frac{E}{X^4} + \text{etc.} \right)$$

$$A = \frac{1}{m+np+1},\ B = \frac{pna}{m-n+np+1} A,\ C = \frac{(p-1)na}{m-2n+np+1} B,$$

$$D = \frac{(p-2)na}{m-3n+np+1} C,\ E = \frac{(p-3)na}{m-4n+np+1} D,\ \text{etc.}$$

$$\int x^m \partial x X^p = x^{m+1} X^p \left[A - B\left(\frac{x^n}{X}\right) + C\left(\frac{x^n}{X}\right)^2 - D\left(\frac{x^n}{X}\right)^3 + E\left(\frac{x^n}{X}\right)^4 - F\left(\frac{x^n}{X}\right)^5 + \text{etc.} \right]$$

$$A = \frac{1}{m+1},\ B = \frac{pnb}{m+n+1} A,\ C = \frac{(p-1)nb}{m+2n+1} B,$$

$$D = \frac{(p-2)nb}{m+3n+1} C,\ E = \frac{(p-3)nb}{m+4n+1} D,\ F = \frac{(p-4)nb}{m+5n+1} E,\ \text{etc.}$$

$$\int x^m \partial x X^p = x^{m-n+1} X^{p+1} \left[A - B\left(\frac{X}{x^n}\right) + C\left(\frac{X}{x^n}\right)^2 - D\left(\frac{X}{x^n}\right)^3 + E\left(\frac{X}{x^n}\right)^4 + F\left(\frac{X}{x^n}\right)^5 + \text{etc.} \right]$$

$$A = \frac{1}{(p+1)nb},\ B = \frac{m-n+1}{(p+2)nb} A,\ C = \frac{m-2n+1}{(p+3)nb} B,$$

$$D = \frac{m-3n+1}{(p+4)nb} C,\ E = \frac{m-4n+1}{(p+5)nb} D,\ F = \frac{m-5n+1}{(p+6)nb} E,\ \text{etc.}$$

Integrations - Methoden. *)

VZ. $F:[x, y, z, t$, etc.$]$ eine rationale Function der Gröſsen x, y, z, t, etc.

I. Das Differential

$$\partial x F : [x, \sqrt[m]{x}, \sqrt[n]{x}, \sqrt[p]{x}, \sqrt[q]{x}, \text{etc.}]$$

wird rational, wenn $x = y^{mnpq \cdots\cdots}$ gesetzt wird; denn dadurch wird $\sqrt[m]{x} = y^{npq \cdots\cdots}$, $\sqrt[n]{x} = y^{mpq \cdots\cdots}$, $\sqrt[p]{x} = y^{mnq \cdots\cdots}$, $\sqrt[q]{x} = y^{mnp \cdots\cdots}$, etc. und $\partial x = (mnpq \cdots\cdots) y^{(mnpq \cdots\cdots)-1} \partial y$. Dahin gehört z. B. das Differential $\frac{x^3 + 2\sqrt[5]{ax^2} + \sqrt[3]{x}}{bx + c\sqrt[4]{dx}} \partial x$; es wird rational, wenn $x = y^{60}$ gesetzt wird.

II. Das Differential

$$\partial x F : [x, \sqrt[n]{(a + bx)}]$$

wird rational, wenn $a + bx = y^n$ gesetzt wird; denn alsdann ist $\sqrt[n]{(a + bx)} = y$, $x = \frac{y^n - a}{b}$, $\partial x = \frac{ny^{n-1}\partial y}{b}$. Dahin gehören z. B. die Differentiale $\frac{x^4 \partial x}{cx^5 + \sqrt[4]{(a + bx)^3}}$, $\frac{x^2 \partial x \sqrt[5]{(a + bx)^3}}{cx + d\sqrt[5]{(a + bx)^2}}$. Das erstere wird rational, wenn $a + bx = y^4$, das zweite, wenn $a + bx = y^5$ gesetzt wird.

III. Das Differential

$$\partial x F : \left[x, \sqrt[n]{\frac{a + bx}{f + gx}}\right]$$

wird rational, wenn $\frac{a + bx}{f + gx} = y^n$ gesetzt wird; denn hierdurch wird

*) Ein Differential soll hier als integrirt angesehen werden, sobald man dasselbe durch irgend eine Verwandlung rational gemacht, oder auf solche irrationale Integrale zurückgeführt hat, welche sich rational machen lassen.

$\sqrt[n]{\frac{a+bx}{f+gx}}=y$, $x=\frac{a-fy^n}{gy^n-b}$, $\partial x=\frac{n(bf-ag)y^{n-1}\partial y}{(gy^n-b)^2}$. Dahin gehören z. B. die Differentiale $x^m\partial x\left(\frac{a+bx}{f+gx}\right)^{\frac{h}{n}}$, $\frac{\partial x\sqrt[n]{(a+bx)}}{x^m\sqrt[n]{(f+gx)}}$, $\frac{\partial x\sqrt[n]{(a+bx)}}{\sqrt[n]{(a+bx)}+\sqrt[n]{(f+gx)}}$. Es lassen sich nämlich diesen Differentialen folgende Formen geben:

$$x^m\partial x\left[\left(\frac{a+bx}{f+gx}\right)^{\frac{1}{n}}\right]^h,\ \frac{\partial x}{x^m}\sqrt[n]{\frac{a+bx}{f+gx}},\ \frac{\partial x}{\sqrt[n]{\frac{a+bx}{f+gx}}+1}\cdot\sqrt[n]{\frac{a+bx}{f+gx}}.$$

IV. Das Differential

$$\partial xF:[x,\ (a+bx)^{\frac{m}{n}},\ (a+bx)^{\frac{p}{q}},\ (a+bx)^{\frac{r}{s}},\ \text{etc.}]$$

wird rational, wenn $a+bx=y^{nqs\cdots}$ gesetzt wird; denn hierdurch wird $(a+bx)^{\frac{m}{n}}=y^{mqs\cdots}$, $(a+bx)^{\frac{p}{q}}=y^{nps\cdots}$, $(a+bx)^{\frac{r}{s}}=y^{nqr\cdots}$, etc., $x=\frac{y^{nqs\cdots}-a}{b}$, $\partial x=\frac{nqs\cdots}{b}y^{(nqs\cdots)-1}\partial y$.

V. Auf eine ähnliche Weise wird das Differential

$$\partial xF:\left[x,\ \left(\frac{a+bx}{f+gx}\right)^{\frac{m}{n}},\ \left(\frac{a+bx}{f+gx}\right)^{\frac{p}{q}},\ \left(\frac{a+bx}{f+gx}\right)^{\frac{r}{s}},\ \text{etc.}\right]$$

rational, wenn man $\frac{a+bx}{f+gx}=y^{nqs\cdots}$ setzt.

VI. Um das Differential

$$\partial xF:[x,\ \sqrt{(a+bx+cx^2)}]$$

rational zu machen, muſs man die beiden Fälle unterscheiden, wo c positiv und wo c negativ ist.

Erster Fall. Das Differential $\partial xF:[x,\ \sqrt{(a+bx+cx^2)}]$ wird rational, wenn $a+bx+cx^2=c(x+y)^2$ gesetzt wird; hieraus erhält man nämlich $x=\frac{a-cy^2}{2cy-b}$, $\partial x=-\frac{2c(cy^2-by+a)\partial y}{(2cy-b)^2}$, $\sqrt{(a+bx+cx^2)}=\frac{(cy^2-by+a)\sqrt{c}}{2cy-b}$.

Zweiter Fall. Es bezeichnen r und r' die beiden Wurzeln der Gleichung $a+bx-cx^2=0$; so ist $\sqrt{(a+bx-cx^2)}=\sqrt{c(x-r)(r'-x)}$. Das Differential $\partial x F:[x, \sqrt{(a+bx-cx^2)}]$ wird daher rational, wenn man $\sqrt{c(x-r)(r'-x)}=(x-r)cy$ setzt; denn hieraus erhält man $x=\frac{cry^2+r'}{cy^2+1}$, $\partial x=\frac{(r-r')2cy\partial y}{(cy^2+1)^2}$, $\sqrt{(a+bx-cx^2)}=\frac{(r'-r)cy}{cy^2+1}$.

Die Wurzeln der Gleichung $a+bx-cx^2=0$ sind nothwendig reell, a und b mögen positiv oder negativ seyn, weil im entgegengesetzten Falle $\sqrt{(a+bx-cx^2)}$ für jeden Werth des x imaginär seyn würde.

VII. Die Differentiale

$$\partial x F:[x, \sqrt{(a+cx^2)}],\ \partial x F:[x, \sqrt{(bx+cx^2)}]$$

sind unter den vorigen begriffen; denn man erhält sie daraus, wenn man $b=0$, oder $a=0$ setzt.

VIII. Um das Differential

$$\partial x F:[x, \sqrt{(a+bx)}, \sqrt{(a'+b'x)}]$$

rational zu machen, setze man zuerst $a+bx=(a'+b'x)y^2$; dies giebt $x=\frac{a-a'y^2}{b'y^2-b}$, $\partial x=\frac{(a'b-ab')2y\partial y}{(b'y^2-b)^2}$, $\sqrt{(a+bx)}=\frac{y\sqrt{(ab'-a'b)}}{\sqrt{(b'y^2-b)}}$, $\sqrt{(a'+b'x)}=\frac{\sqrt{(ab'-a'b)}}{\sqrt{(b'y^2-b)}}$. Durch die Substitution dieser Werthe verwandelt sich das gegebene Differential in ein anderes von der Form $\partial y F':[y, \sqrt{(b'y^2-b)}]$, wenn F' irgend eine andre rationale Function als F bezeichnet; und dieses Differential kann wieder nach dem Vorhergehenden rational gemacht werden.

IX. Das Differential

$$x^{m-1}\partial x(a+bx^n)^{\frac{p}{q}}$$

läſst sich in den beiden Fällen, wo $\frac{m}{n}$ oder $\frac{m}{n}+\frac{p}{q}$ eine ganze positive oder negative Zahl ist, rational machen.

Erster Fall. Man setze $a+bx^n=y^q$, so wird $(a+bx^n)^{\frac{p}{q}}=y^p$,

$$x^n=\frac{y^q-a}{b},\quad x^m=\left(\frac{y^q-a}{b}\right)^{\frac{m}{n}},\quad x^{m-1}\partial x=\frac{qy^{q-1}}{nb}\left(\frac{y^q-a}{b}\right)^{\frac{m-n}{n}}.$$

Durch die Substitution dieser Werthe verwandelt sich das obige Differential in $\frac{q}{nb}y^{p+q-1}\partial y\left(\frac{y^q-a}{b}\right)^{\frac{m-n}{n}}$, und wird daher rational, wenn $\frac{m-n}{n}$, also auch $\frac{m}{n}$ eine ganze Zahl ist.

Zweiter Fall. Man setze $a+bx^n=x^n y^q$; so ist $x^n=\frac{a}{y^q-b}$,

$$a+bx^n=\frac{ay^q}{y^q-b},\quad (a+bx^n)^{\frac{p}{q}}=\frac{a^{\frac{p}{q}}y^p}{(y^q-b)^{\frac{p}{q}}},\quad x^m=\frac{a^{\frac{m}{n}}}{(y^q-b)^{\frac{m}{n}}},$$

$x^{m-1}\partial x=-\frac{qa^{\frac{m}{n}}y^{q-1}}{n(y^q-b)^{\frac{m}{n}+1}}$. Das gegebene Differential verwandelt sich daher in $-\frac{qa^{\frac{m}{n}+\frac{p}{q}}y^{p+q-1}}{n(y^q-b)^{\frac{m}{n}+\frac{p}{q}+1}}$, und wird folglich rational, sobald $\frac{m}{n}+\frac{p}{q}$ eine ganze Zahl ist.

X. In den nämlichen zwey Fällen und durch dieselben Substitutionen wird überhaupt das Differential

$$x^{m-1}\partial x(a+bx^n)^{\frac{p}{q}}F:[x^n]$$

rational. Hieher gehört z. B. das Differential $x^{m+in-1}\partial x(a+bx^n)^{\frac{p}{q}}$, und das noch allgemeinere $\frac{Px^{m-1}\partial x}{Q}(a+bx^n)^{\frac{p}{q}}$, wenn $P=A+Bx^n+Cx^{2n}+Dx^{3n}+\text{etc.}$, $Q=A'+B'x^n+C'x^{2n}+D'x^{3n}+\text{etc.}$

XI. Das Differential

$$x^{m-1}\partial xF:[x^m,\ x^n,\ \sqrt[q]{(a+bx^n)}],$$

welches das in IX und X in sich schliefst, wird rational, wenn $\frac{m}{n}$ eine ganze positive oder negative Zahl ist; denn man setze

$'(a+bx^n) = y$, so wird $x^n = \frac{y^q - a}{b}$, $x^m = \left(\frac{y^q - a}{b}\right)^{\frac{m}{n}}$,

$$^{m-1}\partial x = \frac{qy^{q-1}}{nb}\left(\frac{y^q - a}{b}\right)^{\frac{m}{n}-1}\partial y$$

XII. Es bezeichnen X, X', X'', rationale Functionen von x, › lassen sich die Differentiale

$$\frac{X\partial x}{X' + X''\sqrt{(a+bx+cx^2)}},$$

$$\frac{X\partial x}{X'\sqrt{(a+bx+cx^2)}+X''\sqrt{(a'+b'x+c'x^2)}},$$

ımmer rational machen, wenn man ersteres mit $X' - X''\sqrt{(a+bx+cx^2)}$, nd letzteres mit $X'\sqrt{(a+bx+cx^2)} - X''\sqrt{(a'+b'x+c'x^2)}$ ıultiplicirt; denn hierdurch verwandelt sich das erstere in $\frac{XX'\partial x}{X'^2 - X''^2(a+bx+cx^2)} - \frac{XX''\partial x\sqrt{(a+bx+cx^2)}}{X'^2 - X''^2(a+bx+cx^2)}$, worin ıan blofs noch den zweiten Theil rational zu machen hat, und as zweite in $\frac{XX'\partial x\sqrt{(a+bx+cx^2)}}{X'^2(a+bx+cx^2) - X''^2(a'+b'x+c'x^2)} - \frac{XX''\partial x\sqrt{(a'+b'x+c'x^2)}}{X'^2(a+bx+cx^2) - X''^2(a'+b'x+c'x^2)}$, worin sich jeder Theil esonders rational machen läfst. *)

XIII. Das Differential

$$x^m\partial x F : [x^n,\ \sqrt{(a+bx^n+cx^{2n})}]$$

erwandelt sich, wenn $x^n = y$ gesetzt wird, in

$$\frac{1}{n}\, y^{\frac{m+1}{n}-1}\partial y F : [y,\ \sqrt{(a+by+cy^2)}],$$

ınd kann in dieser Gestalt nach der Methode in VI. rational ge-

*) Es lassen sich überhaupt immer die Wurzelgröfsen aus dem Nenner ines Bruches wegschaffen, (m. s. meine Samml. v. Aufg. a. d. Th. d. Gl. ›eite 213 — 215) und man hat es alsdann blofs mit der Integration von Monomen zu thun.

macht werden, wenn $\frac{m+1}{n}$ eine ganze Zahl ist. Hieher gehört das Differential $x^m \partial x (a + bx^n + cx^{2n})^{\frac{p}{2}}$

XIV. Unter der nämlichen Bedingung, daſs $\frac{m+1}{n}$ eine ganze Zahl sey, läſst sich auch das Differential

$$x^m \partial x F : [x^n,\ hx^n + \sqrt{(a + h^2 x^{2n})}]$$

und das noch allgemeinere

$$x^m \partial x F : [x^n,\ \sqrt{(a + h^2 x^{2n})},\ hx^n + \sqrt{(a + h^2 x^{2n})}]$$

rational machen; denn man setze $hx^n + \sqrt{(a + h^2 x^{2n})} = y$, so ist

$$x^n = \frac{y^2 - a}{2hy},\quad \sqrt{(a + h^2 x^{2n})} = \frac{y^2 + a}{2y},$$

$$x^m \partial x = \frac{1}{n(2h)^{\frac{m+1}{n}}} \left(\frac{y^2 + a}{y}\right) \left(\frac{y^2 - a}{y}\right)^{\frac{m+1}{n} - 1} \partial y.$$

Hieher gehört das weniger umfassende Differential

$$\partial x F : [x,\ \sqrt{(a + h^2 x^2)},\ hx + \sqrt{(a + h^2 x^2)}]$$

wohin die einzelnen Differentiale $[x + \sqrt{(1 + x^2)}]^n \partial x$, $[x + \sqrt{(1 + x^2)}]^n X \partial x$, $[ax + b\sqrt{(1 + x^2)}][x + \sqrt{(1 + x^2)}]^n \partial x$ zu rechnen sind, welche Euler in dem Anhange zu § 125 des ersten Bandes seiner Institutionen anführt.

Integraltafeln

für

ranscendente Differentiale.

Tafel
der Reductionsformeln für das Integral
$$\int \partial\varphi \operatorname{Sin}^m\varphi \operatorname{Cos}^n\varphi$$

I.

$$\int \partial\varphi \operatorname{Sin}^m\varphi \operatorname{Cos}^n\varphi = \frac{\operatorname{Sin}^{m+1}\varphi \operatorname{Cos}^{n-1}\varphi}{m+1} + \frac{n-1}{m+1}\int \partial\varphi \operatorname{Sin}^{m+2}\varphi \operatorname{Cos}^{n-2}\varphi$$

II.

$$\int \partial\varphi \operatorname{Sin}^m\varphi \operatorname{Cos}^n\varphi = -\frac{\operatorname{Sin}^{m-1}\varphi \operatorname{Cos}^{n+1}\varphi}{n+1} + \frac{m-1}{n+1}\int \partial\varphi \operatorname{Sin}^{m-2}\varphi \operatorname{Cos}^{n+2}\varphi$$

III.

$$\int \partial\varphi \operatorname{Sin}^m\varphi \operatorname{Cos}^n\varphi = -\frac{\operatorname{Sin}^{m-1}\varphi \operatorname{Cos}^{n+1}\varphi}{m+n} + \frac{m-1}{m+n}\int \partial\varphi \operatorname{Sin}^{m-2}\varphi \operatorname{Cos}^{n}\varphi$$

IV.

$$\int \partial\varphi \operatorname{Sin}^m\varphi \operatorname{Cos}^n\varphi = \frac{\operatorname{Sin}^{m+1}\varphi \operatorname{Cos}^{n-1}\varphi}{m+n} + \frac{n-1}{m+n}\int \partial\varphi \operatorname{Sin}^{m}\varphi \operatorname{Cos}^{n-2}\varphi$$

V.

$$\int \partial\varphi \operatorname{Sin}^m\varphi \operatorname{Cos}^n\varphi = \frac{\operatorname{Sin}^{m+1}\varphi \operatorname{Cos}^{n+1}\varphi}{m+1} + \frac{m+n+2}{m+1}\int \partial\varphi \operatorname{Sin}^{m+2}\varphi \operatorname{Cos}^{n}\varphi$$

VI.

$$\int \partial\varphi \operatorname{Sin}^m\varphi \operatorname{Cos}^n\varphi = -\frac{\operatorname{Sin}^{m+1}\varphi \operatorname{Cos}^{n+1}\varphi}{n+1} + \frac{m+n+2}{n+1}\int \partial\varphi \operatorname{Sin}^{m}\varphi \operatorname{Cos}^{n+2}\varphi$$

Diese Formeln gelten, m und n mögen positive oder negative, ganze [od]er gebrochene Zahlen, oder auch $=0$ seyn.

Taf. I.

$$\int \partial\varphi \operatorname{Sin}^m \varphi$$

$$\int \partial\varphi \operatorname{Sin}\varphi = -\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi = -\tfrac{1}{2}\operatorname{Sin}\varphi\operatorname{Cos}\varphi + \tfrac{1}{2}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^3\varphi = \left(-\tfrac{1}{3}\operatorname{Sin}^2\varphi - \tfrac{2}{3}\right)\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^4\varphi = \left(-\tfrac{1}{4}\operatorname{Sin}^3\varphi - \tfrac{3}{8}\operatorname{Sin}\varphi\right)\operatorname{Cos}\varphi + \tfrac{3}{8}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^5\varphi = \left(-\tfrac{1}{5}\operatorname{Sin}^4\varphi - \tfrac{4}{15}\operatorname{Sin}^2\varphi - \tfrac{8}{15}\right)\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^6\varphi = \left(-\tfrac{1}{6}\operatorname{Sin}^5\varphi - \tfrac{5}{24}\operatorname{Sin}^3\varphi - \tfrac{5}{16}\operatorname{Sin}\varphi\right)\operatorname{Cos}\varphi + \tfrac{5}{16}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi = \left(-\tfrac{1}{7}\operatorname{Sin}^6\varphi - \tfrac{6}{35}\operatorname{Sin}^4\varphi - \tfrac{8}{35}\operatorname{Sin}^2\varphi - \tfrac{16}{35}\right)\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^8\varphi = \left(-\tfrac{1}{8}\operatorname{Sin}^7\varphi - \tfrac{7}{48}\operatorname{Sin}^5\varphi - \tfrac{35}{192}\operatorname{Sin}^3\varphi - \tfrac{35}{128}\operatorname{Sin}\varphi\right)\operatorname{Cos}\varphi + \tfrac{35}{128}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi = \left(-\tfrac{1}{9}\operatorname{Sin}^8\varphi - \tfrac{8}{63}\operatorname{Sin}^6\varphi - \tfrac{16}{105}\operatorname{Sin}^4\varphi - \tfrac{64}{315}\operatorname{Sin}^2\varphi - \tfrac{128}{315}\right)\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}\varphi = -\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi = -\tfrac{1}{4}\operatorname{Sin}2\varphi + \tfrac{1}{2}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^3\varphi = \tfrac{1}{12}\operatorname{Cos}3\varphi - \tfrac{3}{4}\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^4\varphi = \tfrac{1}{32}\operatorname{Sin}4\varphi - \tfrac{1}{4}\operatorname{Sin}2\varphi + \tfrac{3}{8}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^5\varphi = -\tfrac{1}{80}\operatorname{Cos}5\varphi + \tfrac{5}{48}\operatorname{Cos}3\varphi - \tfrac{5}{8}\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^6\varphi = -\tfrac{1}{192}\operatorname{Sin}6\varphi + \tfrac{3}{64}\operatorname{Sin}4\varphi - \tfrac{15}{64}\operatorname{Sin}2\varphi + \tfrac{5}{16}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi = \tfrac{1}{448}\operatorname{Cos}7\varphi - \tfrac{7}{320}\operatorname{Cos}5\varphi + \tfrac{7}{64}\operatorname{Cos}3\varphi - \tfrac{35}{64}\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^8\varphi = \tfrac{1}{1024}\operatorname{Sin}8\varphi - \tfrac{1}{96}\operatorname{Sin}6\varphi + \tfrac{7}{128}\operatorname{Sin}4\varphi - \tfrac{7}{32}\operatorname{Sin}2\varphi + \tfrac{35}{128}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi = -\tfrac{1}{2304}\operatorname{Cos}9\varphi + \tfrac{9}{1792}\operatorname{Cos}7\varphi - \tfrac{9}{320}\operatorname{Cos}5\varphi + \tfrac{7}{64}\operatorname{Cos}3\varphi - \tfrac{63}{128}\operatorname{Cos}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^m \varphi$$

Taf. II.

$$\int \partial\varphi \operatorname{Cos}\varphi = \operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^2\varphi = \tfrac{1}{2}\operatorname{Sin}\varphi \operatorname{Cos}\varphi + \tfrac{1}{2}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^3\varphi = \left(\tfrac{1}{3}\operatorname{Cos}^2\varphi + \tfrac{2}{3}\right)\operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^4\varphi = \left(\tfrac{1}{4}\operatorname{Cos}^3\varphi + \tfrac{3}{8}\operatorname{Cos}\varphi\right)\operatorname{Sin}\varphi + \tfrac{3}{8}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^5\varphi = \left(\tfrac{1}{5}\operatorname{Cos}^4\varphi + \tfrac{4}{15}\operatorname{Cos}^2\varphi + \tfrac{8}{15}\right)\operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^6\varphi = \left(\tfrac{1}{6}\operatorname{Cos}^5\varphi + \tfrac{5}{24}\operatorname{Cos}^3\varphi + \tfrac{5}{16}\operatorname{Cos}\varphi\right)\operatorname{Sin}\varphi + \tfrac{5}{16}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^7\varphi = \left(\tfrac{1}{7}\operatorname{Cos}^6\varphi + \tfrac{6}{35}\operatorname{Cos}^4\varphi + \tfrac{8}{35}\operatorname{Cos}^2\varphi + \tfrac{16}{35}\right)\operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^8\varphi = \left(\tfrac{1}{8}\operatorname{Cos}^7\varphi + \tfrac{7}{48}\operatorname{Cos}^5\varphi + \tfrac{35}{192}\operatorname{Cos}^3\varphi + \tfrac{35}{128}\operatorname{Cos}\varphi\right)\operatorname{Sin}\varphi + \tfrac{35}{128}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^9\varphi = \left(\tfrac{1}{9}\operatorname{Cos}^8\varphi + \tfrac{8}{63}\operatorname{Cos}^6\varphi + \tfrac{16}{105}\operatorname{Cos}^4\varphi + \tfrac{64}{315}\operatorname{Cos}^2\varphi + \tfrac{128}{315}\right)\operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}\varphi = \operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^2\varphi = \tfrac{1}{4}\operatorname{Sin}2\varphi + \tfrac{1}{2}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^3\varphi = \tfrac{1}{12}\operatorname{Sin}3\varphi + \tfrac{3}{4}\operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^4\varphi = \tfrac{1}{32}\operatorname{Sin}4\varphi + \tfrac{1}{4}\operatorname{Sin}2\varphi + \tfrac{3}{8}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^5\varphi = \tfrac{1}{80}\operatorname{Sin}5\varphi + \tfrac{5}{48}\operatorname{Sin}3\varphi + \tfrac{5}{8}\operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^6\varphi = \tfrac{1}{192}\operatorname{Sin}6\varphi + \tfrac{3}{64}\operatorname{Sin}4\varphi + \tfrac{15}{64}\operatorname{Sin}2\varphi + \tfrac{5}{16}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^7\varphi = \tfrac{1}{448}\operatorname{Sin}7\varphi + \tfrac{7}{320}\operatorname{Sin}5\varphi + \tfrac{7}{64}\operatorname{Sin}3\varphi + \tfrac{35}{64}\operatorname{Sin}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^8\varphi = \tfrac{1}{1024}\operatorname{Sin}8\varphi + \tfrac{1}{96}\operatorname{Sin}6\varphi + \tfrac{7}{128}\operatorname{Sin}4\varphi + \tfrac{7}{52}\operatorname{Sin}2\varphi + \tfrac{35}{128}\varphi$$

$$\int \partial\varphi \operatorname{Cos}^9\varphi = \tfrac{1}{2304}\operatorname{Sin}9\varphi + \tfrac{9}{1792}\operatorname{Sin}7\varphi + \tfrac{9}{320}\operatorname{Sin}5\varphi + \tfrac{7}{64}\operatorname{Sin}3\varphi + \tfrac{63}{128}\operatorname{Sin}\varphi$$

Taf. III. $\int \partial\varphi \operatorname{Sin}\varphi \operatorname{Cos}^n\varphi$

$$\int \partial\varphi \operatorname{Sin}\varphi \operatorname{Cos}^n\varphi = -\frac{1}{n+1}\operatorname{Cos}^{n+1}\varphi$$

$$\operatorname{Cos}\varphi = \operatorname{Cos}\varphi$$

$$\operatorname{Cos}^2\varphi = \tfrac{1}{2}\operatorname{Cos}2\varphi + \tfrac{1}{2}$$

$$\operatorname{Cos}^3\varphi = \tfrac{1}{4}\operatorname{Cos}3\varphi + \tfrac{3}{4}\operatorname{Cos}\varphi$$

$$\operatorname{Cos}^4\varphi = \tfrac{1}{8}\operatorname{Cos}4\varphi + \tfrac{1}{2}\operatorname{Cos}2\varphi + \tfrac{3}{8}$$

$$\operatorname{Cos}^5\varphi = \tfrac{1}{16}\operatorname{Cos}5\varphi + \tfrac{5}{16}\operatorname{Cos}3\varphi + \tfrac{5}{8}\operatorname{Cos}\varphi$$

$$\operatorname{Cos}^6\varphi = \tfrac{1}{32}\operatorname{Cos}6\varphi + \tfrac{3}{16}\operatorname{Cos}4\varphi + \tfrac{15}{32}\operatorname{Cos}2\varphi + \tfrac{5}{16}$$

$$\operatorname{Cos}^7\varphi = \tfrac{1}{64}\operatorname{Cos}7\varphi + \tfrac{7}{64}\operatorname{Cos}5\varphi + \tfrac{21}{64}\operatorname{Cos}3\varphi + \tfrac{35}{64}\operatorname{Cos}\varphi$$

$$\operatorname{Cos}^8\varphi = \tfrac{1}{128}\operatorname{Cos}8\varphi + \tfrac{1}{16}\operatorname{Cos}6\varphi + \tfrac{7}{32}\operatorname{Cos}4\varphi + \tfrac{7}{16}\operatorname{Cos}2\varphi + \tfrac{35}{128}$$

$$\operatorname{Cos}^9\varphi = \tfrac{1}{256}\operatorname{Cos}9\varphi + \tfrac{9}{256}\operatorname{Cos}7\varphi + \tfrac{9}{64}\operatorname{Cos}5\varphi + \tfrac{21}{64}\operatorname{Cos}3\varphi + \tfrac{63}{128}\operatorname{Cos}\varphi$$

$$\operatorname{Cos}^{10}\varphi = \tfrac{1}{512}\operatorname{Cos}10\varphi + \tfrac{5}{256}\operatorname{Cos}8\varphi + \tfrac{45}{512}\operatorname{Cos}6\varphi + \tfrac{15}{64}\operatorname{Cos}4\varphi + \tfrac{105}{256}\operatorname{Cos}2\varphi + \tfrac{63}{256}$$

....

$$\operatorname{Cos}^n\varphi = \frac{1}{2^{n-1}}\Big[\operatorname{Cos}n\varphi + {}^n\mathfrak{A}\operatorname{Cos}(n-2)\varphi + {}^n\mathfrak{B}\operatorname{Cos}(n-4)\varphi + {}^n\mathfrak{C}\operatorname{Cos}(n-6)\varphi + {}^n\mathfrak{D}\operatorname{Cos}(n-8)\varphi + {}^n\mathfrak{E}\operatorname{Cos}(n-10)\varphi + \text{etc.}\Big]$$

[Die Reihe in den Haken so weit fortgesetzt, bis man zu negativen Winkeln kommt, und anstatt $\operatorname{Cos} o\varphi$ nur $\frac{1}{2}\operatorname{Cos} o\varphi = \frac{1}{2}$ gesetzt.]

$\int \partial\varphi \operatorname{Cos}\varphi \operatorname{Sin}^n\varphi$ *Taf. IV.*

$$\int \partial\varphi \operatorname{Cos}\varphi \operatorname{Sin}^n\varphi = \frac{1}{n+1} \operatorname{Sin}^{n+1}\varphi$$

$$\operatorname{Sin}\varphi = \operatorname{Sin}\varphi$$

$$\operatorname{Sin}^2\varphi = -\tfrac{1}{2}\operatorname{Cos}2\varphi + \tfrac{1}{2}$$

$$\operatorname{Sin}^3\varphi = -\tfrac{1}{4}\operatorname{Sin}3\varphi + \tfrac{3}{4}\operatorname{Sin}\varphi$$

$$\operatorname{Sin}^4\varphi = \tfrac{1}{8}\operatorname{Cos}4\varphi - \tfrac{1}{2}\operatorname{Cos}2\varphi + \tfrac{3}{8}$$

$$\operatorname{Sin}^5\varphi = \tfrac{1}{16}\operatorname{Sin}5\varphi - \tfrac{5}{16}\operatorname{Sin}3\varphi + \tfrac{5}{8}\operatorname{Sin}\varphi$$

$$\operatorname{Sin}^6\varphi = -\tfrac{1}{32}\operatorname{Cos}6\varphi + \tfrac{3}{16}\operatorname{Cos}4\varphi - \tfrac{15}{32}\operatorname{Cos}2\varphi + \tfrac{5}{16}$$

$$\operatorname{Sin}^7\varphi = -\tfrac{1}{64}\operatorname{Sin}7\varphi + \tfrac{7}{64}\operatorname{Sin}5\varphi - \tfrac{21}{64}\operatorname{Sin}3\varphi + \tfrac{35}{64}\operatorname{Sin}\varphi$$

$$\operatorname{Sin}^8\varphi = \tfrac{1}{128}\operatorname{Cos}8\varphi - \tfrac{1}{16}\operatorname{Cos}6\varphi + \tfrac{7}{32}\operatorname{Cos}4\varphi - \tfrac{7}{16}\operatorname{Cos}2\varphi + \tfrac{35}{128}$$

$$\operatorname{Sin}^9\varphi = \tfrac{1}{256}\operatorname{Sin}9\varphi - \tfrac{9}{256}\operatorname{Sin}7\varphi + \tfrac{9}{64}\operatorname{Sin}5\varphi - \tfrac{21}{64}\operatorname{Sin}3\varphi + \tfrac{63}{128}\operatorname{Sin}\varphi$$

$$\operatorname{Sin}^{10}\varphi = -\tfrac{1}{512}\operatorname{Cos}10\varphi + \tfrac{5}{256}\operatorname{Cos}8\varphi - \tfrac{45}{512}\operatorname{Cos}6\varphi + \tfrac{15}{64}\operatorname{Cos}4\varphi - \tfrac{105}{256}\operatorname{Cos}2\varphi + \tfrac{63}{256}$$

....

$$\operatorname{Sin}^n\varphi = \pm\frac{1}{2^{n-1}}\Big[\operatorname{Cos}n\varphi - {}^n\mathfrak{A}\operatorname{Cos}(n-2)\varphi + {}^n\mathfrak{B}\operatorname{Cos}(n-4)\varphi - {}^n\mathfrak{C}\operatorname{Cos}(n-6)\varphi + \text{etc.}\Big]$$

$$\operatorname{Sin}^n\varphi = \pm\frac{1}{2^{n-1}}\Big[\operatorname{Sin}n\varphi - {}^n\mathfrak{A}\operatorname{Sin}(n-2)\varphi + {}^n\mathfrak{B}\operatorname{Sin}(n-4)\varphi - {}^n\mathfrak{C}\operatorname{Sin}(n-6)\varphi + \text{etc.}\Big]$$

Die erste Reihe für $\operatorname{Sin}^n\varphi$ mit dem Vorzeichen $+$, wenn n von der Form $4k$, und mit dem Vorzeichen $-$, wenn n von der Form $4k+2$ ist; die zweite Reihe mit dem Vorzeichen $+$, wenn n von der Form $4k+1$, und mit dem Voszeichen $-$, wenn n von der Form $4k+3$ ist. Beide Reihen werden so weit fortgesetzt, bis man zu negativen Winkeln kommt, und anstatt $\operatorname{Cos}0\varphi$ nur $\tfrac{1}{2}\operatorname{Cos}0\varphi = \tfrac{1}{2}$ gesetzt.

Taf. V. $\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^n\varphi$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}\varphi = \frac{1}{3}\operatorname{Sin}^3\varphi$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^2\varphi = \frac{1}{4}\operatorname{Sin}^3\varphi \operatorname{Cos}\varphi - \frac{1}{8}\operatorname{Sin}\varphi \operatorname{Cos}\varphi + \frac{1}{8}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^3\varphi = \left(\frac{1}{5}\operatorname{Cos}^2\varphi + \frac{2}{15}\right)\operatorname{Sin}^3\varphi$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^4\varphi = \frac{1}{6}\operatorname{Sin}^3\varphi \operatorname{Cos}^3\varphi + \frac{1}{2}\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^2\varphi$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^5\varphi = \left(\frac{1}{7}\operatorname{Cos}^4\varphi + \frac{4}{35}\operatorname{Cos}^2\varphi + \frac{8}{105}\right)\operatorname{Sin}^3\varphi$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}\varphi = -\frac{1}{4}\left(\frac{1}{3}\operatorname{Sin}3\varphi - \operatorname{Sin}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^2\varphi = -\frac{1}{8}\left(\frac{1}{4}\operatorname{Sin}4\varphi - \varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^3\varphi = -\frac{1}{16}\left(\frac{1}{5}\operatorname{Sin}5\varphi + \frac{1}{3}\operatorname{Sin}3\varphi - 2\operatorname{Sin}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^4\varphi = -\frac{1}{32}\left(\frac{1}{6}\operatorname{Sin}6\varphi + \frac{1}{2}\operatorname{Sin}4\varphi - \frac{1}{2}\operatorname{Sin}2\varphi - 2\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^5\varphi = -\frac{1}{64}\left(\frac{1}{7}\operatorname{Sin}7\varphi + \frac{3}{5}\operatorname{Sin}5\varphi + \frac{1}{3}\operatorname{Sin}3\varphi - 5\operatorname{Sin}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^6\varphi = -\frac{1}{128}\left(\frac{1}{8}\operatorname{Sin}8\varphi + \frac{2}{3}\operatorname{Sin}6\varphi + \operatorname{Sin}4\varphi - 2\operatorname{Sin}2\varphi - 5\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^7\varphi = -\frac{1}{256}\left(\frac{1}{9}\operatorname{Sin}9\varphi + \frac{5}{7}\operatorname{Sin}7\varphi + \frac{8}{5}\operatorname{Sin}5\varphi - 14\operatorname{Sin}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^8\varphi = -\frac{1}{512}\left(\frac{1}{10}\operatorname{Sin}10\varphi + \frac{3}{4}\operatorname{Sin}8\varphi + \frac{13}{6}\operatorname{Sin}6\varphi + 2\operatorname{Sin}4\varphi - 7\operatorname{Sin}2\varphi - 14\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^9\varphi = -\frac{1}{1024}\left(\frac{1}{11}\operatorname{Sin}11\varphi + \frac{7}{9}\operatorname{Sin}9\varphi + \frac{19}{7}\operatorname{Sin}7\varphi + \frac{21}{5}\operatorname{Sin}5\varphi - 2\operatorname{Sin}3\varphi - 42\operatorname{Sin}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^2\varphi \operatorname{Cos}^{10}\varphi = -\frac{1}{2048}\left(\frac{1}{12}\operatorname{Sin}12\varphi + \frac{4}{5}\operatorname{Sin}10\varphi + \frac{15}{4}\operatorname{Sin}8\varphi + \frac{20}{3}\operatorname{Sin}6\varphi + \frac{15}{4}\operatorname{Sin}4\varphi - 24\operatorname{Sin}2\varphi - 42\varphi\right)$$

$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^n \varphi$ *Taf. VI.*

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}\,\varphi = \tfrac{1}{4} \mathrm{Sin}^4 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^2 \varphi = \left(\tfrac{1}{5} \mathrm{Sin}^4 \varphi - \tfrac{1}{15} \mathrm{Sin}^2 \varphi - \tfrac{2}{15}\right) \mathrm{Cos}\,\varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^3 \varphi = \left(\tfrac{1}{6} \mathrm{Cos}^2 \varphi + \tfrac{1}{12}\right) \mathrm{Sin}^4 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^4 \varphi = \tfrac{1}{7} \mathrm{Sin}^4 \varphi \, \mathrm{Cos}^3 \varphi - \tfrac{3}{7} \int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^2 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}\,\varphi = \tfrac{1}{8}\left(\tfrac{1}{4} \mathrm{Cos}\,4\varphi - \mathrm{Cos}\,2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^2 \varphi = \tfrac{1}{16}\left(\tfrac{1}{5} \mathrm{Cos}\,5\varphi - \tfrac{1}{3} \mathrm{Cos}\,3\varphi - 2\mathrm{Cos}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^3 \varphi = \tfrac{1}{32}\left(\tfrac{1}{6} \mathrm{Cos}\,6\varphi - \tfrac{3}{2} \mathrm{Cos}\,2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^4 \varphi = \tfrac{1}{64}\left(\tfrac{1}{7} \mathrm{Cos}\,7\varphi + \tfrac{1}{5} \mathrm{Cos}\,5\varphi - \mathrm{Cos}\,3\varphi - 3\mathrm{Cos}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^5 \varphi = \tfrac{1}{128}\left(\tfrac{1}{8} \mathrm{Cos}\,8\varphi + \tfrac{1}{3} \mathrm{Cos}\,6\varphi - \tfrac{1}{2} \mathrm{Cos}\,4\varphi - 3\mathrm{Cos}\,2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^6 \varphi = \tfrac{1}{256}\left(\tfrac{1}{9} \mathrm{Cos}\,9\varphi + \tfrac{3}{7} \mathrm{Cos}\,7\varphi - \tfrac{8}{3} \mathrm{Cos}\,3\varphi - 6\mathrm{Cos}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^7 \varphi = \tfrac{1}{512}\left(\tfrac{1}{10} \mathrm{Cos}\,10\varphi + \tfrac{1}{2} \mathrm{Cos}\,8\varphi + \tfrac{1}{2} \mathrm{Cos}\,6\varphi - 2\mathrm{Cos}\,4\varphi - 7\mathrm{Cos}\,2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^8 \varphi = \tfrac{1}{1024}\left(\tfrac{1}{11} \mathrm{Cos}\,11\varphi + \tfrac{5}{9} \mathrm{Cos}\,9\varphi + \mathrm{Cos}\,7\varphi - \mathrm{Cos}\,5\varphi - \tfrac{22}{3} \mathrm{Cos}\,3\varphi - 14\mathrm{Cos}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^9 \varphi = \tfrac{1}{2048}\left(\tfrac{1}{12} \mathrm{Cos}\,12\varphi + \tfrac{3}{5} \mathrm{Cos}\,10\varphi + \tfrac{3}{2} \mathrm{Cos}\,8\varphi + \tfrac{1}{3} \mathrm{Cos}\,6\varphi - \tfrac{27}{4} \mathrm{Cos}\,4\varphi - 18\mathrm{Cos}\,2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^3 \varphi \, \mathrm{Cos}^{10} \varphi = \tfrac{1}{4096}\left(\tfrac{1}{13} \mathrm{Cos}\,13\varphi + \tfrac{7}{11} \mathrm{Cos}\,11\varphi + 2\mathrm{Cos}\,9\varphi + 2\mathrm{Cos}\,7\varphi - 5\mathrm{Cos}\,5\varphi - 21\mathrm{Cos}\,3\varphi - 36\mathrm{Cos}\,\varphi\right)$$

Taf. VII. $\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^n\varphi$

$$\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}\,\varphi = \tfrac{1}{5}\mathrm{Sin}^5\varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^2\varphi = \left(\tfrac{1}{6}\mathrm{Sin}^5\varphi - \tfrac{1}{24}\mathrm{Sin}^3\varphi - \tfrac{1}{16}\mathrm{Sin}\,\varphi\right)\mathrm{Cos}\,\varphi + \tfrac{1}{16}\varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^3\varphi = \left(\tfrac{1}{7}\mathrm{Cos}^2\varphi + \tfrac{2}{35}\right)\mathrm{Sin}^5\varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}\,\varphi = \tfrac{1}{16}\left(\tfrac{1}{5}\mathrm{Sin}\,5\varphi - \mathrm{Sin}\,3\varphi + 2\mathrm{Sin}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^2\varphi = \tfrac{1}{32}\left(\tfrac{1}{6}\mathrm{Sin}\,6\varphi - \tfrac{1}{2}\mathrm{Sin}\,4\varphi - \tfrac{1}{2}\mathrm{Sin}\,2\varphi + 2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^3\varphi = \tfrac{1}{64}\left(\tfrac{1}{7}\mathrm{Sin}\,7\varphi - \tfrac{1}{5}\mathrm{Sin}\,5\varphi - \mathrm{Sin}\,3\varphi + 3\mathrm{Sin}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^4\varphi = \tfrac{1}{128}\left(\tfrac{1}{8}\mathrm{Sin}\,8\varphi - \mathrm{Sin}\,4\varphi + 3\varphi\right)$$

$$\begin{aligned}\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^5\varphi = \tfrac{1}{256}\Big(&\tfrac{1}{9}\mathrm{Sin}\,9\varphi + \tfrac{1}{7}\mathrm{Sin}\,7\varphi - \tfrac{4}{5}\mathrm{Sin}\,5\varphi - \tfrac{4}{3}\mathrm{Sin}\,3\varphi \\ &+ 6\mathrm{Sin}\,\varphi\Big)\end{aligned}$$

$$\begin{aligned}\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^6\varphi = \tfrac{1}{512}\Big(&\tfrac{1}{10}\mathrm{Sin}\,10\varphi + \tfrac{1}{4}\mathrm{Sin}\,8\varphi - \tfrac{1}{2}\mathrm{Sin}\,6\varphi - 2\mathrm{Sin}\,4\varphi \\ &+ \mathrm{Sin}\,2\varphi + 6\varphi\Big)\end{aligned}$$

$$\begin{aligned}\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^7\varphi = \tfrac{1}{1024}\Big(&\tfrac{1}{11}\mathrm{Sin}\,11\varphi + \tfrac{1}{3}\mathrm{Sin}\,9\varphi - \tfrac{1}{7}\mathrm{Sin}\,7\varphi - \tfrac{11}{5}\mathrm{Sin}\,5\varphi \\ &- 2\mathrm{Sin}\,3\varphi + 14\mathrm{Sin}\,\varphi\Big)\end{aligned}$$

$$\begin{aligned}\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^8\varphi = \tfrac{1}{2048}\Big(&\tfrac{1}{12}\mathrm{Sin}\,12\varphi + \tfrac{2}{5}\mathrm{Sin}\,10\varphi + \tfrac{1}{4}\mathrm{Sin}\,8\varphi - 2\mathrm{Sin}\,6\varphi \\ &- \tfrac{17}{4}\mathrm{Sin}\,4\varphi + 4\mathrm{Sin}\,2\varphi + 14\varphi\Big)\end{aligned}$$

$$\begin{aligned}\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^9\varphi = \tfrac{1}{4096}\Big(&\tfrac{1}{13}\mathrm{Sin}\,13\varphi + \tfrac{5}{11}\mathrm{Sin}\,11\varphi + \tfrac{2}{3}\mathrm{Sin}\,9\varphi - \tfrac{10}{7}\mathrm{Sin}\,7\varphi \\ &- \tfrac{29}{5}\mathrm{Sin}\,5\varphi - 3\mathrm{Sin}\,3\varphi + 36\mathrm{Sin}\,\varphi\Big)\end{aligned}$$

$$\begin{aligned}\int \partial\varphi \, \mathrm{Sin}^4\varphi \, \mathrm{Cos}^{10}\varphi = \tfrac{1}{8192}\Big(&\tfrac{1}{14}\mathrm{Sin}\,14\varphi + \tfrac{1}{2}\mathrm{Sin}\,12\varphi + \tfrac{11}{10}\mathrm{Sin}\,10\varphi - \tfrac{1}{2}\mathrm{Sin}\,8\varphi \\ &- \tfrac{13}{2}\mathrm{Sin}\,6\varphi - \tfrac{19}{2}\mathrm{Sin}\,4\varphi + \tfrac{27}{2}\mathrm{Sin}\,2\varphi + 36\varphi\Big)\end{aligned}$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^n \varphi$$ *Taf. VIII.*

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}\, \varphi = \tfrac{1}{6} \mathrm{Sin}^6 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^2 \varphi = \tfrac{1}{7} \mathrm{Sin}^6 \varphi \, \mathrm{Cos}\, \varphi + \tfrac{1}{7} \int \partial\varphi \, \mathrm{Sin}^5 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^3 \varphi = \left(\tfrac{1}{8} \mathrm{Cos}^2 \varphi + \tfrac{1}{24}\right) \mathrm{Sin}^6 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}\, \varphi = -\tfrac{1}{32}\left(\tfrac{1}{6} \mathrm{Cos}\, 6\varphi - \mathrm{Cos}\, 4\varphi + \tfrac{5}{2} \mathrm{Cos}\, 2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^2 \varphi = -\tfrac{1}{64}\left(\tfrac{1}{7} \mathrm{Cos}\, 7\varphi - \tfrac{3}{5} \mathrm{Cos}\, 5\varphi + \tfrac{1}{3} \mathrm{Cos}\, 3\varphi + 5 \mathrm{Cos}\, \varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^3 \varphi = -\tfrac{1}{128}\left(\tfrac{1}{8} \mathrm{Cos}\, 8\varphi - \tfrac{1}{3} \mathrm{Cos}\, 6\varphi - \tfrac{1}{2} \mathrm{Cos}\, 4\varphi + 3 \mathrm{Cos}\, 2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^4 \varphi = -\tfrac{1}{256}\left(\tfrac{1}{9} \mathrm{Cos}\, 9\varphi - \tfrac{1}{7} \mathrm{Cos}\, 7\varphi - \tfrac{4}{5} \mathrm{Cos}\, 5\varphi + \tfrac{4}{3} \mathrm{Cos}\, 3\varphi \right.$$
$$\left. + 6 \mathrm{Cos}\, \varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^5 \varphi = -\tfrac{1}{512}\left(\tfrac{1}{10} \mathrm{Cos}\, 10\varphi - \tfrac{5}{6} \mathrm{Cos}\, 6\varphi + 5 \mathrm{Cos}\, 2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^6 \varphi = -\tfrac{1}{1024}\left(\tfrac{1}{11} \mathrm{Cos}\, 11\varphi + \tfrac{1}{9} \mathrm{Cos}\, 9\varphi - \tfrac{5}{7} \mathrm{Cos}\, 7\varphi - \mathrm{Cos}\, 5\varphi \right.$$
$$\left. + \tfrac{10}{3} \mathrm{Cos}\, 3\varphi + 10 \mathrm{Cos}\, \varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^7 \varphi = -\tfrac{1}{2048}\left(\tfrac{1}{12} \mathrm{Cos}\, 12\varphi + \tfrac{1}{5} \mathrm{Cos}\, 10\varphi - \tfrac{1}{2} \mathrm{Cos}\, 8\varphi - \tfrac{5}{3} \mathrm{Cos}\, 6\varphi \right.$$
$$\left. + \tfrac{5}{4} \mathrm{Cos}\, 4\varphi + 10 \mathrm{Cos}\, 2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^8 \varphi = -\tfrac{1}{4096}\left(\tfrac{1}{13} \mathrm{Cos}\, 13\varphi + \tfrac{3}{11} \mathrm{Cos}\, 11\varphi - \tfrac{2}{9} \mathrm{Cos}\, 9\varphi - 2 \mathrm{Cos}\, 7\varphi \right.$$
$$\left. - \mathrm{Cos}\, 5\varphi + \tfrac{25}{3} \mathrm{Cos}\, 3\varphi + 20 \mathrm{Cos}\, \varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^9 \varphi = -\tfrac{1}{8192}\left(\tfrac{1}{14} \mathrm{Cos}\, 14\varphi + \tfrac{1}{3} \mathrm{Cos}\, 12\varphi + \tfrac{1}{10} \mathrm{Cos}\, 10\varphi - 2 \mathrm{Cos}\, 8\varphi \right.$$
$$\left. - \tfrac{19}{8} \mathrm{Cos}\, 6\varphi + 5 \mathrm{Cos}\, 4\varphi + \tfrac{45}{2} \mathrm{Cos}\, 2\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^5 \varphi \, \mathrm{Cos}^{10} \varphi = -\tfrac{1}{16384}\left(\tfrac{1}{15} \mathrm{Cos}\, 15\varphi + \tfrac{5}{13} \mathrm{Cos}\, 13\varphi + \tfrac{5}{11} \mathrm{Cos}\, 11\varphi \right.$$
$$\left. - \tfrac{5}{3} \mathrm{Cos}\, 9\varphi - 5 \mathrm{Cos}\, 7\varphi + \tfrac{1}{5} \mathrm{Cos}\, 5\varphi + \tfrac{65}{3} \mathrm{Cos}\, 3\varphi + 45 \mathrm{Cos}\, \varphi\right)$$

Taf. IX. $\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^n \varphi$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}\, \varphi = \tfrac{1}{7} \mathrm{Sin}^7 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^2 \varphi = \left(\tfrac{1}{8}\mathrm{Sin}^7\varphi - \tfrac{1}{48}\mathrm{Sin}^5\varphi - \tfrac{5}{192}\mathrm{Sin}^3\varphi - \tfrac{5}{128}\mathrm{Sin}\,\varphi\right)\mathrm{Cos}\,\varphi + \tfrac{5}{128}\varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^3 \varphi = \left(\tfrac{1}{9}\mathrm{Cos}^2\varphi + \tfrac{2}{63}\right)\mathrm{Sin}^7\varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}\, \varphi = -\tfrac{1}{64}\left(\tfrac{1}{7}\mathrm{Sin}\,7\varphi - \mathrm{Sin}\,5\varphi + 3\mathrm{Sin}\,3\varphi - 5\mathrm{Sin}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^2 \varphi = -\tfrac{1}{128}\left(\tfrac{1}{8}\mathrm{Sin}\,8\varphi - \tfrac{2}{3}\mathrm{Sin}\,6\varphi + \mathrm{Sin}\,4\varphi + 2\mathrm{Sin}\,2\varphi - 5\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^3 \varphi = -\tfrac{1}{256}\left(\tfrac{1}{9}\mathrm{Sin}\,9\varphi - \tfrac{3}{7}\mathrm{Sin}\,7\varphi + \tfrac{8}{3}\mathrm{Sin}\,3\varphi - 6\mathrm{Sin}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^4 \varphi = -\tfrac{1}{512}\left(\tfrac{1}{10}\mathrm{Sin}\,10\varphi - \tfrac{1}{4}\mathrm{Sin}\,8\varphi - \tfrac{1}{2}\mathrm{Sin}\,6\varphi + 2\mathrm{Sin}\,4\varphi + \mathrm{Sin}\,2\varphi - 6\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^5 \varphi = -\tfrac{1}{1024}\left(\tfrac{1}{11}\mathrm{Sin}\,11\varphi - \tfrac{1}{9}\mathrm{Sin}\,9\varphi - \tfrac{5}{7}\mathrm{Sin}\,7\varphi + \mathrm{Sin}\,5\varphi + \tfrac{10}{3}\mathrm{Sin}\,3\varphi - 10\mathrm{Sin}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^6 \varphi = -\tfrac{1}{2048}\left(\tfrac{1}{12}\mathrm{Sin}\,12\varphi - \tfrac{3}{4}\mathrm{Sin}\,8\varphi + \tfrac{15}{4}\mathrm{Sin}\,4\varphi - 10\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^7 \varphi = -\tfrac{1}{4096}\left(\tfrac{1}{13}\mathrm{Sin}\,13\varphi + \tfrac{1}{11}\mathrm{Sin}\,11\varphi - \tfrac{2}{3}\mathrm{Sin}\,9\varphi - \tfrac{6}{7}\mathrm{Sin}\,7\varphi + 3\mathrm{Sin}\,5\varphi + 5\mathrm{Sin}\,3\varphi - 20\mathrm{Sin}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^8 \varphi = -\tfrac{1}{8192}\left(\tfrac{1}{14}\mathrm{Sin}\,14\varphi + \tfrac{1}{6}\mathrm{Sin}\,12\varphi - \tfrac{1}{2}\mathrm{Sin}\,10\varphi - \tfrac{3}{2}\mathrm{Sin}\,8\varphi + \tfrac{3}{2}\mathrm{Sin}\,6\varphi + \tfrac{15}{2}\mathrm{Sin}\,4\varphi - \tfrac{5}{2}\mathrm{Sin}\,2\varphi - 20\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^9 \varphi = -\tfrac{1}{16384}\left(\tfrac{1}{15}\mathrm{Sin}\,15\varphi + \tfrac{3}{13}\mathrm{Sin}\,13\varphi - \tfrac{3}{11}\mathrm{Sin}\,11\varphi - \tfrac{17}{9}\mathrm{Sin}\,9\varphi - \tfrac{3}{7}\mathrm{Sin}\,7\varphi + \tfrac{39}{5}\mathrm{Sin}\,5\varphi + \tfrac{25}{3}\mathrm{Sin}\,3\varphi - 45\mathrm{Sin}\,\varphi\right)$$

$$\int \partial\varphi \, \mathrm{Sin}^6 \varphi \, \mathrm{Cos}^{10} \varphi = -\tfrac{1}{32768}\left(\tfrac{1}{16}\mathrm{Sin}\,16\varphi + \tfrac{2}{7}\mathrm{Sin}\,14\varphi - 2\mathrm{Sin}\,10\varphi - \tfrac{5}{2}\mathrm{Sin}\,8\varphi + 6\mathrm{Sin}\,6\varphi + 16\mathrm{Sin}\,4\varphi - 10\mathrm{Sin}\,2\varphi - 45\varphi\right)$$

$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^n\varphi$ *Taf. X.*

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}\varphi = \tfrac{1}{8}\operatorname{Sin}^8\varphi$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^2\varphi = \tfrac{1}{9}\operatorname{Sin}^8\varphi\operatorname{Cos}\varphi + \tfrac{1}{9}\int\partial\varphi\operatorname{Sin}^7\varphi$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^3\varphi = \left(\tfrac{1}{10}\operatorname{Cos}^2\varphi + \tfrac{1}{40}\right)\operatorname{Sin}^8\varphi$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^4\varphi = \left(\tfrac{1}{11}\operatorname{Cos}^3\varphi + \tfrac{1}{33}\operatorname{Cos}\varphi\right)\operatorname{Sin}^8\varphi + \tfrac{1}{33}\int\partial\varphi\operatorname{Sin}^7\varphi$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}\varphi = \tfrac{1}{128}\left(\tfrac{1}{8}\operatorname{Cos}8\varphi - \operatorname{Cos}6\varphi + \tfrac{7}{2}\operatorname{Cos}4\varphi - 7\operatorname{Cos}2\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^2\varphi = \tfrac{1}{256}\left(\tfrac{1}{9}\operatorname{Cos}9\varphi - \tfrac{5}{7}\operatorname{Cos}7\varphi + \tfrac{8}{5}\operatorname{Cos}5\varphi - 14\operatorname{Cos}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^3\varphi = \tfrac{1}{512}\left(\tfrac{1}{10}\operatorname{Cos}10\varphi - \tfrac{1}{2}\operatorname{Cos}8\varphi + \tfrac{1}{2}\operatorname{Cos}6\varphi + 2\operatorname{Cos}4\varphi - 7\operatorname{Cos}2\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^4\varphi = \tfrac{1}{1024}\left(\tfrac{1}{11}\operatorname{Cos}11\varphi - \tfrac{1}{3}\operatorname{Cos}9\varphi - \tfrac{1}{7}\operatorname{Cos}7\varphi + \tfrac{11}{5}\operatorname{Cos}5\varphi - 2\operatorname{Cos}3\varphi - 14\operatorname{Cos}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^5\varphi = \tfrac{1}{2048}\left(\tfrac{1}{12}\operatorname{Cos}12\varphi - \tfrac{1}{5}\operatorname{Cos}10\varphi - \tfrac{1}{2}\operatorname{Cos}8\varphi + \tfrac{5}{3}\operatorname{Cos}6\varphi + \tfrac{5}{4}\operatorname{Cos}4\varphi - 10\operatorname{Cos}2\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^6\varphi = \tfrac{1}{4096}\left(\tfrac{1}{13}\operatorname{Cos}13\varphi - \tfrac{1}{11}\operatorname{Cos}11\varphi - \tfrac{2}{3}\operatorname{Cos}9\varphi + \tfrac{6}{7}\operatorname{Cos}7\varphi + 3\operatorname{Cos}5\varphi - 5\operatorname{Cos}3\varphi - 20\operatorname{Cos}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^7\varphi = \tfrac{1}{8192}\left(\tfrac{1}{14}\operatorname{Cos}14\varphi - \tfrac{7}{10}\operatorname{Cos}10\varphi + \tfrac{7}{2}\operatorname{Cos}6\varphi - \tfrac{35}{2}\operatorname{Cos}2\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^8\varphi = \tfrac{1}{16384}\left(\tfrac{1}{15}\operatorname{Cos}15\varphi + \tfrac{1}{13}\operatorname{Cos}13\varphi - \tfrac{7}{11}\operatorname{Cos}11\varphi - \tfrac{7}{9}\operatorname{Cos}9\varphi + 3\operatorname{Cos}7\varphi + \tfrac{21}{5}\operatorname{Cos}5\varphi - \tfrac{35}{3}\operatorname{Cos}3\varphi - 35\operatorname{Cos}\varphi\right)$$

$$\int \partial\varphi \operatorname{Sin}^7\varphi \operatorname{Cos}^9\varphi = \tfrac{1}{32768}\left(\tfrac{1}{16}\operatorname{Cos}16\varphi + \tfrac{1}{7}\operatorname{Cos}14\varphi - \tfrac{1}{2}\operatorname{Cos}12\varphi - \tfrac{7}{5}\operatorname{Cos}10\varphi + \tfrac{7}{4}\operatorname{Cos}8\varphi + 7\operatorname{Cos}6\varphi - \tfrac{7}{2}\operatorname{Cos}4\varphi - 35\operatorname{Cos}2\varphi\right)$$

Taf. XI. $\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^n \varphi$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}\, \varphi = \frac{1}{9} \mathrm{Sin}^9 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^2 \varphi = \frac{1}{10} \mathrm{Sin}^9 \varphi \, \mathrm{Cos}\,\varphi + \frac{1}{10} \int \partial\varphi \, \mathrm{Sin}^8 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^3 \varphi = \left(\frac{1}{11} \mathrm{Cos}^2 \varphi + \frac{2}{99}\right) \mathrm{Sin}^9 \varphi$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}\,\varphi = \frac{1}{256}\Big(\frac{1}{9}\mathrm{Sin}\, 9\varphi - \mathrm{Sin}\, 7\varphi + 4\mathrm{Sin}\, 5\varphi - \frac{28}{3}\mathrm{Sin}\, 3\varphi + 14\mathrm{Sin}\,\varphi\Big)$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^2 \varphi = \frac{1}{512}\Big(\frac{1}{10}\mathrm{Sin}\, 10\varphi - \frac{5}{4}\mathrm{Sin}\, 8\varphi + \frac{13}{6}\mathrm{Sin}\, 6\varphi - 2\mathrm{Sin}\, 4\varphi - 7\mathrm{Sin}\, 2\varphi + 14\varphi\Big)$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^3 \varphi = \frac{1}{1024}\Big(\frac{1}{11}\mathrm{Sin}\, 11\varphi - \frac{5}{9}\mathrm{Sin}\, 9\varphi + \mathrm{Sin}\, 7\varphi + \mathrm{Sin}\, 5\varphi - \frac{22}{3}\mathrm{Sin}\, 3\varphi + 14\mathrm{Sin}\,\varphi\Big)$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^4 \varphi = \frac{1}{2048}\Big(\frac{1}{12}\mathrm{Sin}\, 12\varphi - \frac{2}{5}\mathrm{Sin}\, 10\varphi + \frac{1}{4}\mathrm{Sin}\, 8\varphi + 2\mathrm{Sin}\, 6\varphi - \frac{17}{4}\mathrm{Sin}\, 4\varphi - 4\mathrm{Sin}\, 2\varphi + 14\varphi\Big)$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^5 \varphi = \frac{1}{4096}\Big(\frac{1}{13}\mathrm{Sin}\, 13\varphi - \frac{3}{11}\mathrm{Sin}\, 11\varphi - \frac{2}{9}\mathrm{Sin}\, 9\varphi + 2\mathrm{Sin}\, 7\varphi - \mathrm{Sin}\, 5\varphi - \frac{25}{3}\mathrm{Sin}\, 3\varphi + 20\mathrm{Sin}\,\varphi\Big)$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^6 \varphi = \frac{1}{8192}\Big(\frac{1}{14}\mathrm{Sin}\, 14\varphi - \frac{1}{6}\mathrm{Sin}\, 12\varphi - \frac{1}{2}\mathrm{Sin}\, 10\varphi + \frac{3}{2}\mathrm{Sin}\, 8\varphi + \frac{3}{2}\mathrm{Sin}\, 6\varphi + \frac{15}{2}\mathrm{Sin}\, 4\varphi - \frac{5}{2}\mathrm{Sin}\, 2\varphi + 20\varphi\Big)$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^7 \varphi = \frac{1}{16384}\Big(\frac{1}{15}\mathrm{Sin}\, 15\varphi - \frac{1}{13}\mathrm{Sin}\, 13\varphi - \frac{7}{11}\mathrm{Sin}\, 11\varphi + \frac{7}{9}\mathrm{Sin}\, 9\varphi + 3\mathrm{Sin}\, 7\varphi - \frac{21}{5}\mathrm{Sin}\, 5\varphi - \frac{35}{3}\mathrm{Sin}\, 3\varphi + 35\mathrm{Sin}\,\varphi\Big)$$

$$\int \partial\varphi \, \mathrm{Sin}^8 \varphi \, \mathrm{Cos}^8 \varphi = \frac{1}{32768}\Big(\frac{1}{16}\mathrm{Sin}\, 16\varphi - \frac{2}{3}\mathrm{Sin}\, 12\varphi + \frac{7}{2}\mathrm{Sin}\, 8\varphi - 14\mathrm{Sin}\, 4\varphi + 35\varphi\Big)$$

$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^n\varphi$ *Taf. XII.*

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}\varphi = \tfrac{1}{10}\operatorname{Sin}^{10}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^2\varphi = \tfrac{1}{11}\operatorname{Sin}^{10}\varphi \operatorname{Cos}\varphi + \tfrac{1}{11}\int \partial\varphi \operatorname{Sin}^9\varphi$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^3\varphi = \left(\tfrac{1}{12}\operatorname{Cos}^2\varphi + \tfrac{1}{60}\right)\operatorname{Sin}^{10}\varphi$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}\varphi = -\tfrac{1}{512}\Big(\tfrac{1}{10}\operatorname{Cos}10\varphi - \operatorname{Cos}8\varphi + \tfrac{9}{2}\operatorname{Cos}6\varphi - 12\operatorname{Cos}4\varphi + 21\operatorname{Cos}2\varphi\Big)$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^2\varphi = -\tfrac{1}{1024}\Big(\tfrac{1}{11}\operatorname{Cos}11\varphi - \tfrac{7}{9}\operatorname{Cos}9\varphi + \tfrac{19}{7}\operatorname{Cos}7\varphi - \tfrac{21}{5}\operatorname{Cos}5\varphi - 2\operatorname{Cos}3\varphi + 42\operatorname{Cos}\varphi\Big)$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^3\varphi = -\tfrac{1}{2048}\Big(\tfrac{1}{12}\operatorname{Cos}12\varphi - \tfrac{3}{5}\operatorname{Cos}10\varphi + \tfrac{3}{2}\operatorname{Cos}8\varphi - \tfrac{1}{3}\operatorname{Cos}6\varphi - \tfrac{27}{4}\operatorname{Cos}4\varphi + 18\operatorname{Cos}2\varphi\Big)$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^4\varphi = -\tfrac{1}{4096}\Big(\tfrac{1}{13}\operatorname{Cos}13\varphi - \tfrac{5}{11}\operatorname{Cos}11\varphi + \tfrac{2}{3}\operatorname{Cos}9\varphi + \tfrac{10}{7}\operatorname{Cos}7\varphi - \tfrac{29}{5}\operatorname{Cos}5\varphi + 3\operatorname{Cos}3\varphi + 36\operatorname{Cos}\varphi\Big)$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^5\varphi = -\tfrac{1}{8192}\Big(\tfrac{1}{14}\operatorname{Cos}14\varphi - \tfrac{1}{3}\operatorname{Cos}12\varphi + \tfrac{1}{10}\operatorname{Cos}10\varphi + 2\operatorname{Cos}8\varphi - \tfrac{19}{6}\operatorname{Cos}6\varphi - 5\operatorname{Cos}4\varphi + \tfrac{45}{2}\operatorname{Cos}2\varphi\Big)$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^6\varphi = -\tfrac{1}{16384}\Big(\tfrac{1}{15}\operatorname{Cos}15\varphi - \tfrac{3}{13}\operatorname{Cos}13\varphi - \tfrac{3}{11}\operatorname{Cos}11\varphi + \tfrac{17}{9}\operatorname{Cos}9\varphi - \tfrac{3}{7}\operatorname{Cos}7\varphi - \tfrac{39}{5}\operatorname{Cos}5\varphi + \tfrac{25}{3}\operatorname{Cos}3\varphi + 45\operatorname{Cos}\varphi\Big)$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^7\varphi = -\tfrac{1}{32768}\Big(\tfrac{1}{16}\operatorname{Cos}16\varphi - \tfrac{1}{7}\operatorname{Cos}14\varphi - \tfrac{1}{2}\operatorname{Cos}12\varphi + \tfrac{7}{5}\operatorname{Cos}10\varphi + \tfrac{7}{4}\operatorname{Cos}8\varphi - 7\operatorname{Cos}6\varphi - \tfrac{7}{2}\operatorname{Cos}4\varphi + 35\operatorname{Cos}2\varphi\Big)$$

$$\int \partial\varphi \operatorname{Sin}^9\varphi \operatorname{Cos}^8\varphi = -\tfrac{1}{65536}\Big(\tfrac{1}{17}\operatorname{Cos}17\varphi - \tfrac{1}{15}\operatorname{Cos}15\varphi - \tfrac{8}{13}\operatorname{Cos}13\varphi + \tfrac{8}{11}\operatorname{Cos}11\varphi + \tfrac{28}{9}\operatorname{Cos}9\varphi - 4\operatorname{Cos}7\varphi - \tfrac{56}{5}\operatorname{Cos}5\varphi + \tfrac{56}{3}\operatorname{Cos}3\varphi + 70\operatorname{Cos}\varphi\Big)$$

Taf. XIII.

$$\int\frac{\partial\varphi}{\mathrm{Sin}^n\varphi},\quad \int\frac{\partial\varphi}{\mathrm{Cos}^n\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi} = \log\mathrm{Tang}\,\frac{\varphi}{2}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^2\varphi} = -\frac{\mathrm{Cos}\,\varphi}{\mathrm{Sin}\,\varphi} = -\,\mathrm{Cot}\,\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^3\varphi} = -\frac{\mathrm{Cos}\,\varphi}{2\mathrm{Sin}^2\varphi} + \tfrac{1}{2}\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^4\varphi} = \left(-\frac{1}{3\mathrm{Sin}^3\varphi} - \frac{2}{3\mathrm{Sin}\,\varphi}\right)\mathrm{Cos}\,\varphi = \mathrm{Cot}\,\varphi - \tfrac{1}{3}\mathrm{Cot}^3\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi} = \left(-\frac{1}{4\mathrm{Sin}^4\varphi} - \frac{3}{8\mathrm{Sin}^2\varphi}\right)\mathrm{Cos}\,\varphi + \tfrac{3}{8}\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi} = \left(-\frac{1}{5\mathrm{Sin}^5\varphi} - \frac{4}{15\mathrm{Sin}^3\varphi} - \frac{8}{15\mathrm{Sin}\,\varphi}\right)\mathrm{Cos}\,\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^7\varphi} = \left(-\frac{1}{6\mathrm{Sin}^6\varphi} - \frac{5}{24\mathrm{Sin}^4\varphi} - \frac{5}{16\mathrm{Sin}^2\varphi}\right)\mathrm{Cos}\,\varphi + \tfrac{5}{16}\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^8\varphi} = \left(-\frac{1}{7\mathrm{Sin}^7\varphi} - \frac{6}{35\mathrm{Sin}^5\varphi} - \frac{8}{35\mathrm{Sin}^3\varphi} - \frac{16}{35\mathrm{Sin}\,\varphi}\right)\mathrm{Cos}\,\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi} = \log\mathrm{Tang}\left(45^\circ + \frac{\varphi}{2}\right)$$

$$\int\frac{\partial\varphi}{\mathrm{Cos}^2\varphi} = \frac{\mathrm{Sin}\,\varphi}{\mathrm{Cos}\,\varphi} = \mathrm{Tang}\,\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Cos}^3\varphi} = \frac{\mathrm{Sin}\,\varphi}{2\mathrm{Cos}^2\varphi} + \tfrac{1}{2}\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Cos}^4\varphi} = \left(\frac{1}{3\mathrm{Cos}^3\varphi} + \frac{2}{3\mathrm{Cos}\,\varphi}\right)\mathrm{Sin}\,\varphi = \mathrm{Tang}\,\varphi + \tfrac{1}{3}\mathrm{Tang}^3\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Cos}^5\varphi} = \left(\frac{1}{4\mathrm{Cos}^4\varphi} + \frac{3}{8\mathrm{Cos}^2\varphi}\right)\mathrm{Sin}\,\varphi + \tfrac{3}{8}\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Cos}^6\varphi} = \left(\frac{1}{5\mathrm{Cos}^5\varphi} + \frac{4}{15\mathrm{Cos}^3\varphi} + \frac{8}{15\mathrm{Cos}\,\varphi}\right)\mathrm{Sin}\,\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Cos}^7\varphi} = \left(\frac{1}{6\mathrm{Cos}^6\varphi} + \frac{5}{24\mathrm{Cos}^4\varphi} + \frac{5}{16\mathrm{Cos}^2\varphi}\right)\mathrm{Sin}\,\varphi + \tfrac{5}{16}\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Cos}^8\varphi} = \left(\frac{1}{7\mathrm{Cos}^7\varphi} + \frac{6}{35\mathrm{Cos}^5\varphi} + \frac{8}{35\mathrm{Cos}^3\varphi} + \frac{16}{35\mathrm{Cos}\,\varphi}\right)\mathrm{Sin}\,\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^n\varphi}{\mathrm{Cos}\,\varphi},\quad \int\frac{\partial\varphi\,\mathrm{Cos}^n\varphi}{\mathrm{Sin}\,\varphi}$$ *Taf. XIV.*

$$\int\frac{\partial\varphi\,\mathrm{Sin}\,\varphi}{\mathrm{Cos}\,\varphi}=-\log\mathrm{Cos}\,\varphi=\log\mathrm{Sec}\,\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^2\varphi}{\mathrm{Cos}\,\varphi}=-\mathrm{Sin}\,\varphi+\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^3\varphi}{\mathrm{Cos}\,\varphi}=-\frac{\mathrm{Sin}^2\varphi}{2}+\int\frac{\partial\varphi\,\mathrm{Sin}\,\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^4\varphi}{\mathrm{Cos}\,\varphi}=-\frac{\mathrm{Sin}^3\varphi}{3}-\mathrm{Sin}\,\varphi+\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^5\varphi}{\mathrm{Cos}\,\varphi}=-\frac{\mathrm{Sin}^4\varphi}{4}-\frac{\mathrm{Sin}^2\varphi}{2}+\int\frac{\partial\varphi\,\mathrm{Sin}\,\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^6\varphi}{\mathrm{Cos}\,\varphi}=-\frac{\mathrm{Sin}^5\varphi}{5}-\frac{\mathrm{Sin}^3\varphi}{3}-\mathrm{Sin}\,\varphi+\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^7\varphi}{\mathrm{Cos}\,\varphi}=-\frac{\mathrm{Sin}^6\varphi}{6}-\frac{\mathrm{Sin}^4\varphi}{4}-\frac{\mathrm{Sin}^2\varphi}{2}+\int\frac{\partial\varphi\,\mathrm{Sin}\,\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^8\varphi}{\mathrm{Cos}\,\varphi}=-\frac{\mathrm{Sin}^7\varphi}{7}-\frac{\mathrm{Sin}^5\varphi}{5}-\frac{\mathrm{Sin}^3\varphi}{3}-\mathrm{Sin}\,\varphi+\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}\,\varphi}{\mathrm{Sin}\,\varphi}=\log\mathrm{Sin}\,\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^2\varphi}{\mathrm{Sin}\,\varphi}=\mathrm{Cos}\,\varphi+\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^3\varphi}{\mathrm{Sin}\,\varphi}=\frac{\mathrm{Cos}^2\varphi}{2}+\int\frac{\partial\varphi\,\mathrm{Cos}\,\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^4\varphi}{\mathrm{Sin}\,\varphi}=\frac{\mathrm{Cos}^3\varphi}{3}+\mathrm{Cos}\,\varphi+\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^5\varphi}{\mathrm{Sin}\,\varphi}=\frac{\mathrm{Cos}^4\varphi}{4}+\frac{\mathrm{Cos}^2\varphi}{2}+\int\frac{\partial\varphi\,\mathrm{Cos}\,\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^6\varphi}{\mathrm{Sin}\,\varphi}=\frac{\mathrm{Cos}^5\varphi}{5}+\frac{\mathrm{Cos}^3\varphi}{3}+\mathrm{Cos}\,\varphi+\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^7\varphi}{\mathrm{Sin}\,\varphi}=\frac{\mathrm{Cos}^6\varphi}{6}+\frac{\mathrm{Cos}^4\varphi}{4}+\frac{\mathrm{Cos}^2\varphi}{2}+\int\frac{\partial\varphi\,\mathrm{Cos}\,\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^8\varphi}{\mathrm{Sin}\,\varphi}=\frac{\mathrm{Cos}^7\varphi}{7}+\frac{\mathrm{Cos}^5\varphi}{5}+\frac{\mathrm{Cos}^3\varphi}{3}+\mathrm{Cos}\,\varphi+\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

Taf. XV. $\int \frac{\partial\varphi \operatorname{Sin}^n \varphi}{\operatorname{Cos}^2 \varphi}, \int \frac{\partial\varphi \operatorname{Cos}^n \varphi}{\operatorname{Sin}^2 \varphi}$

$$\int \frac{\partial\varphi \operatorname{Sin}\varphi}{\operatorname{Cos}^2\varphi} = \frac{1}{\operatorname{Cos}\varphi} = \operatorname{Sec}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^2\varphi}{\operatorname{Cos}^2\varphi} = \frac{\operatorname{Sin}\varphi}{\operatorname{Cos}\varphi} - \varphi = \operatorname{Tang}\varphi - \varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^3\varphi}{\operatorname{Cos}^2\varphi} = \left(-\operatorname{Sin}^2\varphi + 2\right)\frac{1}{\operatorname{Cos}\varphi} = \operatorname{Cos}\varphi + \operatorname{Sec}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^4\varphi}{\operatorname{Cos}^2\varphi} = \left(-\frac{1}{2}\operatorname{Sin}^3\varphi + \frac{3}{2}\operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}\varphi} - \frac{3}{2}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^5\varphi}{\operatorname{Cos}^2\varphi} = \left(-\frac{1}{3}\operatorname{Sin}^4\varphi - \frac{4}{3}\operatorname{Sin}^2\varphi + \frac{8}{3}\right)\frac{1}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^6\varphi}{\operatorname{Cos}^2\varphi} = \left(-\frac{1}{4}\operatorname{Sin}^5\varphi - \frac{5}{8}\operatorname{Sin}^3\varphi + \frac{15}{8}\operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}\varphi} - \frac{15}{8}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^7\varphi}{\operatorname{Cos}^2\varphi} = \left(-\frac{1}{5}\operatorname{Sin}^6\varphi - \frac{2}{5}\operatorname{Sin}^4\varphi - \frac{8}{5}\operatorname{Sin}^2\varphi + \frac{16}{5}\right)\frac{1}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^8\varphi}{\operatorname{Cos}^2\varphi} = \left(-\frac{1}{6}\operatorname{Sin}^7\varphi - \frac{7}{24}\operatorname{Sin}^5\varphi - \frac{35}{48}\operatorname{Sin}^3\varphi + \frac{35}{16}\operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}\varphi} - \frac{35}{16}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}\varphi}{\operatorname{Sin}^2\varphi} = -\frac{1}{\operatorname{Sin}\varphi} = -\operatorname{Cosec}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^2\varphi}{\operatorname{Sin}^2\varphi} = -\frac{\operatorname{Cos}\varphi}{\operatorname{Sin}\varphi} - \varphi = -\operatorname{Cot}\varphi - \varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^3\varphi}{\operatorname{Sin}^2\varphi} = \left(\operatorname{Cos}^2\varphi - 2\right)\frac{1}{\operatorname{Sin}\varphi} = -\operatorname{Sin}\varphi - \operatorname{Cosec}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^4\varphi}{\operatorname{Sin}^2\varphi} = \left(\frac{1}{2}\operatorname{Cos}^3\varphi - \frac{3}{2}\operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}\varphi} - \frac{3}{2}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^5\varphi}{\operatorname{Sin}^2\varphi} = \left(\frac{1}{3}\operatorname{Cos}^4\varphi + \frac{4}{3}\operatorname{Cos}^2\varphi - \frac{8}{3}\right)\frac{1}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^6\varphi}{\operatorname{Sin}^2\varphi} = \left(\frac{1}{4}\operatorname{Cos}^5\varphi + \frac{5}{8}\operatorname{Cos}^3\varphi - \frac{15}{8}\operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}\varphi} - \frac{15}{8}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^7\varphi}{\operatorname{Sin}^2\varphi} = \left(\frac{1}{5}\operatorname{Cos}^6\varphi + \frac{2}{5}\operatorname{Cos}^4\varphi + \frac{8}{5}\operatorname{Cos}^2\varphi - \frac{16}{5}\right)\frac{1}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^8\varphi}{\operatorname{Sin}^2\varphi} = \left(\frac{1}{6}\operatorname{Cos}^7\varphi + \frac{7}{24}\operatorname{Cos}^5\varphi + \frac{35}{48}\operatorname{Cos}^3\varphi - \frac{35}{16}\operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}\varphi} - \frac{35}{16}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^n\varphi}{\operatorname{Cos}^3\varphi},\quad \int \frac{\partial\varphi \operatorname{Cos}^n\varphi}{\operatorname{Sin}^3\varphi}$$ *Taf. XVI.*

$$\int \frac{\partial\varphi \operatorname{Sin}\varphi}{\operatorname{Cos}^3\varphi} = \frac{1}{2\operatorname{Cos}^2\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^2\varphi}{\operatorname{Cos}^3\varphi} = \frac{\operatorname{Sin}\varphi}{2\operatorname{Cos}^2\varphi} - \frac{1}{2}\int \frac{\partial\varphi}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^3\varphi}{\operatorname{Cos}^3\varphi} = \frac{1}{2\operatorname{Cos}^2\varphi} + \log\operatorname{Cos}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^4\varphi}{\operatorname{Cos}^3\varphi} = \left(-\operatorname{Sin}^3\varphi + \frac{3}{2}\operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}^2\varphi} - \frac{3}{2}\int \frac{\partial\varphi}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^5\varphi}{\operatorname{Cos}^3\varphi} = \left(-\frac{1}{2}\operatorname{Sin}^4\varphi + 1\right)\frac{1}{\operatorname{Cos}^2\varphi} + 2\log\operatorname{Cos}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^6\varphi}{\operatorname{Cos}^3\varphi} = \left(-\frac{1}{3}\operatorname{Sin}^5\varphi - \frac{5}{3}\operatorname{Sin}^3\varphi + \frac{5}{2}\operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}^2\varphi} - \frac{5}{2}\int \frac{\partial\varphi}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^7\varphi}{\operatorname{Cos}^3\varphi} = \left(-\frac{1}{4}\operatorname{Sin}^6\varphi - \frac{3}{4}\operatorname{Sin}^4\varphi + \frac{3}{2}\right)\frac{1}{\operatorname{Cos}^2\varphi} + 3\log\operatorname{Cos}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^8\varphi}{\operatorname{Cos}^3\varphi} = \left(-\frac{1}{5}\operatorname{Sin}^7\varphi - \frac{7}{15}\operatorname{Sin}^5\varphi - \frac{7}{3}\operatorname{Sin}^3\varphi + \frac{7}{2}\operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}^2\varphi} - \frac{7}{2}\int \frac{\partial\varphi}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}\varphi}{\operatorname{Sin}^3\varphi} = -\frac{1}{2\operatorname{Sin}^2\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^2\varphi}{\operatorname{Sin}^3\varphi} = -\frac{\operatorname{Cos}\varphi}{2\operatorname{Sin}^2\varphi} - \frac{1}{2}\int \frac{\partial\varphi}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^3\varphi}{\operatorname{Sin}^3\varphi} = -\frac{1}{2\operatorname{Sin}^2\varphi} - \log\operatorname{Sin}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^4\varphi}{\operatorname{Sin}^3\varphi} = \left(\operatorname{Cos}^3\varphi - \frac{3}{2}\operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}^2\varphi} - \frac{3}{2}\int \frac{\partial\varphi}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^5\varphi}{\operatorname{Sin}^3\varphi} = \left(\frac{1}{2}\operatorname{Cos}^4\varphi - 1\right)\frac{1}{\operatorname{Sin}^2\varphi} - 2\log\operatorname{Sin}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^6\varphi}{\operatorname{Sin}^3\varphi} = \left(\frac{1}{3}\operatorname{Cos}^5\varphi + \frac{5}{3}\operatorname{Cos}^3\varphi - \frac{5}{2}\operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}^2\varphi} - \frac{5}{2}\int \frac{\partial\varphi}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^7\varphi}{\operatorname{Sin}^3\varphi} = \left(\frac{1}{4}\operatorname{Cos}^6\varphi + \frac{3}{4}\operatorname{Cos}^4\varphi - \frac{3}{2}\right)\frac{1}{\operatorname{Sin}^2\varphi} - 3\log\operatorname{Sin}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^8\varphi}{\operatorname{Sin}^3\varphi} = \left(\frac{1}{5}\operatorname{Cos}^7\varphi + \frac{7}{15}\operatorname{Cos}^5\varphi + \frac{7}{3}\operatorname{Cos}^3\varphi - \frac{7}{2}\operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}^2\varphi} - \frac{7}{2}\int \frac{\partial\varphi}{\operatorname{Sin}\varphi}$$

Taf. XVII. $\int \frac{\partial\varphi \operatorname{Sin}^n \varphi}{\operatorname{Cos}^4 \varphi}, \int \frac{\partial\varphi \operatorname{Cos}^n \varphi}{\operatorname{Sin}^4 \varphi}$

$$\int \frac{\partial\varphi \operatorname{Sin}\varphi}{\operatorname{Cos}^4\varphi} = \frac{1}{3\operatorname{Cos}^3\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^2\varphi}{\operatorname{Cos}^4\varphi} = \frac{\operatorname{Sin}^3\varphi}{3\operatorname{Cos}^3\varphi} = \tfrac{1}{3}\operatorname{Tang}^3\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^3\varphi}{\operatorname{Cos}^4\varphi} = \left(\operatorname{Sin}^2\varphi - \tfrac{2}{3}\right)\frac{1}{\operatorname{Cos}^3\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^4\varphi}{\operatorname{Cos}^4\varphi} = \left(\tfrac{4}{3}\operatorname{Sin}^3\varphi - \operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}^3\varphi} + \varphi = \tfrac{1}{3}\operatorname{Tang}^3\varphi - \operatorname{Tang}\varphi + \varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^5\varphi}{\operatorname{Cos}^4\varphi} = \left(-\operatorname{Sin}^4\varphi + 4\operatorname{Sin}^2\varphi - \tfrac{8}{3}\right)\frac{1}{\operatorname{Cos}^3\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^6\varphi}{\operatorname{Cos}^4\varphi} = \left(-\tfrac{1}{2}\operatorname{Sin}^5\varphi + \tfrac{10}{3}\operatorname{Sin}^3\varphi - \tfrac{5}{2}\operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}^3\varphi} + \tfrac{5}{2}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^7\varphi}{\operatorname{Cos}^4\varphi} = \left(-\tfrac{1}{3}\operatorname{Sin}^6\varphi - 2\operatorname{Sin}^4\varphi + 8\operatorname{Sin}^2\varphi - \tfrac{16}{3}\right)\frac{1}{\operatorname{Cos}^3\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^8\varphi}{\operatorname{Cos}^4\varphi} = \left(-\tfrac{1}{4}\operatorname{Sin}^7\varphi - \tfrac{7}{8}\operatorname{Sin}^5\varphi + \tfrac{35}{6}\operatorname{Sin}^3\varphi - \tfrac{35}{8}\operatorname{Sin}\varphi\right)\frac{1}{\operatorname{Cos}^3\varphi} + \tfrac{35}{8}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}\varphi}{\operatorname{Sin}^4\varphi} = -\frac{1}{3\operatorname{Sin}^3\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^2\varphi}{\operatorname{Sin}^4\varphi} = -\frac{\operatorname{Cos}^3\varphi}{3\operatorname{Sin}^3\varphi} = -\tfrac{1}{3}\operatorname{Cot}^3\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^3\varphi}{\operatorname{Sin}^4\varphi} = \left(-\operatorname{Cos}^2\varphi + \tfrac{2}{3}\right)\frac{1}{\operatorname{Sin}^3\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^4\varphi}{\operatorname{Sin}^4\varphi} = \left(-\tfrac{4}{3}\operatorname{Cos}^3\varphi + \operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}^3\varphi} + \varphi = -\tfrac{1}{3}\operatorname{Cot}^3\varphi + \operatorname{Cot}\varphi + \varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^5\varphi}{\operatorname{Sin}^4\varphi} = \left(\operatorname{Cos}^4\varphi - 4\operatorname{Cos}^2\varphi + \tfrac{8}{3}\right)\frac{1}{\operatorname{Sin}^3\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^6\varphi}{\operatorname{Sin}^4\varphi} = \left(\tfrac{1}{2}\operatorname{Cos}^5\varphi - \tfrac{10}{3}\operatorname{Cos}^3\varphi + \tfrac{5}{2}\operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}^3\varphi} + \tfrac{5}{2}\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^7\varphi}{\operatorname{Sin}^4\varphi} = \left(\tfrac{1}{3}\operatorname{Cos}^6\varphi + 2\operatorname{Cos}^4\varphi - 8\operatorname{Cos}^2\varphi + \tfrac{16}{3}\right)\frac{1}{\operatorname{Sin}^3\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^8\varphi}{\operatorname{Sin}^4\varphi} = \left(\tfrac{1}{4}\operatorname{Cos}^7\varphi + \tfrac{7}{8}\operatorname{Cos}^5\varphi - \tfrac{35}{6}\operatorname{Cos}^3\varphi + \tfrac{35}{8}\operatorname{Cos}\varphi\right)\frac{1}{\operatorname{Sin}^3\varphi} + \tfrac{35}{8}\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^n\varphi}{\mathrm{Cos}^5\varphi},\quad\int\frac{\partial\varphi\,\mathrm{Cos}^n\varphi}{\mathrm{Sin}^5\varphi}$$ *Taf. XVIII.*

$$\int\frac{\partial\varphi\,\mathrm{Sin}\,\varphi}{\mathrm{Cos}^5\varphi}=\frac{1}{4\mathrm{Cos}^4\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^2\varphi}{\mathrm{Cos}^5\varphi}=\left(\tfrac{1}{8}\mathrm{Sin}^3\varphi+\tfrac{1}{8}\mathrm{Sin}\,\varphi\right)\frac{1}{\mathrm{Cos}^4\varphi}-\tfrac{1}{8}\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^3\varphi}{\mathrm{Cos}^5\varphi}=\frac{\mathrm{Sin}^4\varphi}{4\mathrm{Cos}^4\varphi}=\tfrac{1}{4}\mathrm{Tang}^4\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^4\varphi}{\mathrm{Cos}^5\varphi}=\left(\tfrac{5}{8}\mathrm{Sin}^3\varphi-\tfrac{3}{8}\mathrm{Sin}\,\varphi\right)\frac{1}{\mathrm{Cos}^4\varphi}+\tfrac{3}{8}\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^5\varphi}{\mathrm{Cos}^5\varphi}=\left(\tfrac{3}{4}\mathrm{Sin}^4\varphi-\tfrac{1}{2}\mathrm{Sin}^2\varphi\right)\frac{1}{\mathrm{Cos}^4\varphi}-\log\mathrm{Cos}\,\varphi$$

$$=\tfrac{1}{4}\mathrm{Tang}^4\varphi-\tfrac{1}{2}\mathrm{Tang}^2\varphi-\log\mathrm{Cos}\,\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^6\varphi}{\mathrm{Cos}^5\varphi}=\left(-\mathrm{Sin}^5\varphi+\tfrac{25}{8}\mathrm{Sin}^3\varphi-\tfrac{15}{8}\mathrm{Sin}\,\varphi\right)\frac{1}{\mathrm{Cos}^4\varphi}+\tfrac{15}{8}\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Sin}^7\varphi}{\mathrm{Cos}^5\varphi}=\left(-\tfrac{1}{2}\mathrm{Sin}^6\varphi+\tfrac{9}{4}\mathrm{Sin}^4\varphi-\tfrac{3}{2}\mathrm{Sin}^2\varphi\right)\frac{1}{\mathrm{Cos}^4\varphi}-3\log\mathrm{Cos}\,\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}\,\varphi}{\mathrm{Sin}^5\varphi}=-\frac{1}{4\mathrm{Sin}^4\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^2\varphi}{\mathrm{Sin}^5\varphi}=\left(-\tfrac{1}{8}\mathrm{Cos}^3\varphi-\tfrac{1}{8}\mathrm{Cos}\,\varphi\right)\frac{1}{\mathrm{Sin}^4\varphi}-\tfrac{1}{8}\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^3\varphi}{\mathrm{Sin}^5\varphi}=-\frac{\mathrm{Cos}^4\varphi}{4\mathrm{Sin}^4\varphi}=-\tfrac{1}{4}\mathrm{Cot}^4\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^4\varphi}{\mathrm{Sin}^5\varphi}=\left(-\tfrac{5}{8}\mathrm{Cos}^3\varphi+\tfrac{3}{8}\mathrm{Cos}\,\varphi\right)\frac{1}{\mathrm{Sin}^4\varphi}+\tfrac{3}{8}\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^5\varphi}{\mathrm{Sin}^5\varphi}=\left(-\tfrac{3}{4}\mathrm{Cos}^4\varphi+\tfrac{1}{2}\mathrm{Cos}^2\varphi\right)\frac{1}{\mathrm{Sin}^4\varphi}+\log\mathrm{Sin}\,\varphi$$

$$=-\tfrac{1}{4}\mathrm{Cot}^4\varphi+\tfrac{1}{2}\mathrm{Cot}^2\varphi+\log\mathrm{Sin}\,\varphi$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^6\varphi}{\mathrm{Sin}^5\varphi}=\left(\mathrm{Cos}^5\varphi-\tfrac{25}{8}\mathrm{Cos}^3\varphi+\tfrac{15}{8}\mathrm{Cos}\,\varphi\right)\frac{1}{\mathrm{Sin}^4\varphi}+\tfrac{15}{8}\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi\,\mathrm{Cos}^7\varphi}{\mathrm{Sin}^5\varphi}=\left(\tfrac{1}{2}\mathrm{Cos}^5\varphi-\tfrac{9}{4}\mathrm{Cos}^4\varphi+\tfrac{3}{2}\mathrm{Cos}^2\varphi\right)\frac{1}{\mathrm{Sin}^4\varphi}+3\log\mathrm{Sin}\,\varphi$$

Taf. XIX. $\int \frac{\partial\varphi \operatorname{Sin}^n \varphi}{\operatorname{Cos}^6 \varphi}, \int \frac{\partial\varphi \operatorname{Cos}^n \varphi}{\operatorname{Sin}^6 \varphi}$

$$\int \frac{\partial\varphi \operatorname{Sin}\varphi}{\operatorname{Cos}^6 \varphi} = \frac{1}{5\operatorname{Cos}^5 \varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^2\varphi}{\operatorname{Cos}^6 \varphi} = \left(-\tfrac{2}{15}\operatorname{Sin}^5\varphi + \tfrac{1}{3}\operatorname{Sin}^3\varphi\right)\frac{1}{\operatorname{Cos}^5\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^3\varphi}{\operatorname{Cos}^6 \varphi} = \left(\tfrac{1}{3}\operatorname{Sin}^2\varphi - \tfrac{2}{15}\right)\frac{1}{\operatorname{Cos}^5\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^4\varphi}{\operatorname{Cos}^6 \varphi} = \tfrac{1}{5}\operatorname{Tang}^5\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^5\varphi}{\operatorname{Cos}^6 \varphi} = \left(\operatorname{Sin}^4\varphi - \tfrac{4}{3}\operatorname{Sin}^2\varphi + \tfrac{8}{15}\right)\frac{1}{\operatorname{Cos}^5\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^6\varphi}{\operatorname{Cos}^6 \varphi} = \tfrac{1}{5}\operatorname{Tang}^5\varphi - \tfrac{1}{3}\operatorname{Tang}^3\varphi + \operatorname{Tang}\varphi - \varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^7\varphi}{\operatorname{Cos}^6 \varphi} = \left(-\operatorname{Sin}^6\varphi + 6\operatorname{Sin}^4\varphi - 8\operatorname{Sin}^2\varphi + \tfrac{16}{5}\right)\frac{1}{\operatorname{Cos}^5\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^8\varphi}{\operatorname{Cos}^6 \varphi} = -\frac{\operatorname{Sin}^7\varphi}{2\operatorname{Cos}^5\varphi} + \frac{7}{2}\int \frac{\partial\varphi \operatorname{Sin}^6\varphi}{\operatorname{Cos}^6\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}\varphi}{\operatorname{Sin}^6 \varphi} = -\frac{1}{5\operatorname{Sin}^5 \varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^2\varphi}{\operatorname{Sin}^6 \varphi} = \left(\tfrac{2}{15}\operatorname{Cos}^5\varphi - \tfrac{1}{3}\operatorname{Cos}^3\varphi\right)\frac{1}{\operatorname{Sin}^5\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^3\varphi}{\operatorname{Sin}^6 \varphi} = \left(-\tfrac{1}{3}\operatorname{Cos}^2\varphi + \tfrac{2}{15}\right)\frac{1}{\operatorname{Sin}^5\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^4\varphi}{\operatorname{Sin}^6 \varphi} = -\tfrac{1}{5}\operatorname{Cot}^5\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^5\varphi}{\operatorname{Sin}^6 \varphi} = \left(-\operatorname{Cos}^4\varphi + \tfrac{4}{3}\operatorname{Cos}^2\varphi - \tfrac{8}{15}\right)\frac{1}{\operatorname{Sin}^5\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^6\varphi}{\operatorname{Sin}^6 \varphi} = -\tfrac{1}{5}\operatorname{Cot}^5\varphi + \tfrac{1}{3}\operatorname{Cot}^3\varphi - \operatorname{Cot}\varphi - \varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^7\varphi}{\operatorname{Sin}^6 \varphi} = \left(\operatorname{Cos}^6\varphi - 6\operatorname{Cos}^4\varphi + 8\operatorname{Cos}^2\varphi - \tfrac{16}{5}\right)\frac{1}{\operatorname{Sin}^5\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^8\varphi}{\operatorname{Sin}^6 \varphi} = \frac{\operatorname{Cos}^7\varphi}{2\operatorname{Sin}^5\varphi} + \frac{7}{2}\int \frac{\partial\varphi \operatorname{Cos}^6\varphi}{\operatorname{Sin}^6\varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Sin}^n \varphi}{\mathrm{Cos}^7 \varphi}, \quad \int \frac{\partial\varphi \, \mathrm{Cos}^n \varphi}{\mathrm{Sin}^7 \varphi}$$

Taf. XX.

$$\int \frac{\partial\varphi \, \mathrm{Sin}\, \varphi}{\mathrm{Cos}^7 \varphi} = \frac{1}{6\mathrm{Cos}^6 \varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Sin}^2 \varphi}{\mathrm{Cos}^7 \varphi} = \left(-\tfrac{1}{16}\mathrm{Sin}^5 \varphi + \tfrac{1}{6}\mathrm{Sin}^3 \varphi + \tfrac{1}{16}\mathrm{Sin}\,\varphi\right)\frac{1}{\mathrm{Cos}^6 \varphi} - \frac{1}{16}\int \frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Sin}^3 \varphi}{\mathrm{Cos}^7 \varphi} = \left(\tfrac{1}{4}\mathrm{Sin}^2 \varphi - \tfrac{1}{12}\right)\frac{1}{\mathrm{Cos}^6 \varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Sin}^4 \varphi}{\mathrm{Cos}^7 \varphi} = \left(\tfrac{1}{16}\mathrm{Sin}^5 \varphi + \tfrac{1}{6}\mathrm{Sin}^3 \varphi - \tfrac{1}{16}\mathrm{Sin}\,\varphi\right)\frac{1}{\mathrm{Cos}^6 \varphi} + \frac{1}{16}\int \frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Sin}^5 \varphi}{\mathrm{Cos}^7 \varphi} = \tfrac{1}{6}\mathrm{Tang}^6 \varphi$$

$$\int \frac{\partial\varphi \, \mathrm{Sin}^6 \varphi}{\mathrm{Cos}^7 \varphi} = \left(\tfrac{11}{16}\mathrm{Sin}^5 \varphi - \tfrac{5}{6}\mathrm{Sin}^3 \varphi + \tfrac{5}{16}\mathrm{Sin}\,\varphi\right)\frac{1}{\mathrm{Cos}^6 \varphi} - \frac{5}{16}\int \frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Sin}^7 \varphi}{\mathrm{Cos}^7 \varphi} = \tfrac{1}{6}\mathrm{Tang}^6 \varphi - \tfrac{1}{4}\mathrm{Tang}^4 \varphi + \tfrac{1}{2}\mathrm{Tang}^2 \varphi + \log \mathrm{Cos}\,\varphi$$

$$\int \frac{\partial\varphi \, \mathrm{Sin}^8 \varphi}{\mathrm{Cos}^7 \varphi} = \left(-\mathrm{Sin}^7 \varphi + \tfrac{77}{16}\mathrm{Sin}^5 \varphi - \tfrac{35}{6}\mathrm{Sin}^3 \varphi + \tfrac{35}{16}\mathrm{Sin}\,\varphi\right)\frac{1}{\mathrm{Cos}^6 \varphi} - \frac{35}{16}\int \frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Cos}\,\varphi}{\mathrm{Sin}^7 \varphi} = -\frac{1}{6\mathrm{Sin}^6 \varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Cos}^2 \varphi}{\mathrm{Sin}^7 \varphi} = \left(\tfrac{1}{16}\mathrm{Cos}^5 \varphi - \tfrac{1}{6}\mathrm{Cos}^3 \varphi - \tfrac{1}{16}\mathrm{Cos}\,\varphi\right)\frac{1}{\mathrm{Sin}^6 \varphi} - \frac{1}{16}\int \frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Cos}^3 \varphi}{\mathrm{Sin}^7 \varphi} = \left(-\tfrac{1}{4}\mathrm{Cos}^2 \varphi + \tfrac{1}{12}\right)\frac{1}{\mathrm{Sin}^6 \varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Cos}^4 \varphi}{\mathrm{Sin}^7 \varphi} = \left(-\tfrac{1}{16}\mathrm{Cos}^5 \varphi - \tfrac{1}{6}\mathrm{Cos}^3 \varphi + \tfrac{1}{16}\mathrm{Cos}\,\varphi\right)\frac{1}{\mathrm{Sin}^6 \varphi} + \frac{1}{16}\int \frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Cos}^5 \varphi}{\mathrm{Sin}^7 \varphi} = -\tfrac{1}{6}\mathrm{Cot}^6 \varphi$$

$$\int \frac{\partial\varphi \, \mathrm{Cos}^6 \varphi}{\mathrm{Sin}^7 \varphi} = \left(-\tfrac{11}{16}\mathrm{Cos}^5 \varphi + \tfrac{5}{6}\mathrm{Cos}^3 \varphi - \tfrac{5}{16}\mathrm{Cos}\,\varphi\right)\frac{1}{\mathrm{Sin}^6 \varphi} - \frac{5}{16}\int \frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int \frac{\partial\varphi \, \mathrm{Cos}^7 \varphi}{\mathrm{Sin}^7 \varphi} = -\tfrac{1}{6}\mathrm{Cot}^6 \varphi + \tfrac{1}{4}\mathrm{Cot}^4 \varphi - \tfrac{1}{2}\mathrm{Cot}^2 \varphi - \log \mathrm{Sin}\,\varphi$$

$$\int \frac{\partial\varphi \, \mathrm{Cos}^8 \varphi}{\mathrm{Sin}^7 \varphi} = \left(\mathrm{Cos}^7 \varphi - \tfrac{77}{16}\mathrm{Cos}^5 \varphi + \tfrac{35}{6}\mathrm{Cos}^3 \varphi - \tfrac{35}{16}\mathrm{Cos}\,\varphi\right)\frac{1}{\mathrm{Sin}^6 \varphi} - \frac{35}{16}\int \frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

Taf. XXI $\int \frac{\partial\varphi \operatorname{Sin}^n\varphi}{\operatorname{Cos}^8\varphi}$, $\int \frac{\partial\varphi \operatorname{Cos}^n\varphi}{\operatorname{Sin}^8\varphi}$

$$\int \frac{\partial\varphi \operatorname{Sin}\varphi}{\operatorname{Cos}^8\varphi} = \frac{1}{7\operatorname{Cos}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^2\varphi}{\operatorname{Cos}^8\varphi} = \left(\frac{8}{105}\operatorname{Sin}^7\varphi - \frac{4}{15}\operatorname{Sin}^5\varphi + \frac{1}{3}\operatorname{Sin}^3\varphi\right)\frac{1}{\operatorname{Cos}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^3\varphi}{\operatorname{Cos}^8\varphi} = \left(\frac{1}{5}\operatorname{Sin}\ \varphi - \frac{2}{35}\right)\frac{1}{\operatorname{Cos}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^4\varphi}{\operatorname{Cos}^8\varphi} = \left(-\frac{2}{35}\operatorname{Sin}^7\varphi + \frac{1}{5}\operatorname{Sin}^5\varphi\right)\frac{1}{\operatorname{Cos}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^5\varphi}{\operatorname{Cos}^8\varphi} = \left(\frac{1}{3}\operatorname{Sin}^4\varphi - \frac{4}{15}\operatorname{Sin}^2\varphi + \frac{8}{105}\right)\frac{1}{\operatorname{Cos}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^6\varphi}{\operatorname{Cos}^8\varphi} = \frac{1}{7}\operatorname{Tang}^7\varphi$$

$$\int \frac{\partial\varphi \operatorname{Sin}^7\varphi}{\operatorname{Cos}^8\varphi} = \left(\operatorname{Sin}^6\varphi - 2\operatorname{Sin}^4\varphi + \frac{8}{5}\operatorname{Sin}^2\varphi - \frac{16}{35}\right)\frac{1}{\operatorname{Cos}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Sin}^8\varphi}{\operatorname{Cos}^8\varphi} = \frac{1}{7}\operatorname{Tang}^7\varphi - \frac{1}{5}\operatorname{Tang}^5\varphi + \frac{1}{3}\operatorname{Tang}^3\varphi - \operatorname{Tang}\varphi + \varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}\varphi}{\operatorname{Sin}^8\varphi} = -\frac{1}{7\operatorname{Sin}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^2\varphi}{\operatorname{Sin}^8\varphi} = \left(-\frac{8}{105}\operatorname{Cos}^7\varphi + \frac{4}{15}\operatorname{Cos}^5\varphi - \frac{1}{3}\operatorname{Cos}^3\varphi\right)\frac{1}{\operatorname{Sin}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^3\varphi}{\operatorname{Sin}^8\varphi} = \left(-\frac{1}{5}\operatorname{Cos}^2\varphi + \frac{2}{35}\right)\frac{1}{\operatorname{Sin}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^4\varphi}{\operatorname{Sin}^8\varphi} = \left(\frac{2}{35}\operatorname{Cos}^7\varphi - \frac{1}{5}\operatorname{Cos}^5\varphi\right)\frac{1}{\operatorname{Sin}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^5\varphi}{\operatorname{Sin}^8\varphi} = \left(-\frac{1}{3}\operatorname{Cos}^4\varphi + \frac{4}{15}\operatorname{Cos}^2\varphi - \frac{8}{105}\right)\frac{1}{\operatorname{Sin}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^6\varphi}{\operatorname{Sin}^8\varphi} = -\frac{1}{7}\operatorname{Cot}^7\varphi$$

$$\int \frac{\partial\varphi \operatorname{Cos}^7\varphi}{\operatorname{Sin}^8\varphi} = \left(-\operatorname{Cos}^6\varphi + 2\operatorname{Cos}^4\varphi - \frac{8}{5}\operatorname{Cos}^2\varphi + \frac{16}{35}\right)\frac{1}{\operatorname{Sin}^7\varphi}$$

$$\int \frac{\partial\varphi \operatorname{Cos}^8\varphi}{\operatorname{Sin}^8\varphi} = -\frac{1}{7}\operatorname{Cot}^7\varphi + \frac{1}{5}\operatorname{Cot}^5\varphi - \frac{1}{3}\operatorname{Cot}^3\varphi + \operatorname{Cot}\varphi + \varphi$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}^n\varphi}, \int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}^n\varphi}$$ *Taf. XXII.*

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}\varphi} = \log \operatorname{Tang}\varphi$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}^2\varphi} = \frac{1}{\operatorname{Cos}\varphi} + \int \frac{\partial\varphi}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}^3\varphi} = \frac{1}{2\operatorname{Cos}^2\varphi} + \log \operatorname{Tang}\varphi$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}^4\varphi} = \frac{1}{3\operatorname{Cos}^3\varphi} + \frac{1}{\operatorname{Cos}\varphi} + \int \frac{\partial\varphi}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}^5\varphi} = \frac{1}{4\operatorname{Cos}^4\varphi} + \frac{1}{2\operatorname{Cos}^2\varphi} + \log \operatorname{Tang}\varphi$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}^6\varphi} = \frac{1}{5\operatorname{Cos}^5\varphi} + \frac{1}{3\operatorname{Cos}^3\varphi} + \frac{1}{\operatorname{Cos}\varphi} + \int \frac{\partial\varphi}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}^7\varphi} = \frac{1}{6\operatorname{Cos}^6\varphi} + \frac{1}{4\operatorname{Cos}^4\varphi} + \frac{1}{2\operatorname{Cos}^2\varphi} + \log \operatorname{Tang}\varphi$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}\varphi \operatorname{Cos}^8\varphi} = \frac{1}{7\operatorname{Cos}^7\varphi} + \frac{1}{5\operatorname{Cos}^5\varphi} + \frac{1}{3\operatorname{Cos}^3\varphi} + \frac{1}{\operatorname{Cos}\varphi} + \int \frac{\partial\varphi}{\operatorname{Sin}\varphi}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}\varphi} = -\frac{1}{\operatorname{Sin}\varphi} + \int \frac{\partial\varphi}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}^2\varphi} = -2\operatorname{Cot} 2\varphi$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}^3\varphi} = \left(\frac{1}{2\operatorname{Cos}^2\varphi} - \frac{3}{2}\right)\frac{1}{\operatorname{Sin}\varphi} + \frac{3}{2}\int \frac{\partial\varphi}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}^4\varphi} = \frac{1}{3\operatorname{Sin}\varphi \operatorname{Cos}^3\varphi} - \frac{8}{3}\operatorname{Cot} 2\varphi$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}^5\varphi} = \left(\frac{1}{4\operatorname{Cos}^4\varphi} + \frac{5}{8\operatorname{Cos}^2\varphi} - \frac{15}{8}\right)\frac{1}{\operatorname{Sin}\varphi} + \frac{15}{8}\int \frac{\partial\varphi}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}^6\varphi} = \left(\frac{1}{5\operatorname{Cos}^5\varphi} + \frac{2}{5\operatorname{Cos}^3\varphi}\right)\frac{1}{\operatorname{Sin}\varphi} - \frac{16}{5}\operatorname{Cot} 2\varphi$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}^7\varphi} = \left(\frac{1}{6\operatorname{Cos}^6\varphi} + \frac{7}{24\operatorname{Cos}^4\varphi} + \frac{35}{48\operatorname{Cos}^2\varphi} - \frac{35}{16}\right)\frac{1}{\operatorname{Sin}\varphi} + \frac{35}{16}\int \frac{\partial\varphi}{\operatorname{Cos}\varphi}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^2\varphi \operatorname{Cos}^8\varphi} = \left(\frac{1}{7\operatorname{Cos}^7\varphi} + \frac{8}{35\operatorname{Cos}^5\varphi} + \frac{16}{35\operatorname{Cos}^3\varphi}\right)\frac{1}{\operatorname{Sin}\varphi} - \frac{128}{35}\operatorname{Cot} 2\varphi$$

Taf. XXIII. $\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^n\varphi},\ \int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^n\varphi}$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}\,\varphi} = -\frac{1}{2\mathrm{Sin}^2\varphi} + \log \mathrm{Tang}\,\varphi$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^2\varphi} = \frac{1}{\mathrm{Sin}^2\varphi\,\mathrm{Cos}\,\varphi} + 3\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^3\varphi} = -\frac{2\mathrm{Cos}\,2\varphi}{\mathrm{Sin}^2 2\varphi} + 2\log \mathrm{Tang}\,\varphi$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^4\varphi} = \left(\frac{1}{3\mathrm{Cos}^3\varphi} + \frac{5}{3\mathrm{Cos}\,\varphi}\right)\frac{1}{\mathrm{Sin}^2\varphi} + 5\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^5\varphi} = \frac{1}{4\mathrm{Sin}^2\varphi\,\mathrm{Cos}^4\varphi} + \frac{3}{2}\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^3\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^6\varphi} = \left(\frac{1}{5\mathrm{Cos}^5\varphi} + \frac{7}{15\mathrm{Cos}^3\varphi} + \frac{7}{3\mathrm{Cos}\,\varphi}\right)\frac{1}{\mathrm{Sin}^2\varphi} + 7\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^7\varphi} = \left(\frac{1}{6\mathrm{Cos}^6\varphi} + \frac{1}{3\mathrm{Cos}^4\varphi}\right)\frac{1}{\mathrm{Sin}^2\varphi} + 2\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^3\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^8\varphi} = \left(\frac{1}{7\mathrm{Cos}^7\varphi} + \frac{9}{35\mathrm{Cos}^5\varphi} + \frac{3}{5\mathrm{Cos}^3\varphi} + \frac{3}{\mathrm{Cos}\,\varphi}\right)\frac{1}{\mathrm{Sin}^2\varphi} + 9\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}\,\varphi} = -\frac{1}{3\mathrm{Sin}^3\varphi} - \frac{1}{\mathrm{Sin}\,\varphi} + \int \frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^2\varphi} = -\frac{1}{3\mathrm{Cos}\,\varphi\,\mathrm{Sin}^3\varphi} - \frac{8}{3}\mathrm{Cot}\,2\varphi$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^3\varphi} = \frac{1}{2\mathrm{Cos}^2\varphi\,\mathrm{Sin}^3\varphi} + \frac{5}{2}\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^4\varphi} = \left(-\frac{8}{3\mathrm{Sin}^3 2\varphi} - \frac{16}{3\mathrm{Sin}\,2\varphi}\right)\mathrm{Cos}\,2\varphi$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^5\varphi} = \left(\frac{1}{4\mathrm{Cos}^4\varphi} + \frac{7}{8\mathrm{Cos}^2\varphi}\right)\frac{1}{\mathrm{Sin}^3\varphi} + \frac{35}{8}\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^6\varphi} = \frac{1}{5\mathrm{Cos}^5\varphi\,\mathrm{Sin}^3\varphi} + \frac{8}{5}\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^4\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^7\varphi} = \left(\frac{1}{6\mathrm{Cos}^6\varphi} + \frac{3}{8\mathrm{Cos}^4\varphi} + \frac{21}{16\mathrm{Cos}^2\varphi}\right)\frac{1}{\mathrm{Sin}^3\varphi} + \frac{105}{16}\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^8\varphi} = \left(\frac{1}{7\mathrm{Cos}^7\varphi} + \frac{2}{7\mathrm{Cos}^5\varphi}\right)\frac{1}{\mathrm{Sin}^3\varphi} + \frac{16}{7}\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^4\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^n\varphi},\ \int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^n\varphi}$$ *Taf. XXIV.*

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}\,\varphi}=-\frac{1}{4\mathrm{Sin}^4\varphi}-\frac{1}{2\mathrm{Sin}^2\varphi}+\log\mathrm{Tang}\,\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^2\varphi}=\left(-\frac{1}{4\mathrm{Sin}^4\varphi}-\frac{5}{8\mathrm{Sin}^2\varphi}+\frac{15}{8}\right)\frac{1}{\mathrm{Cos}\,\varphi}+\frac{15}{8}\int\frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^3\varphi}=-\frac{1}{4\mathrm{Cos}^2\varphi\,\mathrm{Sin}^4\varphi}+\frac{3}{2}\int\frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^3\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^4\varphi}=\frac{1}{3\mathrm{Sin}^4\varphi\,\mathrm{Cos}^3\varphi}+\frac{7}{5}\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^2\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^5\varphi}=\left(-\frac{4}{\mathrm{Sin}^4 2\varphi}-\frac{6}{\mathrm{Sin}^2 2\varphi}\right)\mathrm{Cos}\,2\varphi+6\log\mathrm{Tang}\,\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^6\varphi}=\left(\frac{1}{5\mathrm{Cos}^5\varphi}+\frac{3}{5\mathrm{Cos}^3\varphi}\right)\frac{1}{\mathrm{Sin}^4\varphi}+\frac{21}{5}\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^2\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^7\varphi}=\frac{1}{6\mathrm{Sin}^4\varphi\,\mathrm{Cos}^6\varphi}+\frac{5}{3}\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^5\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^8\varphi}=\left(\frac{1}{7\mathrm{Cos}^7\varphi}+\frac{11}{35\mathrm{Cos}^5\varphi}+\frac{33}{35\mathrm{Cos}^3\varphi}\right)\frac{1}{\mathrm{Sin}^4\varphi}+\frac{33}{5}\int\frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^2\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}\,\varphi}=-\frac{1}{5\mathrm{Sin}^5\varphi}-\frac{1}{3\mathrm{Sin}^3\varphi}-\frac{1}{\mathrm{Sin}\,\varphi}+\int\frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^2\varphi}=\left(-\frac{1}{5\mathrm{Sin}^5\varphi}-\frac{2}{5\mathrm{Sin}^3\varphi}\right)\frac{1}{\mathrm{Cos}\,\varphi}-\frac{16}{5}\mathrm{Cot}\,2\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^3\varphi}=\left(-\frac{1}{5\mathrm{Sin}^5\varphi}-\frac{7}{15\mathrm{Sin}^3\varphi}-\frac{7}{3\mathrm{Sin}\,\varphi}\right)\frac{1}{\mathrm{Cos}^2\varphi}+7\int\frac{\partial\varphi}{\mathrm{Cos}^3\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^4\varphi}=-\frac{1}{5\mathrm{Sin}^5\varphi\,\mathrm{Cos}^3\varphi}+\frac{8}{5}\int\frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^4\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^5\varphi}=\left(-\frac{1}{5\mathrm{Sin}^5\varphi}-\frac{3}{5\mathrm{Sin}^3\varphi}\right)\frac{1}{\mathrm{Cos}^4\varphi}+\frac{21}{5}\int\frac{\partial\varphi}{\mathrm{Sin}^2\varphi\,\mathrm{Cos}^5\varphi}$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^6\varphi}=\left(-\frac{32}{5\mathrm{Sin}^5 2\varphi}-\frac{128}{15\mathrm{Sin}^3 2\varphi}-\frac{256}{15\mathrm{Sin}\,2\varphi}\right)\mathrm{Cos}\,2\varphi$$

$$\int\frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^7\varphi}=\frac{1}{6\mathrm{Sin}^5\varphi\,\mathrm{Cos}^6\varphi}-\left(\frac{11}{30\mathrm{Sin}^5\varphi}+\frac{11}{10\mathrm{Sin}^3\varphi}\right)\frac{1}{\mathrm{Cos}^4\varphi}$$
$$-\frac{77}{10}\int\frac{\partial\varphi}{\mathrm{Sin}^2\varphi\,\mathrm{Cos}^5\varphi}$$

Taf. XXV. $\int \frac{\partial\varphi}{\mathrm{Sin}^7\varphi\,\mathrm{Cos}^n\varphi},\ \int \frac{\partial\varphi}{\mathrm{Sin}^8\varphi\,\mathrm{Cos}^n\varphi}$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^7\varphi\,\mathrm{Cos}\,\varphi} = -\frac{1}{6\mathrm{Sin}^6\varphi} - \frac{1}{4\mathrm{Sin}^4\varphi} - \frac{1}{2\mathrm{Sin}^2\varphi} + \log\mathrm{Tang}\,\varphi$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^7\varphi\,\mathrm{Cos}^2\varphi} = \left(-\frac{1}{6\mathrm{Sin}^6\varphi} - \frac{7}{24\mathrm{Sin}^4\varphi} - \frac{35}{48\mathrm{Sin}^2\varphi} + \frac{35}{16}\right)\frac{1}{\mathrm{Cos}\,\varphi} + \frac{35}{16}\int \frac{\partial\varphi}{\mathrm{Sin}\,\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^7\varphi\,\mathrm{Cos}^3\varphi} = \left(-\frac{1}{6\mathrm{Sin}^6\varphi} - \frac{1}{3\mathrm{Sin}^4\varphi}\right)\frac{1}{\mathrm{Cos}^2\varphi} + 2\int \frac{\partial\varphi}{\mathrm{Sin}^3\varphi\,\mathrm{Cos}^3\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^7\varphi\,\mathrm{Cos}^4\varphi} = \left(-\frac{1}{6\mathrm{Sin}^6\varphi} - \frac{3}{8\mathrm{Sin}^4\varphi} - \frac{21}{16\mathrm{Sin}^2\varphi}\right)\frac{1}{\mathrm{Cos}^3\varphi} + \frac{105}{16}\int \frac{\partial\varphi}{\mathrm{Sin}\,\varphi\,\mathrm{Cos}^4\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^7\varphi\,\mathrm{Cos}^5\varphi} = -\frac{1}{6\mathrm{Cos}^4\varphi\,\mathrm{Sin}^6\varphi} + \frac{5}{3}\int \frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^5\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^7\varphi\,\mathrm{Cos}^6\varphi} = -\frac{1}{6\mathrm{Cos}^5\varphi\,\mathrm{Sin}^6\varphi} + \left(\frac{11}{30\mathrm{Cos}^5\varphi} + \frac{11}{10\mathrm{Cos}^3\varphi}\right)\frac{1}{\mathrm{Sin}^4\varphi}$$
$$+ \frac{77}{10}\int \frac{\partial\varphi}{\mathrm{Sin}^5\varphi\,\mathrm{Cos}^2\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^7\varphi\,\mathrm{Cos}^7\varphi} = \left(-\frac{32}{3\mathrm{Sin}^6 2\varphi} - \frac{40}{3\mathrm{Sin}^4 2\varphi} - \frac{20}{\mathrm{Sin}^2 2\varphi}\right)\mathrm{Cos}\,2\varphi + 20\log\mathrm{Tang}\,\varphi$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^8\varphi\,\mathrm{Cos}\,\varphi} = -\frac{1}{7\mathrm{Sin}^7\varphi} - \frac{1}{5\mathrm{Sin}^5\varphi} - \frac{1}{3\mathrm{Sin}^3\varphi} - \frac{1}{\mathrm{Sin}\,\varphi} + \int \frac{\partial\varphi}{\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^8\varphi\,\mathrm{Cos}^2\varphi} = \left(-\frac{1}{7\mathrm{Sin}^7\varphi} - \frac{8}{35\mathrm{Sin}^5\varphi} - \frac{16}{35\mathrm{Sin}^3\varphi}\right)\frac{1}{\mathrm{Cos}\,\varphi} - \frac{128}{35}\mathrm{Cot}\,2\varphi$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^8\varphi\,\mathrm{Cos}^3\varphi} = \left(-\frac{1}{7\mathrm{Sin}^7\varphi} - \frac{9}{35\mathrm{Sin}^5\varphi} - \frac{3}{5\mathrm{Sin}^3\varphi} - \frac{3}{\mathrm{Sin}\,\varphi}\right)\frac{1}{\mathrm{Cos}^2\varphi} + 9\int \frac{\partial\varphi}{\mathrm{Cos}^3\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^8\varphi\,\mathrm{Cos}^4\varphi} = \left(-\frac{1}{7\mathrm{Sin}^7\varphi} - \frac{2}{7\mathrm{Sin}^5\varphi}\right)\frac{1}{\mathrm{Cos}^3\varphi} + \frac{16}{7}\int \frac{\partial\varphi}{\mathrm{Sin}^4\varphi\,\mathrm{Cos}^4\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^8\varphi\,\mathrm{Cos}^5\varphi} = \left(-\frac{1}{7\mathrm{Sin}^7\varphi} - \frac{11}{35\mathrm{Sin}^5\varphi} - \frac{33}{35\mathrm{Sin}^3\varphi}\right)\frac{1}{\mathrm{Cos}^4\varphi}$$
$$+ \frac{33}{5}\int \frac{\partial\varphi}{\mathrm{Sin}^2\varphi\,\mathrm{Cos}^5\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^8\varphi\,\mathrm{Cos}^6\varphi} = -\frac{1}{7\mathrm{Sin}^7\varphi\,\mathrm{Cos}^5\varphi} + \frac{12}{7}\int \frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^6\varphi}$$

$$\int \frac{\partial\varphi}{\mathrm{Sin}^8\varphi\,\mathrm{Cos}^7\varphi} = -\frac{1}{7\mathrm{Sin}^7\varphi\,\mathrm{Cos}^6\varphi} + \frac{13}{6}\int \frac{\partial\varphi}{\mathrm{Sin}^6\varphi\,\mathrm{Cos}^7\varphi}$$

Bemerkungen zu den vorhergehenden Tafeln.

1) Die Formeln S. 261 — S. 286 für das Integral $\int \partial\varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi$ lassen sich auch bey dem Integral $\int \partial\varphi \operatorname{Sin}^m (k\varphi + l) \operatorname{Cos}^n (k\varphi + l)$ anwenden, wenn k und l constante Gröſsen sind. Man darf in den gegebenen Formeln nur $k\varphi + l$ für φ setzen, und hierauf das Ganze mit $\frac{1}{k}$ multipliciren, jedoch muſs vorher das Integralzeichen weggeschaft, und die Formel völlig entwickelt dargestellt werden. Man findet so z. B.

$$\int \partial\varphi \operatorname{Cos}(k\varphi + l) = \frac{1}{k} \operatorname{Sin}(k\varphi + l)$$

$$\int \partial\varphi \operatorname{Sin}(k\varphi + l) = -\frac{1}{k} \operatorname{Cos}(k\varphi + l)$$

$$\int \partial\varphi \operatorname{Cos}(k\varphi + l) \operatorname{Sin}^n (k\varphi + l) = \frac{\operatorname{Sin}^{n+1}(k\varphi + l)}{k(n+1)}$$

$$\int \partial\varphi \operatorname{Sin}(k\varphi + l) \operatorname{Cos}^n (k\varphi + l) = -\frac{\operatorname{Cos}^{n+1}(k\varphi + l)}{k(n+1)}$$

$$\int \frac{\partial\varphi}{\operatorname{Sin}^3(k\varphi + l) \operatorname{Cos}^2(k\varphi + l)} = \frac{1}{k \operatorname{Sin}^2(k\varphi + l) \operatorname{Cos}(k\varphi + l)}$$
$$- \frac{3 \operatorname{Cos}(k\varphi + l)}{2k \operatorname{Sin}^2(k\varphi + l)} + \frac{3}{2k} \log \operatorname{Tang} \tfrac{1}{2}(k\varphi + l).$$

2) Differentialformeln wie diese: $\partial\varphi \operatorname{Tang}^m \varphi$, $\partial\varphi \operatorname{Sec}^m \varphi \operatorname{Cot}^n \varphi$, $\partial\varphi \operatorname{Sec}^m \varphi \operatorname{Tang}^n \varphi \operatorname{Cosec}^p \varphi$, etc., lassen sich auf die Form $\partial\varphi \operatorname{Sin}^m \varphi \operatorname{Cos}^n \varphi$ bringen, wenn anstatt $\operatorname{Tang}\varphi$, $\operatorname{Cot}\varphi$, $\operatorname{Sec}\varphi$, $\operatorname{Cosec}\varphi$, ihre Werthe $\frac{\operatorname{Sin}\varphi}{\operatorname{Cos}\varphi}$, $\frac{\operatorname{Cos}\varphi}{\operatorname{Sin}\varphi}$, $\frac{1}{\operatorname{Cos}\varphi}$, $\frac{1}{\operatorname{Sin}\varphi}$ gesetzt werden.

3) Die folgenden Formeln sind, ihres häufigen Gebrauches wegen, noch zu bemerken:

$$\int \partial\varphi \operatorname{Sin}(k\varphi + l) \operatorname{Cos}(k'\varphi + l') = -\frac{\operatorname{Cos}[(k + k')\varphi + l + l']}{2(k + k')}$$
$$- \frac{\operatorname{Cos}[(k - k')\varphi + l - l']}{2(k - k')}$$

$$\int \partial\varphi \operatorname{Sin}(k\varphi + l) \operatorname{Sin}(k'\varphi + l') = \frac{\operatorname{Sin}[(k - k')\varphi + l - l']}{2(k - k')}$$
$$- \frac{\operatorname{Sin}[(k + k')\varphi + l + l']}{2(k + k')}$$

$$\int \partial\varphi \operatorname{Cos}(k\varphi + l) \operatorname{Cos}(k'\varphi + l') = \frac{\operatorname{Sin}[(k + k')\varphi + l + l']}{2(k + k')}$$
$$+ \frac{\operatorname{Sin}[(k - k')\varphi + l - l']}{2(k - k')}$$

Taf. XXVI.

$$\int \varphi^n \partial\varphi \operatorname{Sin}\varphi$$

Allgemeine Formel.

$$\int \varphi^n \partial\varphi \operatorname{Sin}\varphi = -\varphi^n \operatorname{Cos}\varphi + n\varphi^{n-1}\operatorname{Sin}\varphi + n(n-1)\varphi^{n-2}\operatorname{Cos}\varphi$$
$$- n(n-1)(n-2)\varphi^{n-3}\operatorname{Sin}\varphi$$
$$- n(n-1)(n-2)(n-3)\varphi^{n-4}\operatorname{Cos}\varphi ++--$$

Einzelne Fälle.

$$\int \varphi \partial\varphi \operatorname{Sin}\varphi = -\varphi \operatorname{Cos}\varphi + \operatorname{Sin}\varphi$$

$$\int \varphi^2 \partial\varphi \operatorname{Sin}\varphi = -\varphi^2 \operatorname{Cos}\varphi + 2\varphi \operatorname{Sin}\varphi + 2\operatorname{Cos}\varphi$$

$$\int \varphi^3 \partial\varphi \operatorname{Sin}\varphi = -\varphi^3 \operatorname{Cos}\varphi + 3\varphi^2 \operatorname{Sin}\varphi + 6\varphi \operatorname{Cos}\varphi - 6\operatorname{Sin}\varphi$$

$$\int \varphi^4 \partial\varphi \operatorname{Sin}\varphi = -\varphi^4 \operatorname{Cos}\varphi + 4\varphi^3 \operatorname{Sin}\varphi + 12\varphi^2 \operatorname{Cos}\varphi - 24\varphi \operatorname{Sin}\varphi$$
$$- 24\operatorname{Cos}\varphi$$

$$\int \varphi^5 \partial\varphi \operatorname{Sin}\varphi = -\varphi^5 \operatorname{Cos}\varphi + 5\varphi^4 \operatorname{Sin}\varphi + 20\varphi^3 \operatorname{Cos}\varphi - 60\varphi^2 \operatorname{Sin}\varphi$$
$$- 120\varphi \operatorname{Cos}\varphi + 120\operatorname{Sin}\varphi$$

$$\int \varphi^n \partial\varphi \operatorname{Cos}\varphi$$

Allgemeine Formel.

$$\int \varphi^n \partial\varphi \operatorname{Cos}\varphi = \varphi^n \operatorname{Sin}\varphi + n\varphi^{n-1}\operatorname{Cos}\varphi - n(n-1)\varphi^{n-2}\operatorname{Sin}\varphi$$
$$- n(n-1)(n-2)\varphi^{n-3}\operatorname{Cos}\varphi ++--$$

Einzelne Fälle.

$$\int \varphi \partial\varphi \operatorname{Cos}\varphi = \varphi \operatorname{Sin}\varphi + \operatorname{Cos}\varphi$$

$$\int \varphi^2 \partial\varphi \operatorname{Cos}\varphi = \varphi^2 \operatorname{Sin}\varphi + 2\varphi \operatorname{Cos}\varphi - 2\operatorname{Sin}\varphi$$

$$\int \varphi^3 \partial\varphi \operatorname{Cos}\varphi = \varphi^3 \operatorname{Sin}\varphi + 3\varphi^2 \operatorname{Cos}\varphi - 6\varphi \operatorname{Sin}\varphi - 6\operatorname{Cos}\varphi$$

$$\int \varphi^4 \partial\varphi \operatorname{Cos}\varphi = \varphi^4 \operatorname{Sin}\varphi + 4\varphi^3 \operatorname{Cos}\varphi - 12\varphi^2 \operatorname{Sin}\varphi - 24\varphi \operatorname{Cos}\varphi$$
$$+ 24\operatorname{Sin}\varphi$$

$$\int \varphi^5 \partial\varphi \operatorname{Cos}\varphi = \varphi^5 \operatorname{Sin}\varphi + 5\varphi^4 \operatorname{Cos}\varphi - 20\varphi^3 \operatorname{Sin}\varphi - 60\varphi^2 \operatorname{Cos}\varphi$$
$$+ 120\varphi \operatorname{Cos}\varphi + 120\operatorname{Sin}\varphi$$

$\int X\varphi\partial x$ *Taf. XXVII.*

[X eine algebraische Function von x; $\varphi = \text{Arc Sin}\, x$, $\text{Arc Cos}\, x$, $\text{Arc Tang}\, x$, etc.]

Allgemeine Formeln.

$$\int X\partial x\, \text{Arc Sin}\, x = \text{Arc Sin}\, x \cdot \int X\partial x - \int \frac{\partial x \int X\partial x}{\sqrt{(1-x^2)}}$$

$$\int X\partial x\, \text{Arc Cos}\, x = \text{Arc Cos}\, x \cdot \int X\partial x + \int \frac{\partial x \int X\partial x}{\sqrt{(1-x^2)}}$$

$$\int X\partial x\, \text{Arc Tang}\, x = \text{Arc Tang}\, x \cdot \int X\partial x - \int \frac{\partial x \int X\partial x}{1+x^2}$$

$$\int X\partial x\, \text{Arc Cot}\, x = \text{Arc Cot}\, x \cdot \int X\partial x + \int \frac{\partial x \int X\partial x}{1+x^2}$$

$$\int X\partial x\, \text{Arc Sec}\, x = \text{Arc Sec}\, x \cdot \int X\partial x - \int \frac{\partial x \int X\partial x}{x\sqrt{(x^2-1)}}$$

$$\int X\partial x\, \text{Arc Cosec}\, x = \text{Arc Cosec}\, x \cdot \int X\partial x + \int \frac{\partial x \int X\partial x}{x\sqrt{(x^2-1)}}$$

$$\int X\partial x\, \text{Arc Sin vers}\, x = \text{Arc Sin v.}\, x \cdot \int X\partial x - \int \frac{\partial x \int X\partial x}{\sqrt{(2x-x^2)}}$$

Einzelne Fälle.

$$\int \partial x\, \text{Arc Sin}\, x = x\, \text{Arc Sin}\, x - \int \frac{x\partial x}{\sqrt{(1-x^2)}}$$

$$\int x^m\partial x\, \text{Arc Sin}\, x = \frac{x^{m+1}}{m+1}\text{Arc Sin}\, x - \frac{1}{m+1}\int \frac{x^{m+1}\partial x}{\sqrt{(1-x^2)}}$$

$$\int \frac{\partial x}{\sqrt{(1-x^2)}}\text{Arc Sin}\, x = \frac{1}{2}\left(\text{Arc Sin}\, x\right)^2$$

$$\int \frac{x\partial x}{\sqrt{(1-x^2)}}\text{Arc Sin}\, x = -\text{Arc Sin}\, x \cdot \sqrt{(1-x^2)} + x$$

$$\int \frac{x^2\partial x}{\sqrt{(1-x^2)}}\text{Arc Sin}\, x = \left(-\frac{1}{2}x\sqrt{(1-x^2)} + \frac{1}{4}\text{Arc Sin}\, x\right)\text{Arc Sin}\, x + \frac{1}{4}x^2$$

$$\int \frac{x^3\partial x}{\sqrt{(1-x^2)}}\text{Arc Sin}\, x = -\left(\frac{1}{3}x^2 + \frac{2}{3}\right)\sqrt{(1-x^2)} \cdot \text{Arc Sin}\, x + \frac{1}{9}x^3 + \frac{2}{3}x$$

$$\int \frac{x^4\partial x}{\sqrt{(1-x^2)}}\text{Arc Sin}\, x = \left[-\left(\frac{1}{4}x^3 + \frac{3}{8}x\right)\sqrt{(1-x^2)} + \frac{3}{16}\text{Arc Sin}\, x\right] \times \text{Arc Sin}\, x + \frac{1}{16}x^4 + \frac{3}{16}x^2$$

$$\int \frac{x^5 \partial x}{\sqrt{(1-x^2)}} \operatorname{Arc Sin} x = -\left(\tfrac{1}{5}x^4 + \tfrac{4}{15}x^2 + \tfrac{8}{15}\right)\sqrt{X} \cdot \operatorname{Arc Sin} x$$
$$+ \tfrac{1}{25}x^5 + \tfrac{4}{45}x^3 + \tfrac{8}{15}x$$

$$\int \frac{\partial x}{(1-x^2)^{\frac{3}{2}}} \operatorname{Arc Sin} x = \frac{x \operatorname{Arc Sin} x}{\sqrt{(1-x^2)}} + \tfrac{1}{2}\log(1-x^2)$$

$$\int \frac{x\partial x}{(1-x^2)^{\frac{3}{2}}} \operatorname{Arc Sin} x = \frac{\operatorname{Arc Sin} x}{\sqrt{(1-x^2)}} + \tfrac{1}{2}\log\frac{1-x}{1+x}$$

$$\int x^m \partial x \operatorname{Arc Cos} x = \frac{x^{m+1}}{m+1}\operatorname{Arc Cos} x + \frac{1}{m+1}\int \frac{x^{m+1}\partial x}{\sqrt{(1-x^2)}}$$

$$\int x^m \partial x \operatorname{Arc Tang} x = \frac{x^{m+1}}{m+1}\operatorname{Arc Tang} x - \frac{1}{m+1}\int \frac{x^{m+1}\partial x}{1+x^2}$$

$$\int x^m \partial x \operatorname{Arc Cot} x = \frac{x^{m+1}}{m+1}\operatorname{Arc Cot} x + \frac{1}{m+1}\int \frac{x^{m+1}\partial x}{1+x^2}$$

$$\int x^m \partial x \operatorname{Arc Sec} x = \frac{x^{m+1}}{m+1}\operatorname{Arc Sec} x - \frac{1}{m+1}\int \frac{x^m\partial x}{\sqrt{(x^2-1)}}$$

$$\int x^m \partial x \operatorname{Arc Cosec} x = \frac{x^{m+1}}{m+1}\operatorname{Arc Cosec} x + \frac{1}{m+1}\int \frac{x^m\partial x}{\sqrt{(x^2-1)}}$$

$$\int x^m \partial x \operatorname{Arc Sin vers} x = \frac{x^{m+1}}{m+1}\operatorname{Arc Sin v.} x - \frac{1}{m+1}\int \frac{x^{m+1}\partial x}{\sqrt{(2x-x^2)}}$$

$$\int \frac{\partial x}{1+x^2}\operatorname{Arc Tang} x = \tfrac{1}{2}\left(\operatorname{Arc Tang} x\right)^2$$

$$\int \frac{x^2\partial x}{1+x^2}\operatorname{Arc Tang} x = \left(x - \tfrac{1}{2}\operatorname{Arc Tang} x\right)\operatorname{Arc Tang} x - \tfrac{1}{2}\log(1+x^2)$$

$$\int \frac{\partial x}{(1+x^2)^2}\operatorname{Arc Tang} x = \left(\frac{x}{2(1+x^2)} + \tfrac{1}{4}\operatorname{Arc Tang} x\right)\operatorname{Arc Tang} x$$
$$+ \frac{1}{4(1+x^2)}$$

$$\int \frac{\partial x}{\sqrt{(1-x^2)}}\operatorname{Arc Cos} x = -\tfrac{1}{2}\left(\operatorname{Arc Cos} x\right)^2$$

$$\int \frac{\partial x}{1+x^2}\operatorname{Arc Cot} x = -\tfrac{1}{2}\left(\operatorname{Arc Cot} x\right)^2$$

$$\int \frac{\partial x}{\sqrt{(2x-x^2)}}\operatorname{Arc Sin vers} x = \tfrac{1}{2}\left(\operatorname{Arc Sin v.} x\right)^2$$

$\int X\partial x \log Z$ *Taf. XXVIII.*

(X, Z, algebraische Functionen von x)

Allgemeine Formel.

$$\int X\partial x \log Z = \log Z \cdot \int X\partial x - \int \frac{\partial Z \int X\partial x}{Z}$$

Einzelne Fälle.

$$\int X\partial x \log x = \log x \cdot \int X\partial x - \int \frac{\partial x \int X\partial x}{x}$$

$$\int x^m \partial x \log x = \frac{x^{m+1}}{m+1}\left(\log x - \frac{1}{m+1}\right)$$

$$\int (a+bx)^m \partial x \log x = \frac{(a+bx)^{m+1}}{(m+1)b} \log x - \frac{1}{(m+1)b}\int \frac{\partial x(a+bx)^{m+1}}{x}$$

$$\int x^{-1}\partial x \log x = \int \frac{\partial x}{x} \log x = \tfrac{1}{2}\log^2 x$$

$$\int \frac{\partial x}{a+bx}\log x = \frac{1}{b}\log x \cdot \log(a+bx) - \frac{1}{b}\int \frac{\partial x}{x}\log(a+bx)$$

Hieraus erhält man entweder *)

$$\int \frac{\partial x}{a+bx}\log x = \frac{1}{b}\log x \cdot \log \frac{a+bx}{a} - \frac{x}{a} + \frac{bx^2}{2^2 a^2} - \frac{b^2 x^3}{3^2 a^3} + \text{etc.}$$

oder

$$\int \frac{\partial x}{a+bx}\log x = \frac{1}{b}\log x \cdot \log(a+bx) - \frac{1}{2b}(\log bx)^2 + \frac{a}{b^2 x} - \frac{a^2}{2^2 b^3 x^2} + \frac{a^3}{3^2 b^4 x^3} - \frac{a^4}{4^2 b^5 x^4} + \text{etc.}$$

$$\int x^m \partial x \log(a+bx) = \frac{x^{m+1}}{m+1}\log(a+bx) - \frac{b}{m+1}\int \frac{x^{m+1}\partial x}{a+bx}$$

$$\int \frac{\partial x}{x}\log(a+bx) = \log a \cdot \log x + \frac{bx}{a} - \frac{b^2x^2}{2^2a^2} + \frac{b^3x^3}{3^2a^3} - \text{etc.}$$

$$\int \frac{\partial x}{x}\log(a+bx) = \tfrac{1}{2}(\log bx)^2 - \frac{a}{bx} + \frac{a^2}{2^2b^2x^2} - \frac{a^3}{3^2b^3x^3} + \text{etc.}$$

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*) Man s. die beiden letzten Formeln auf dieser Seite. Es wird nämlich $\log(a+bx)$ in eine nach den Potenzen von x steigende oder fallende Reihe verwandelt, mit $\frac{\partial x}{x}$ multiplicirt und hieraus integrirt.

Taf. XXIX.

$$\int X\partial x \log^n x$$

Allgemeine Formel.

$$\int X\partial x \log^n x = X'\log^n x - nX''\log^{n-1} x + n(n-1)X'''\log^{n-2} x$$
$$+ n(n-1)(n-2)X''''\log^{n-3} x + \text{etc.}$$

$$X = \int X\partial x,\ X'' = \int \frac{X'\partial x}{x},\ X''' = \int \frac{X''\partial x}{x},\ \text{etc.}$$

Einzelne Fälle.

$$\int x^m \partial x \log^n x = \frac{x^{m+1}}{m+1}\Big(\log^n x - \frac{n}{m+1}\log^{n-1} x + \frac{n(n-1)}{(m+1)^2}\log^{n-2} x$$
$$- \frac{n(n-1)(n-2)}{(m+1)^3}\log^{n-3} x + \text{etc. *)}\Big)$$

$$\int x^{-1}\partial x \log^n x = \int \frac{\partial x}{x}\log^n x = \frac{1}{n+1}\log^{n+1} x$$

$$\int x^m \partial x \log x = \frac{x^{m+1}}{m+1}\left(\log x - \frac{1}{m+1}\right)$$

$$\int x^m \partial x \log^2 x = \frac{x^{m+1}}{m+1}\left(\log^2 x - \frac{2}{m+1}\log x + \frac{2\cdot 1}{(m+1)^2}\right)$$

$$\int x^m \partial x \log^3 x = \frac{x^{m+1}}{m+1}\left(\log^3 x - \frac{3}{m+1}\log^2 x + \frac{3\cdot 2}{(m+1)^2}\log x - \frac{3\cdot 2\cdot 1}{(m+1)^3}\right)$$

$$\int \frac{x^m \partial x}{\sqrt{\log x}} = \frac{x^{m+1}}{(m+1)\sqrt{\log x}}\Big(1 + \frac{1}{(2m+2)\log x} + \frac{1\cdot 3}{[(2m+2)\log x]^2}$$
$$+ \frac{1\cdot 3\cdot 5}{[(2m+2)\log x]^3} + \frac{1\cdot 3\cdot 5\cdot 7}{[(2m+2)\log x]^4} + \textit{in infinit.}\Big)$$

$$\int \frac{x^m \partial x}{\sqrt{\log \frac{1}{x}}} = \frac{x^{m+1}}{(m+1)\sqrt{\log \frac{1}{x}}}\Big(1 + \frac{1}{(2m+2)\log x} + \frac{1\cdot 3}{[(2m+2)\log x]^2}$$
$$+ \frac{1\cdot 3\cdot 5}{[(2m+2)\log x]^3} + \frac{1\cdot 3\cdot 5\cdot 7}{[(2m+2)\log x]^4} + \textit{in infinit.}\Big)$$

Die erste von den beiden Integralformeln $\int \frac{x^m \partial x}{\sqrt{\log x}}$, $\int \frac{x^m \partial x}{\sqrt{\log \frac{1}{x}}}$ wird imaginär, wenn x zwischen 0 und 1 fällt, die zweite wird es, wenn $x > 1$.

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*) Die Reihe bricht ab, wenn n eine ganze positive Zahl ist. Wenn n eine ganze negative Zahl ist, läſst sich ebenfalls eine endliche Reihe finden. M. s. die folg. Seite.

$$\int\frac{X\partial x}{\log^n x}$$ *Taf. XXX.*

Allgemeine Formel.

$$\int\frac{X\partial x}{\log^n x}=-\frac{Xx}{(n-1)\log^{n-1}x}-\frac{X'x}{(n-1)(n-2)\log^{n-2}x}-\frac{X''x}{(n-1)(n-2)(n-3)\log^{n-3}x}-\text{etc.}$$

$$X'=\frac{\partial(Xx)}{\partial x},\quad X''=\frac{\partial(X'x)}{\partial x},\quad X'''=\frac{\partial(X''x)}{\partial x},\ \text{etc.}$$

Einzelne Fälle.

$$\int\frac{x^m\partial x}{\log^n x}=-\frac{x^{m+1}}{(n-1)\log^{n-1}x}-\frac{(m+1)x^{m+1}}{(n-1)(n-2)\log^{n-2}x}-\frac{(m+1)^2x^{m+1}}{(n-1)(n-2)(n-3)\log^{n-3}x}-\ldots\ldots\ldots\ldots\ldots\ldots-\frac{(m+1)^{n-2}x^{m+1}}{(n-1)(n-2)(n-3)\cdots2\cdot1\log x}+\frac{(m+1)^{n-1}}{(n-1)(n-2)\cdots2\cdot1}\int\frac{x^m\partial x}{\log x}$$

$$\int\frac{\partial x}{x}\log^n x=\frac{1}{n+1}\log^{n+1}x$$

$$\int\frac{\partial x}{\log x}=\log\log x+\frac{\log x}{1}+\frac{1}{2}\cdot\frac{\log^2 x}{1\cdot2}+\frac{1}{3}\cdot\frac{\log^3 x}{1\cdot2\cdot3}+\frac{1}{4}\cdot\frac{\log^4 x}{1\cdot2\cdot3\cdot4}+\frac{1}{5}\cdot\frac{\log^5 x}{1\cdot2\cdot3\cdot4\cdot5}+\textit{in infinit.}$$

$$\int\frac{x^m\partial x}{\log x}=\int\frac{\partial y}{\log y}\ \text{für}\ y=x^{m+1}$$

$$\int\frac{x^m\partial x}{\log^2 x}=-\frac{x^{m+1}}{\log x}+\frac{m+1}{1}\int\frac{x^m\partial x}{\log x}$$

$$\int\frac{x^m\partial x}{\log^3 x}=-\frac{x^{m+1}}{2\log^2 x}-\frac{(m+1)x^{m+1}}{2\cdot1\log x}+\frac{(m+1)^2}{2\cdot1}\int\frac{x^m\partial x}{\log x}$$

$$\int\frac{\partial x}{\log\frac{1}{x}}=\log\log x-\frac{\log x}{1}+\frac{1}{2}\cdot\frac{\log^2 x}{1\cdot2}-\frac{1}{3}\cdot\frac{\log^3 x}{1\cdot2\cdot3}+\frac{1}{4}\cdot\frac{\log^4 x}{1\cdot2\cdot3\cdot4}-\frac{1}{5}\cdot\frac{\log^5 x}{1\cdot2\cdot3\cdot4\cdot5}+-+-\textit{in infinit.}$$

$$\int'\frac{\partial x}{\sqrt{\log\frac{1}{x}}}=\sqrt{\pi}$$ [Das Integral von $x=0$ bis $x=1$ genommen. (Euler *Comment. Acad. Petrop. Tom. XVI. p.* 111.)]

$$\int'\partial x\left(\log\frac{1}{x}\right)^{\frac{2n+1}{2}}=\frac{1\cdot3\cdot5\cdot7\cdots\cdots(2n+1)}{2^{n+1}}\sqrt{\pi}$$ (Ebend.)

Taf. XXXI. $\int a^x X\partial x$

Allgemeine Formeln.

$$\int a^x X\partial x = \frac{a^x X}{\log a} - \frac{a^x X'}{\log^2 a} + \frac{a^x X''}{\log^3 a} - \frac{a^x X'''}{\log^4 a} + - + - \ldots\ldots\ldots$$

$$X' = \frac{\partial X}{\partial x},\; X'' = \frac{\partial X'}{\partial x},\; X''' = \frac{\partial X''}{\partial x},\text{ etc.}$$

$$\int a^x X\partial x = a^x X_1 - a^x X_2 \log a + a^x X_3 \log^2 a - a^x X_4 \log^3 a + a^x X_5 \log^4 a - + - + \ldots\ldots\ldots$$

$$X_1 = \int X\partial x,\; X_2 = \int X_1\partial x,\; X_3 = \int X_2\partial x,\text{ etc.}$$

Einzelne Fälle.

$$\int a^x x^n\partial x = \frac{a^x x^n}{\log a} - \frac{n a^x x^{n-1}}{\log^2 a} + \frac{n(n-1)a^x x^{n-2}}{\log^3 a} - \frac{n(n-1)(n-2)a^x x^{n-3}}{\log^4 a} + \ldots\ldots \pm \frac{n(n-1)(n-2)\cdots 2\cdot 1\, a^x}{\log^{n+1} a}$$

$$\int \frac{a^x\partial x}{x^n} = -\frac{a^x}{(n-1)x^{n-1}} - \frac{a^x \log a}{(n-1)(n-2)x^{n-2}} - \frac{a^x \log^2 a}{(n-1)(n-2)(n-3)x^{n-3}} - \frac{a^x \log^3 a}{(n-1)(n-2)(n-3)(n-4)x^{n-4}} - \ldots\ldots - \frac{a^x \log^{n-2} a}{(n-1)(n-2)\cdots 2\cdot 1\, x} + \frac{\log^{n-1} a}{(n-1)(n-2)\cdots\cdots 2\cdot 1}\int \frac{a^x\partial x}{x}$$

$$\int a^x\partial x = \frac{a^x}{\log a},\; \int a^{mx}\partial x = \frac{a^{mx}}{m\log a},\; \int e^{mx}\partial x = \frac{e^{mx}}{m}$$

$$\int a^x x\partial x = \frac{a^x x}{\log a} - \frac{a^x}{\log^2 a}$$

$$\int a^x x^2\partial x = \frac{a^x x^2}{\log a} - \frac{2a^x x}{\log^2 a} + \frac{2\cdot 1\, a^x}{\log^3 a}$$

$$\int a^x x^3\partial x = \frac{a^x x^3}{\log a} - \frac{3a^x x^2}{\log^2 a} + \frac{3\cdot 2\, a^x x}{\log^3 a} - \frac{3\cdot 2\cdot 1\, a^x}{\log^4 a}$$

$$\int \frac{a^x\partial x}{x} = \log x + \frac{x\log a}{1} + \frac{x^2\log^2 a}{1\cdot 2\cdot 2} + \frac{x^3\log^3 a}{1\cdot 2\cdot 3\cdot 3} + \frac{x^4\log^4 a}{1\cdot 2\cdot 3\cdot 4\cdot 4} + \frac{x^5\log^5 a}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 5} + \textit{in infinit.}$$

$$\int \frac{a^x\partial x}{x^2} = -\frac{a^x}{x} + \log a\int \frac{a^x\partial x}{x}$$

$$\int \frac{a^x \partial x}{x^3} = -\frac{a^x}{2x^2} - \frac{a^x \log a}{2 \cdot 1 x} + \frac{\log^2 a}{2 \cdot 1} \int \frac{a^x \partial x}{x}$$

$$\int \frac{a^x \partial x}{x^4} = -\frac{a^x}{3x^3} - \frac{a^x \log a}{3 \cdot 2 x^2} - \frac{a^x \log^2 a}{3 \cdot 2 \cdot 1 x} + \frac{\log^3 a}{3 \cdot 2 \cdot 1} \int \frac{a^x \partial x}{x}$$

$$\int \frac{a^x \partial x}{\sqrt{x}} = \frac{a^x}{\sqrt{x}} \left(\frac{1}{\log a} + \frac{1}{2x \log^2 a} + \frac{1 \cdot 3}{2^2 x^2 \log^3 a} + \frac{1 \cdot 3 \cdot 5}{2^3 x^3 \log^4 a} + \text{etc.} \right)$$

$$\int \frac{a^x \partial x}{\sqrt{x}} = \frac{a^x}{\sqrt{x}} \left(\frac{2x}{1} - \frac{2^2 x^2 \log a}{1 \cdot 3} + \frac{2^3 x^3 \log^2 a}{1 \cdot 3 \cdot 5} - \frac{2^4 x^4 \log^3 a}{1 \cdot 3 \cdot 5 \cdot 7} + \text{etc.} \right)$$

$$\int \frac{a^x \partial x}{1-x} = a^x \Big[\frac{1}{(1-x) \log a} - \frac{1}{(1-x)^2 \log^2 a} + \frac{1 \cdot 2}{(1-x)^3 \log^3 a}$$
$$- \frac{1 \cdot 2 \cdot 3}{(1-x)^4 \log^4 a} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{(1-x)^5 \log^5 a} - \text{etc.} \Big]$$

$$\int a^{mx} x^n \partial x = \frac{1}{m^{n+1}} \int a^y y^n \partial y \quad \text{für } y = mx$$

$$\int x^{nx} x^m \partial x = \int \left(1 + \frac{nx \log x}{1} + \frac{n^2 x^2 \log^2 x}{1 \cdot 2} + \frac{n^3 x^3 \log^3 x}{1 \cdot 2 \cdot 3} + \text{etc.} \right) x^m \partial x$$

$$= {}^{*)} \quad x^{m+1} \left(\frac{1}{m+1} - \frac{nx}{(m+2)^2} + \frac{n^2 x^2}{(m+3)^3} - \frac{n^3 x^3}{(m+4)^4} + \text{etc.} \right)$$
$$+ \frac{n x^{m+2} \log x}{1} \left(\frac{1}{m+2} - \frac{nx}{(m+3)^2} + \frac{n^2 x^2}{(m+4)^3} - \frac{n^3 x^3}{(m+5)^4} + \text{etc.} \right)$$
$$+ \frac{n^2 x^{m+3} \log^2 x}{1 \cdot 2} \left(\frac{1}{m+3} - \frac{nx}{(m+4)^2} + \frac{n^2 x^2}{(m+5)^3} - \frac{n^3 x^3}{(m+6)^4} + \text{etc.} \right)$$
$$+ \frac{n^3 x^{m+4} \log^3 x}{1 \cdot 2 \cdot 3} \left(\frac{1}{m+4} - \frac{nx}{(m+5)^2} + \frac{n^2 x^2}{(m+6)^3} - \frac{n^3 x^3}{(m+7)^4} + \text{etc.} \right)$$
$$\ldots \ldots \text{etc.} \ldots \ldots \text{etc.} \ldots \ldots \text{etc.} \ldots \ldots$$

$$\int' e^{-x^2} \partial x = \sqrt{\pi} \quad \left[\begin{array}{l} \text{Das Integral von } x = -\infty \text{ bis } x = +\infty. \\ \text{Laplace, } \textit{Mécan. cél. livre X. No. 5.} \end{array} \right]$$

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*) Durch Integrirung der Differentiale $x^m \partial x$, $x^{m+1} \partial x \log x$, $x^{m+2} \partial x \log^2 x$, $x^{m+3} \partial x \log^3 x$, etc. Der Werth des Integrals $\int x^{nx} x^m \partial x$ zwischen den Grenzen 0 und 1 genommen, jedoch unter der Voraussetzung, dafs $m+1$ eine positive Zahl sey, ist

$$= \frac{1}{m+1} - \frac{n}{(m+2)^2} + \frac{n^2}{(m+3)^3} - \frac{n^3}{(m+4)^4} + \frac{n^4}{(m+5)^5} - \text{etc.}$$

Taf. XXXII. $\int e^{ax}\partial x \,\mathrm{Sin}^n x, \quad \int e^{ax}\partial x \,\mathrm{Cos}^n x$

Reductionsformeln.

$$\int e^{ax}\partial x\,\mathrm{Sin}^n x = \frac{e^{ax}\mathrm{Sin}^{n-1}x(a\,\mathrm{Sin}\,x - n\,\mathrm{Cos}\,x)}{a^2+n^2} + \frac{n(n-1)}{a^2+n^2}\int e^{ax}\partial x\,\mathrm{Sin}^{n-2}x$$

$$\int e^{ax}\partial x\,\mathrm{Cos}^n x = \frac{e^{ax}\mathrm{Cos}^{n-1}x(a\,\mathrm{Cos}\,x + n\,\mathrm{Sin}\,x)}{a^2+n^2} + \frac{n(n-1)}{a^2+n^2}\int e^{ax}\partial x\,\mathrm{Cos}^{n-2}x$$

Einzelne Formeln.

$$\int e^{ax}\partial x\,\mathrm{Sin}\,x = \frac{e^{ax}(a\,\mathrm{Sin}\,x - \mathrm{Cos}\,x)}{a^2+1}$$

$$\int e^{ax}\partial x\,\mathrm{Sin}^2 x = \frac{e^{ax}\mathrm{Sin}\,x(a\,\mathrm{Sin}\,x - 2\mathrm{Cos}\,x)}{a^2+4} + \frac{1\cdot 2}{a(a^2+4)}e^{ax}$$

$$\int e^{ax}\partial x\,\mathrm{Sin}^3 x = \frac{e^{ax}\mathrm{Sin}^2 x(a\,\mathrm{Sin}\,x - 3\mathrm{Cos}\,x)}{a^2+9} + \frac{2\cdot 3\,e^{ax}(a\,\mathrm{Sin}\,x - \mathrm{Cos}\,x)}{(a^2+1)(a^2+9)}$$

. . . .

$$\int e^{ax}\partial x\,\mathrm{Cos}\,x = \frac{e^{ax}(a\,\mathrm{Cos}\,x + \mathrm{Sin}\,x)}{a^2+1}$$

$$\int e^{ax}\partial x\,\mathrm{Cos}^2 x = \frac{e^{ax}\mathrm{Cos}\,x(a\,\mathrm{Cos}\,x + 2\mathrm{Sin}\,x)}{a^2+4} + \frac{1\cdot 2}{a(a^2+4)}e^{ax}$$

$$\int e^{ax}\partial x\,\mathrm{Cos}^3 x = \frac{e^{ax}\mathrm{Cos}^2 x(a\,\mathrm{Cos}\,x + 3\mathrm{Sin}\,x)}{a^2+9} + \frac{2\cdot 3\,e^{ax}(a\,\mathrm{Cos}\,x + \mathrm{Sin}\,x)}{(a^2+1)(a^2+9)}$$

. . . .

$$\int e^{ax}\partial x\,\mathrm{Sin}\,kx = \frac{e^{ax}(a\,\mathrm{Sin}\,kx - k\,\mathrm{Cos}\,kx)}{a^2+k^2}$$

$$\int e^{ax}\partial x\,\mathrm{Cos}\,kx = \frac{e^{ax}(a\,\mathrm{Cos}\,kx + k\,\mathrm{Sin}\,kx)}{a^2+k^2}$$

Mit Hülfe der beiden letzten Formeln kann auch das Integral $\int e^{ax}\partial x\,\mathrm{Sin}^m x\,\mathrm{Cos}^n x$ gefunden werden, wenn nämlich $\mathrm{Sin}^m x\,\mathrm{Cos}^n x$ nach Sinus und Cosinus der vielfachen Winkel entwickelt wird, wodurch man lauter Monomen von der Form $e^{ax}\partial x\,\mathrm{Sin}\,kx$, $e^{ax}\partial x\,\mathrm{Cos}\,kx$, $e^{ax}\partial x$, erhält.

$$\int \frac{(f+g\,\mathrm{Cos}\,\varphi)\,\partial\varphi}{(a+b\,\mathrm{Cos}\,\varphi)^n}$$ *Taf.* XXXIII.

Reductionsformel.

$$\int \frac{(f+g\,\mathrm{Cos}\,\varphi)\,\partial\varphi}{(a+b\,\mathrm{Cos}\,\varphi)^n} = \frac{(ag-bf)\,\mathrm{Sin}\,\varphi}{(n-1)(a^2-b^2)(a+b\,\mathrm{Cos}\,\varphi)^{n-1}}$$
$$+\frac{1}{(n-1)(a^2-b^2)}\int \frac{[(n-1)(af-bg)+(n-2)(ag-bf)\,\mathrm{Cos}\,\varphi]\,\partial\varphi}{(a+b\,\mathrm{Cos}\,\varphi)^{n-1}}$$

Einzelne Fälle.

$$\int \frac{\partial\varphi}{a+b\,\mathrm{Cos}\,\varphi} = \frac{2}{\sqrt{(a^2-b^2)}}\,\mathrm{Arc\,Tang}\,\frac{(a-b)\,\mathrm{Tang}\,\frac{1}{2}\varphi}{\sqrt{(a^2-b^2)}}$$
$$= \frac{1}{\sqrt{(a^2-b^2)}}\,\mathrm{Arc\,Tang}\,\frac{\mathrm{Sin}\,\varphi\,\sqrt{(a^2-b^2)}}{b+a\,\mathrm{Cos}\,\varphi}$$
$$= \frac{1}{\sqrt{(a^2-b^2)}}\,\mathrm{Arc\,Sin}\,\frac{\mathrm{Sin}\,\varphi\,\sqrt{(a^2-b^2)}}{a+b\,\mathrm{Cos}\,\varphi}$$
$$= \frac{1}{\sqrt{(a^2-b^2)}}\,\mathrm{Arc\,Cos}\,\frac{b+a\,\mathrm{Cos}\,\varphi}{a+b\,\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi}{a+b\,\mathrm{Cos}\,\varphi} = \frac{1}{\sqrt{(b^2-a^2)}}\,\log\,\frac{b+a\,\mathrm{Cos}\,\varphi+\mathrm{Sin}\,\varphi\,\sqrt{(b^2-a^2)}}{a+b\,\mathrm{Cos}\,\varphi}$$

Der erste von diesen beiden Werthen für $b<a$, der zweite, für $b>a$; für $b=a$ ist

$$\int \frac{\partial\varphi}{a+a\,\mathrm{Cos}\,\varphi} = \frac{1}{a}\int \frac{\partial\varphi}{1+\mathrm{Cos}\,\varphi} = \frac{1}{a}\,\mathrm{Tang}\,\frac{1}{2}\varphi$$

$$\int \frac{\partial\varphi\,\mathrm{Sin}\,\varphi}{a+b\,\mathrm{Cos}\,\varphi} = -\frac{1}{b}\log(a+b\,\mathrm{Cos}\,\varphi)$$

$$\int \frac{\partial\varphi\,\mathrm{Cos}\,\varphi}{a+b\,\mathrm{Cos}\,\varphi} = \frac{\varphi}{b} - \frac{a}{b}\int \frac{\partial\varphi}{a+b\,\mathrm{Cos}\,\varphi}$$

$$\int \frac{\partial\varphi}{(a+b\,\mathrm{Cos}\,\varphi)^2} = \frac{1}{a^2-b^2}\left(\frac{-b\,\mathrm{Sin}\,\varphi}{a+b\,\mathrm{Cos}\,\varphi} + a\int \frac{\partial\varphi}{a+b\,\mathrm{Cos}\,\varphi}\right)$$

$$\int \frac{\partial\varphi\,\mathrm{Cos}\,\varphi}{(a+b\,\mathrm{Cos}\,\varphi)^2} = \frac{1}{a^2-b^2}\left(\frac{a\,\mathrm{Sin}\,\varphi}{a+b\,\mathrm{Cos}\,\varphi} - b\int \frac{\partial\varphi}{a+b\,\mathrm{Cos}\,\varphi}\right)$$

Taf. XXXIV.

Entwickelung des Integrals

$$\int \partial\varphi (1 + n \operatorname{Cos}\varphi)^p$$

nach vielfachen Winkeln. *)

I. p *positiv.* $p = + m$

Allgemeine Formel.

$$\int \partial\varphi (1 + n \operatorname{Cos}\varphi)^m = A\varphi + B \operatorname{Sin}\varphi + \tfrac{1}{2} C \operatorname{Sin} 2\varphi + \tfrac{1}{3} D \operatorname{Sin} 3\varphi$$
$$+ \tfrac{1}{4} E \operatorname{Sin} 4\varphi + \tfrac{1}{5} F \operatorname{Sin} 5\varphi + \text{etc.}$$

$$A = 1 + \frac{1}{2}\,{}^{m}\mathfrak{B} n^2 + \frac{1 \cdot 3}{2 \cdot 4}\,{}^{m}\mathfrak{D} n^4 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\,{}^{m}\mathfrak{F} n^6$$
$$+ \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\,{}^{m}\mathfrak{H} n^8 + \text{etc.}$$

$$B = 2n\Big(\frac{1}{2}\,{}^{m}\mathfrak{A} + \frac{1 \cdot 3}{2 \cdot 4}\,{}^{m}\mathfrak{C} n^2 + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\,{}^{m}\mathfrak{E} n^4$$
$$+ \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\,{}^{m}\mathfrak{G} n^6 + \text{etc.}\Big)$$

$$C = \frac{2mnA - 2B}{(m+2)n}, \quad D = \frac{(m-1)nB - 4C}{(m+3)n}$$

$$E = \frac{(m-2)nC - 6D}{(m+4)n}, \quad F = \frac{(m-3)nD - 8E}{(m+5)n}$$

$$G = \frac{(m-4)nE - 10F}{(m+6)n}, \quad H = \frac{(m-5)nF - 12G}{(m+7)n}$$

etc.

[Die Reihen für A und B brechen ab, wenn m eine ganze Zahl ist.]

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*) Das Integral $\int \partial\varphi (a + b \operatorname{Cos}\varphi)^p$ läſst sich auf dieses zurückführen, wenn $\frac{b}{a} = n$ gesetzt wird: denn es ist

$$\int \partial\varphi (a + b \operatorname{Cos}\varphi)^p = a^p \int \partial\varphi (1 + n \operatorname{Cos}\varphi)^p$$

Einzelne Fälle.

Für $m=1$ ist

$$A=1,\ B=n,\ (C,\ D,\ E,\ \text{etc.}=0).$$

Für $m=2$ ist

$$A=1+\tfrac{1}{2}n^2,\ B=2n,\ C=\tfrac{1}{2}n^2,\ (D,\ E,\ \text{etc.}=0).$$

Für $m=3$ ist

$$A=1+\tfrac{5}{2}n^2,\ B=3n+\tfrac{5}{4}n^3,\ C=\tfrac{5}{2}n^2,$$

$$D=\tfrac{1}{4}n^3,\ (E,\ F,\ \text{etc.}=0).$$

Für $m=4$ ist

$$A=1+3n^2+\tfrac{5}{8}n^4,\ B=4n+3n^3,\ C=3n^2+\tfrac{1}{2}n^4,$$

$$D=n^3,\ E=\tfrac{1}{8}n^4,\ (F,\ G,\ \text{etc.}=0).$$

Für $m=\tfrac{1}{2}$ ist

$$A=1-\frac{1\cdot 1}{4\cdot 4}n^2-\frac{1\cdot 1\cdot 3\cdot 5}{4\cdot 4\cdot 8\cdot 8}n^4-\frac{1\cdot 1\cdot 3\cdot 5\cdot 7\cdot 9}{4\cdot 4\cdot 8\cdot 8\cdot 12\cdot 12}n^6-\text{etc.}$$

$$B=\frac{1}{2}n+\frac{1\cdot 1\cdot 3}{2\cdot 4\cdot 8}n^3+\frac{1\cdot 1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 8\cdot 8\cdot 12}n^5+\text{etc.}$$

$$C=\frac{2nA-4B}{5n},\quad D=\frac{-nB-8C}{7n},\ \text{etc.}$$

II. *p negativ.* $p=-m$

Reductionsformel.

Es werde gesetzt

$$\int\partial\varphi(1+n\operatorname{Cos}\varphi)^{-m}=A\varphi+B\operatorname{Sin}\varphi+\tfrac{1}{2}C\operatorname{Sin}2\varphi+\tfrac{1}{3}D\operatorname{Sin}3\varphi$$
$$+\tfrac{1}{4}E\operatorname{Sin}4\varphi+\tfrac{1}{5}F\operatorname{Sin}5\varphi+\text{etc.}$$

$$\int\partial\varphi(1+n\operatorname{Cos}\varphi)^{-m-1}=A'\varphi+B'\operatorname{Sin}\varphi+\tfrac{1}{2}C'\operatorname{Sin}2\varphi+\tfrac{1}{3}D'\operatorname{Sin}3\varphi$$
$$+\tfrac{1}{4}E'\operatorname{Sin}4\varphi+\tfrac{1}{5}F'\operatorname{Sin}5\varphi+\text{etc.}$$

so ist

$$A' = \frac{2mA-(m-1)nB}{2m(1-n^2)} = A + \frac{n\partial A}{m\partial n}$$

$$B' = \frac{2(A-A')}{n} = B + \frac{n\partial B}{m\partial n}$$

$$C' = \frac{2(B-B')-2nA'}{n} = C + \frac{n\partial C}{m\partial n}$$

$$D' = \frac{2(C-C')-nB'}{n} = D + \frac{n\partial D}{m\partial n}$$

$$E' = \frac{2(D-D')-nC'}{n} = E + \frac{n\partial E}{m\partial n}$$

etc. etc.

Mit Hülfe dieser doppelten Reductionsformeln, welche einander wechselseitig zur Prüfung dienen können, lassen sich die Coefficienten A, B, C, D, etc., für die Werthe $p = -2$, -3, -4, etc., aus den Werthen derselben für $p = -1$ bestimmen. Für $p = -1$ ist aber

$$A = \frac{1}{\sqrt{(1-n^2)}}$$

$$B = \frac{2-2\sqrt{(1-n^2)}}{n\sqrt{(1-n^2)}}$$

$$C = \frac{4-2n^2-4\sqrt{(1-n^2)}}{n^2\sqrt{(1-n^2)}}$$

$$D = \frac{8-6n^2-2(4-n^2)\sqrt{(1-n^2)}}{n^3\sqrt{(1-n^2)}}$$

$$E = \frac{16-16n^2+2n^4-2(8-4n^2)\sqrt{(1-n^2)}}{n^4\sqrt{(1-n^2)}}$$

. . . .

$$\overset{\mu}{A} = \frac{2}{\sqrt{(1-n^2)}}\left(\frac{1-\sqrt{(1-n^2)}}{n}\right)^{\mu}$$

Die Werthe von A, B, C, D, etc., für $p = -\frac{1}{2}$, $-\frac{3}{2}$, $-\frac{5}{2}$, etc., lassen sich durch die nämlichen Reductionsformeln aus den Werthen derselben für $p = \frac{1}{2}$ (S. 299) herleiten; sie lassen sich aber nicht wohl anders als durch Reihen ausdrücken.

Mit der Auflösung der Winkelfunction $(1+n\,\mathrm{Cos}\,\varphi)^p$ in eine Reihe von der Form $A+B\,\mathrm{Cos}\,\varphi+C\,\mathrm{Cos}\,2\varphi+D\,\mathrm{Cos}\,3\varphi+$ etc. worauf es bei der Integration des Differentials $\partial\varphi(1+n\,\mathrm{Cos}\,\varphi)^p$ einzig und allein ankömmt, haben sich die Analysten vielfältig beschäftigt. Sie hat vorzüglich in der Astronomie ihren Nutzen, wo sie in der Form $(r^2+r'^2-rr'\,\mathrm{Cos}\,\varphi)^p$, oder in dieser etwas einfacheren $(1+\alpha^2-\alpha\,\mathrm{Cos}\,\varphi)^p$ vorkommt. Die ausführlichste Belehrung darüber findet man in Eulers *Instit. calc. integr.* und in dem *Traité du calc. diff. et integr.* von Lacroix. Laplace giebt im zweiten Buche seiner *Mécanique celeste* die folgenden Reductionsformeln.

Es sey nach seiner Bezeichnung

$$(1+\alpha^2-\alpha\,\mathrm{Cos}\,\theta)^{-s}=\tfrac{1}{2}b_s^{(0)}+b_s^{(1)}\mathrm{Cos}\,\theta+b_s^{(2)}\mathrm{Cos}\,2\theta+\text{etc.}$$

$$(1+\alpha^2-\alpha\,\mathrm{Cos}\,\theta)^{-s-1}=\tfrac{1}{2}b_{s+1}^{(0)}+b_{s+1}^{(1)}\mathrm{Cos}\,\theta+b_{s+1}^{(2)}\mathrm{Cos}\,2\theta+\text{etc.}$$

so ist

$$b_s^{(i)}=\frac{(i-1)(1+\alpha^2)b_s^{(i-1)}-(i+s-2)\alpha b_s^{(i-2)}}{(i-s)\alpha}$$

$$b_{s+1}^{(i)}=\frac{(s+i)(1+\alpha^2)b_s^{(i)}-2(i-s+1)\alpha b_s^{(i+1)}}{s(1-\alpha^2)^2}$$

$$b_{s+1}^{(i)}=\frac{(s-i)(1+\alpha^2)b_s^{(i)}+2(i+s-1)\alpha b_s^{(i-1)}}{s(1-\alpha^2)^2}$$

Für die Werthe von $b_s^{(0)}$, $b_s^{(1)}$, giebt er folgende Reihen:

$$b_s^{(0)}=2\left[1+s^2\cdot\alpha^2+\left(\frac{s(s+1)}{1\cdot2}\right)^2\cdot\alpha^4+\left(\frac{s(s+1)(s+2)}{1\cdot2\cdot3}\right)^2\cdot\alpha^6+\text{etc.}\right]$$

$$b_s^{(1)}=2\alpha\left[s+s\cdot\frac{s(s+1)}{1\cdot2}\alpha^2+\frac{s(s+1)}{1\cdot2}\cdot\frac{s(s+1)(s+2)}{1\cdot2\cdot3}\alpha^4+\text{etc.}\right]$$

Hieraus erhält man, wenn $s=-\frac{1}{2}$ gesetzt wird, mithin für $p=\frac{1}{2}$ folgende Reihen:

$$\frac{1}{2}b^{(0)}_{-\frac{1}{2}} = 1 + \left(\frac{1}{2}\right)^2 \alpha^2 + \left(\frac{1\cdot 1}{2\cdot 4}\right)^2 \alpha^4 + \left(\frac{1\cdot 1\cdot 3}{2\cdot 4\cdot 6}\right)^2 \alpha^6 + \text{etc.}$$

$$b^{(1)}_{-\frac{1}{2}} = -\alpha\left(1 - \frac{1\cdot 1}{2\cdot 4}\alpha^2 - \frac{1}{4}\cdot\frac{1\cdot 1\cdot 3}{2\cdot 4\cdot 6}\alpha^4 - \frac{1\cdot 3}{4\cdot 6}\cdot\frac{1\cdot 1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}\alpha^6 - \text{etc.}\right)$$

Diese Reihen convergiren sehr schnell, wenn α ein etwas kleiner Bruch ist. Mit Hülfe derselben und der angegebenen Reductionsformeln lassen sich nun die Werthe von $b^{(2)}_{-\frac{1}{2}}$, $b^{(3)}_{-\frac{1}{2}}$, etc., $b^{(2)}_{-\frac{3}{2}}$, $b^{(3)}_{-\frac{3}{2}}$, etc., wie auch ihre Differentiale in Beziehung auf α, wenn es, wie in dem angeführten Werke erfordert wird, sehr leicht finden.

Berlin, gedruckt bei Aug. Wilh. Petsch.

Druckfehler und Verbesserungen.

Seite 20 Zeile 4 v. u. statt $\int x^{m-1} dx X^{p-i}$ lies $\int x^{m-1} \partial x X^{p-i}$

— 21 — 6 v. u. statt $\int x^{m-1} dx X^{p}$ lies $\int x^{m-1} \partial x X^{p}$

* — 47 — 3 v. o. statt $\frac{1}{\sqrt{ab}}$ l. $\frac{1}{a\sqrt{\frac{b}{a}}}$ und statt $\frac{1}{2\sqrt{-ab}}$ l. $\frac{1}{2a\sqrt{-\frac{b}{a}}}$

— 62 — 4 v. u. statt $\left(\frac{b^2}{c^2} - \frac{2ab}{c^2}\right)$ l. $\left(\frac{b^3}{c^3} - \frac{2ab}{c^2}\right)$

* — 77 — 5 v. o. statt $-\frac{1}{2\sqrt{ab}} \text{Arc Tang} \frac{\sqrt{a}}{x^2\sqrt{b}}$ l. $\frac{1}{2bk^2} \text{Arc Tang}\, x^2\sqrt{\frac{b}{a}}$

* — 83 — 6 v. o. statt $\frac{1}{3\sqrt{ab}}$ l. $\frac{1}{3bk^3}$

— 86 — 10 v. u. statt $\frac{1}{2ch}\Big[$ l. $\frac{1}{2h}\Big[$

— 93 — 6 v. o. statt bcx^6 l. bcx^7

— 122 folgen in den Formeln für $\int \frac{\partial x}{X^{\frac{5}{2}}}$, $\int \frac{x\partial x}{X^{\frac{5}{2}}}$, ... $\int \frac{x^9 \partial x}{X^{\frac{5}{2}}}$ die Potenzen von b in der Ordnung $b, b^2, b^3, b^3, b^4, b^5, b^6, b^7, b^8, b^9$; sie sollten aber so folgen: $b, b^2, b^3, b^4, b^5, b^6, b^7, b^8, b^9, b^{10}$.

— 127 — 8 v. u. statt $\int \partial^6 \partial x \sqrt{X}$ l. $\int x^6 \partial x \sqrt{X}$

— 131 — 2 v. u. statt $-\frac{9}{20} a X^8$ l. $-\frac{9}{23} a X^8$

— 175 — 1 v. o. statt $\int x^m \partial x (a + bx^2)^{\frac{3}{2}}$ l. $\int x^m \partial x (ax + bx^2)^{\frac{3}{2}}$

— 183 — 10 v. o. statt $2\sqrt{c} \,.\, \sqrt{X^2}$ l. $2\sqrt{c} \,.\, \sqrt{X}$

— 231 — 4 v. o. statt $A = \frac{1}{2m+1}$ l. $A = \frac{1}{(2m+1)b}$

— 250 — 2 v. u. hat sich das Integral $\int \frac{x^{m-1}\partial x}{1+x^n}$ eingeschlichen; es erhält aber dasselbe den dort angegebenen Werth nicht zwischen den Grenzen 0 und 1, sondern zwischen den Grenzen 0 und ∞.

— 256 — 2. 8. 9 v. o. ist ∂y ausgelassen.

*) Die mit Sternchen bezeichneten fehlerhaften Ausdrücke sind zwar den verbesserten an sich gleich, aber in der zweiten Form sind sie der Vorzeichen wegen erst allgemein gültig.